cuSOLVER Library

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Chapter 1. Introduction

The cuSolver library is a high-level package based on the cuBLAS and cuSPARSE libraries. It consists of two modules corresponding to two sets of API:

1. The cuSolver API on a single GPU
2. The cuSolverMG API on a single node multiGPU

Each of these can be used independently or in concert with other toolkit libraries. To simplify the notation, cuSolver denotes single GPU API and cuSolverMg denotes multiGPU API.

The intent of cuSolver is to provide useful LAPACK-like features, such as common matrix factorization and triangular solve routines for dense matrices, a sparse least-squares solver and an eigenvalue solver. In addition cuSolver provides a new refactorization library useful for solving sequences of matrices with a shared sparsity pattern.

cuSolver combines three separate components under a single umbrella. The first part of cuSolver is called cuSolverDN, and deals with dense matrix factorization and solve routines such as LU, QR, SVD and LDLT, as well as useful utilities such as matrix and vector permutations.

Next, cuSolverSP provides a new set of sparse routines based on a sparse QR factorization. Not all matrices have a good sparsity pattern for parallelism in factorization, so the cuSolverSP library also provides a CPU path to handle those sequential-like matrices. For those matrices with abundant parallelism, the GPU path will deliver higher performance. The library is designed to be called from C and C++.

The final part is cuSolverRF, a sparse re-factorization package that can provide very good performance when solving a sequence of matrices where only the coefficients are changed but the sparsity pattern remains the same.

The GPU path of the cuSolver library assumes data is already in the device memory. It is the responsibility of the developer to allocate memory and to copy data between GPU memory and CPU memory using standard CUDA runtime API routines, such as `cudaMalloc()`, `cudaFree()`, `cudaMemcpy()`, and `cudaMemcpyAsync()`.

cuSolverMg is GPU-accelerated ScaLAPACK. By now, cuSolverMg supports 1-D column block cyclic layout and provides symmetric eigenvalue solver.

Note: The cuSolver library requires hardware with a CUDA compute capability (CC) of at least 2.0 or higher. Please see the *CUDA C++ Programming Guide* for a list of the Compute Capabilities corresponding to all NVIDIA GPUs.

1.1. cuSolverDN: Dense LAPACK

The cuSolverDN library was designed to solve dense linear systems of the form

$$Ax = b$$

where the coefficient matrix $A \in R^{nxn}$, right-hand-side vector $b \in R^n$ and solution vector $x \in R^n$

The cuSolverDN library provides QR factorization and LU with partial pivoting to handle a general matrix `A`, which may be non-symmetric. Cholesky factorization is also provided for symmetric/Hermitian matrices. For symmetric indefinite matrices, we provide Bunch-Kaufman (LDL) factorization.

The cuSolverDN library also provides a helpful bidiagonalization routine and singular value decomposition (SVD).

The cuSolverDN library targets computationally-intensive and popular routines in LAPACK, and provides an API compatible with LAPACK. The user can accelerate these time-consuming routines with cuSolverDN and keep others in LAPACK without a major change to existing code.

1.2. cuSolverSP: Sparse LAPACK

The cuSolverSP library was mainly designed to a solve sparse linear system

$$Ax = b$$

and the least-squares problem

$$x = \arg\min ||Az - b||$$

where sparse matrix $A \in R^{mxn}$, right-hand-side vector $b \in R^m$ and solution vector $x \in R^n$. For a linear system, we require `m=n`.

The core algorithm is based on sparse QR factorization. The matrix `A` is accepted in CSR format. If matrix `A` is symmetric/Hermitian, the user has to provide a full matrix, ie fill missing lower or upper part.

If matrix `A` is symmetric positive definite and the user only needs to solve $Ax = b$, Cholesky factorization can work and the user only needs to provide the lower triangular part of `A`.

On top of the linear and least-squares solvers, the `cuSolverSP` library provides a simple eigenvalue solver based on shift-inverse power method, and a function to count the number of eigenvalues contained in a box in the complex plane.

1.3. cuSolverRF: Refactorization

The cuSolverRF library was designed to accelerate solution of sets of linear systems by fast re-factorization when given new coefficients in the same sparsity pattern

$$A_i x_i = f_i$$

where a sequence of coefficient matrices $A_i \in R^{nxn}$, right-hand-sides $f_i \in R^n$ and solutions $x_i \in R^n$ are given for `i=1,...,k`.

The cuSolverRF library is applicable when the sparsity pattern of the coefficient matrices A_i as well as the reordering to minimize fill-in and the pivoting used during the LU factorization remain the same across these linear systems. In that case, the first linear system (`i=1`) requires a full LU factorization, while the subsequent linear systems (`i=2,...,k`) require only the LU re-factorization. The later can be performed using the cuSolverRF library.

Notice that because the sparsity pattern of the coefficient matrices, the reordering and pivoting remain the same, the sparsity pattern of the resulting triangular factors L_i and U_i also remains the same. Therefore, the real difference between the full LU factorization and LU re-factorization is that the required memory is known ahead of time.

1.4. Naming Conventions

The cuSolverDN library provides two different APIs; `legacy` and `generic`.

The functions in the legacy API are available for data types `float`, `double`, `cuComplex`, and `cuDoubleComplex`. The naming convention for the legacy API is as follows:

```
cusolverDn<t><operation>
```

where `<t>` can be `S`, `D`, `C`, `Z`, or `X`, corresponding to the data types `float`, `double`, `cuComplex`, `cuDoubleComplex`, and the generic type, respectively. `<operation>` can be Cholesky factorization (`potrf`), LU with partial pivoting (`getrf`), QR factorization (`geqrf`) and Bunch-Kaufman factorization (`sytrf`).

The functions in the generic API provide a single entry point for each routine and support for 64-bit integers to define matrix and vector dimensions. The naming convention for the generic API is data-agnostic and is as follows:

```
cusolverDn<operation>
```

where `<operation>` can be Cholesky factorization (`potrf`), LU with partial pivoting (`getrf`) and QR factorization (`geqrf`).

The cuSolverSP library functions are available for data types `float`, `double`, `cuComplex`, and `cuDoubleComplex`. The naming convention is as follows:

```
cusolverSp[Host]<t>[<matrix data
format>]<operation>[<output matrix data format>]<based on>
```

where `cuSolverSp` is the GPU path and `cusolverSpHost` is the corresponding CPU path. `<t>` can be `S`, `D`, `C`, `Z`, or `X`, corresponding to the data types `float`, `double`, `cuComplex`, `cuDoubleComplex`, and the generic type, respectively.

The `<matrix data format>` is `csr`, compressed sparse row format.

The `<operation>` can be `ls`, `lsq`, `eig`, `eigs`, corresponding to linear solver, least-square solver, eigenvalue solver and number of eigenvalues in a box, respectively.

The <output matrix data format> can be v or m, corresponding to a vector or a matrix.

<based on> describes which algorithm is used. For example, qr (sparse QR factorization) is used in linear solver and least-square solver.

All of the functions have the return type cusolverStatus_t and are explained in more detail in the chapters that follow.

Table 1. cuSolverSP API

Routine	Data format	Operation	Output format	Based on
csrlsvlu	csr	linear solver (ls)	vector (v)	LU (lu) with partial pivoting
csrlsvqr	csr	linear solver (ls)	vector (v)	QR factorization (qr)
csrlsvchol	csr	linear solver (ls)	vector (v)	Cholesky factorization (chol)
csrlsqvqr	csr	least-square solver (lsq)	vector (v)	QR factorization (qr)
csreigvsi	csr	eigenvalue solver (eig)	vector (v)	shift-inverse
csreigs	csr	number of eigenvalues in a box (eigs)		
csrsymrcm	csr	Symmetric Reverse Cuthill-McKee (symrcm)		

The cuSolverRF library routines are available for data type double. Most of the routines follow the naming convention:

cusolverRf_<operation>_[[Host]](...)

where the trailing optional Host qualifier indicates the data is accessed on the host versus on the device, which is the default. The <operation> can be Setup, Analyze, Refactor, Solve, ResetValues, AccessBundledFactors and ExtractSplitFactors.

Finally, the return type of the cuSolverRF library routines is cusolverStatus_t.

1.5. Asynchronous Execution

The cuSolver library functions prefer to keep asynchronous execution as much as possible. Developers can always use the cudaDeviceSynchronize() function to ensure that the execution of a particular cuSolver library routine has completed.

A developer can also use the `cudaMemcpy()` routine to copy data from the device to the host and vice versa, using the `cudaMemcpyDeviceToHost` and `cudaMemcpyHostToDevice` parameters, respectively. In this case there is no need to add a call to `cudaDeviceSynchronize()` because the call to `cudaMemcpy()` with the above parameters is blocking and completes only when the results are ready on the host.

1.6. Library Property

The `libraryPropertyType` data type is an enumeration of library property types. (ie. CUDA version X.Y.Z would yield `MAJOR_VERSION=X`, `MINOR_VERSION=Y`, `PATCH_LEVEL=Z`)

```
typedef enum libraryPropertyType_t
{
        MAJOR_VERSION,
        MINOR_VERSION,
        PATCH_LEVEL
} libraryPropertyType;
```

The following code can show the version of cusolver library.

```
    int major=-1,minor=-1,patch=-1;
    cusolverGetProperty(MAJOR_VERSION, &major);
    cusolverGetProperty(MINOR_VERSION, &minor);
    cusolverGetProperty(PATCH_LEVEL, &patch);
    printf("CUSOLVER Version (Major,Minor,PatchLevel): %d.%d.%d\n",
 major,minor,patch);
```

1.7. High Precision Package

The `cusolver` library uses high precision for iterative refinement when necessary.

Chapter 2. Using the CUSOLVER API

2.1. General Description

This chapter describes how to use the cuSolver library API. It is not a reference for the cuSolver API data types and functions; that is provided in subsequent chapters.

2.1.1. Thread Safety

The library is thread-safe, and its functions can be called from multiple host threads.

2.1.2. Scalar Parameters

In the cuSolver API, the scalar parameters can be passed by reference on the host.

2.1.3. Parallelism with Streams

If the application performs several small independent computations, or if it makes data transfers in parallel with the computation, then CUDA streams can be used to overlap these tasks.

The application can conceptually associate a stream with each task. To achieve the overlap of computation between the tasks, the developer should:

1. Create CUDA streams using the function `cudaStreamCreate()`, and
2. Set the stream to be used by each individual cuSolver library routine by calling, for example, `cusolverDnSetStream()`, just prior to calling the actual cuSolverDN routine.

The computations performed in separate streams would then be overlapped automatically on the GPU, when possible. This approach is especially useful when the computation performed by a single task is relatively small, and is not enough to fill the GPU with work, or when there is a data transfer that can be performed in parallel with the computation.

2.1.4. How to Link cusolver Library

`cusolver` library provides dynamic library `libcusolver.so` and static library `libcusolver_static.a`. If the user links the application with `libcusolver.so`, `libcublas.so` and `libcublasLt.so` are also required. If the user links the application with

libcusolver_static.a, the following libraries are also needed, libcudart_static.a, libculibos.a libcusolver_lapack_static.a, libmetis_static.a, libcublas_static.a and libcusparse_static.a.

Note: The libmetis_static.a library is deprecated and will be removed in the next major release. Use the libcusolver_metis_static.a instead.

2.1.5. Link Third-party LAPACK Library

Starting with CUDA 10.1 update 2, NVIDIA LAPACK library libcusolver_lapack_static.a is a subset of LAPACK and only contains GPU accelerated stedc and bdsqr. The user has to link libcusolver_static.a with libcusolver_lapack_static.a in order to build the application successfully. Prior to CUDA 10.1 update 2, the user can replace libcusolver_lapack_static.a with a third-party LAPACK library, for example, MKL. In CUDA 10.1 update 2, the third-party LAPACK library no longer affects the behavior of cusolver library, neither functionality nor performance. Furthermore the user cannot use libcusolver_lapack_static.a as a standalone LAPACK library because it is only a subset of LAPACK.

Note: The libcusolver_lapack_static.a library, which is the binary of CLAPACK-3.2.1, is a new feature of CUDA 10.0.

- If you use libcusolver_static.a, then you must link with libcusolver_lapack_static.a explicitly, otherwise the linker will report missing symbols. No conflict of symbols between libcusolver_lapack_static.a and other third-party LAPACK library, you are free to link the latter to your application.
- The libcusolver_lapack_static.a is built inside libcusolver.so. Hence, if you use libcusolver.so, then you don't need to specify a LAPACK library. The libcusolver.so will not pick up any routines from the third-party LAPACK library even you link the application with it.

2.1.6. Convention of info

Each LAPACK routine returns an info which indicates the position of invalid parameter. If info = -i, then i-th parameter is invalid. To be consistent with base-1 in LAPACK, cusolver does not report invalid handle into info. Instead, cusolver returns CUSOLVER_STATUS_NOT_INITIALIZED for invalid handle.

2.1.7. Usage of _bufferSize

There is no cudaMalloc inside cuSolver library, the user must allocate the device workspace explicitly. The routine xyz_bufferSize is to query the size of workspace of the routine xyz, for example xyz = potrf. To make the API simple, xyz_bufferSize follows almost the same signature of xyz even it only depends on some parameters, for example, device pointer is not used to decide the size of workspace. In most cases, xyz_bufferSize is called in the beginning before actual device data (pointing by a device pointer) is prepared or before the

device pointer is allocated. In such case, the user can pass null pointer to `xyz_bufferSize` without breaking the functionality.

2.1.8. cuSolverDN Logging

cuSOLVERDn logging mechanism can be enabled by setting the following environment variables before launching the target application:

CUSOLVERDN_LOG_LEVEL=<level> - while level is one of the following levels:

- "0" - Off - logging is disabled (default)
- "1" - Error - only errors will be logged
- "2" - Trace - API calls that launch CUDA kernels will log their parameters and important information
- "3" - Hints - hints that can potentially improve the application's performance
- "4" - Info - provides general information about the library execution, may contain details about heuristic status
- "5" - API Trace - API calls will log their parameter and important information

CUSOLVERDN_LOG_MASK=<mask> - while mask is a combination of the following masks:

- "0" - Off
- "1" - Error
- "2" - Trace
- "4" - Hints
- "8" - Info
- "16" - API Trace

CUSOLVERDN_LOG_FILE=<file_name> - while file name is a path to a logging file. File name may contain %i, that will be replaced with the process id. E.g "<file_name>_%i.log".

If CUSOLVERDN_LOG_FILE is not defined, the log messages are printed to stdout.

Another option is to use the experimental cusolverDn logging API. See:

cusolverDnLoggerSetCallback(), cusolverDnLoggerSetFile(), cusolverDnLoggerOpenFile(), cusolverDnLoggerSetLevel(), cusolverDnLoggerSetMask(), cusolverDnLoggerForceDisable()

2.2. cuSolver Types Reference

2.2.1. cuSolverDN Types

The `float`, `double`, `cuComplex`, and `cuDoubleComplex` data types are supported. The first two are standard C data types, while the last two are exported from `cuComplex.h`. In addition, cuSolverDN uses some familiar types from cuBLAS.

2.2.1.1. cusolverDnHandle_t

This is a pointer type to an opaque cuSolverDN context, which the user must initialize by calling `cusolverDnCreate()` prior to calling any other library function. An un-initialized Handle object will lead to unexpected behavior, including crashes of cuSolverDN. The handle created and returned by `cusolverDnCreate()` must be passed to every cuSolverDN function.

2.2.1.2. cublasFillMode_t

The type indicates which part (lower or upper) of the dense matrix was filled and consequently should be used by the function.

Value	Meaning
`CUBLAS_FILL_MODE_LOWER`	The lower part of the matrix is filled.
`CUBLAS_FILL_MODE_UPPER`	The upper part of the matrix is filled.
`CUBLAS_FILL_MODE_FULL`	The full the matrix is filled.

Notice that BLAS implementations often use Fortran characters `'L'` or `'l'` (lower) and `'U'` or `'u'` (upper) to describe which part of the matrix is filled.

2.2.1.3. cublasOperation_t

The `cublasOperation_t` type indicates which operation needs to be performed with the dense matrix.

Value	Meaning
`CUBLAS_OP_N`	The non-transpose operation is selected.
`CUBLAS_OP_T`	The transpose operation is selected.
`CUBLAS_OP_C`	The conjugate transpose operation is selected.

Notice that BLAS implementations often use Fortran characters `'N'` or `'n'` (non-transpose), `'T'` or `'t'` (transpose) and `'C'` or `'c'` (conjugate transpose) to describe which operations needs to be performed with the dense matrix.

2.2.1.4. cusolverEigType_t

The `cusolverEigType_t` type indicates which type of eigenvalue the solver is.

Value	Meaning
CUSOLVER_EIG_TYPE_1	A*x = lambda*B*x
CUSOLVER_EIG_TYPE_2	A*B*x = lambda*x
CUSOLVER_EIG_TYPE_3	B*A*x = lambda*x

Notice that LAPACK implementations often use Fortran integer 1 (A*x = lambda*B*x), 2 (A*B*x = lambda*x), 3 (B*A*x = lambda*x) to indicate which type of eigenvalue the solver is.

2.2.1.5. cusolverEigMode_t

The cusolverEigMode_t type indicates whether or not eigenvectors are computed.

Value	Meaning
CUSOLVER_EIG_MODE_NOVECTOR	Only eigenvalues are computed.
CUSOLVER_EIG_MODE_VECTOR	Both eigenvalues and eigenvectors are computed.

Notice that LAPACK implementations often use Fortran character 'N' (only eigenvalues are computed), 'V' (both eigenvalues and eigenvectors are computed) to indicate whether or not eigenvectors are computed.

2.2.1.6. cusolverIRSRefinement_t

The cusolverIRSRefinement_t type indicates which solver type would be used for the specific cusolver function. Most of our experimentation shows that CUSOLVER_IRS_REFINE_GMRES is the best option.

More details about the refinement process can be found in Azzam Haidar, Stanimire Tomov, Jack Dongarra, and Nicholas J. Higham. 2018. Harnessing GPU tensor cores for fast FP16 arithmetic to speed up mixed-precision iterative refinement solvers. In Proceedings of the International Conference for High Performance Computing, Networking, Storage, and Analysis (SC '18). IEEE Press, Piscataway, NJ, USA, Article 47, 11 pages.

Value	Meaning
CUSOLVER_IRS_REFINE_NOT_SET	Solver is not set; this value is what is set when creating the params structure. IRS solver will return an error.
CUSOLVER_IRS_REFINE_NONE	No refinement solver, the IRS solver performs a factorisation followed by a solve without any refinement. For example if the IRS solver was cusolverDnIRSXgesv(), this is equivalent to a Xgesv routine without refinement and where the factorisation is carried out in the lowest precision. If for example the main precision was CUSOLVER_R_64F and the lowest was CUSOLVER_R_64F as well, then this is equivalent to a call to cusolverDnDgesv().
CUSOLVER_IRS_REFINE_CLASSICAL	Classical iterative refinement solver. Similar to the one used in LAPACK routines.

Value	Meaning
CUSOLVER_IRS_REFINE_GMRES	GMRES (Generalized Minimal Residual) based iterative refinement solver. In recent study, the GMRES method has drawn the scientific community attention for its ability to be used as refinement solver that outperforms the classical iterative refinement method. based on our experimentation, we recommend this setting.
CUSOLVER_IRS_REFINE_CLASSICAL_GMRES	Classical iterative refinement solver that uses the GMRES (Generalized Minimal Residual) internally to solve the correction equation at each iteration. We call the *classical refinement iteration* the outer iteration while the *GMRES* is called inner iteration. Note that if the tolerance of the inner GMRES is set very low, lets say to machine precision, then the outer *classical refinement iteration* will performs only one iteration and thus this option will behave like CUSOLVER_IRS_REFINE_GMRES.
CUSOLVER_IRS_REFINE_GMRES_GMRES	Similar to CUSOLVER_IRS_REFINE_CLASSICAL_GMRES which consists of classical refinement process that uses GMRES to solve the inner correction system; here it is a GMRES (Generalized Minimal Residual) based iterative refinement solver that uses another GMRES internally to solve the preconditioned system.

2.2.1.7. cusolverDnIRSParams_t

This is a pointer type to an opaque cusolverDnIRSParams_t structure, which holds parameters for the iterative refinement linear solvers such as cusolverDnXgesv(). Use corresponding helper functions described below to either Create/Destroy this structure or Set/Get solver parameters.

2.2.1.8. cusolverDnIRSInfos_t

This is a pointer type to an opaque cusolverDnIRSInfos_t structure, which holds information about the performed call to an iterative refinement linear solver (e.g., cusolverDnXgesv()). Use corresponding helper functions described below to either Create/Destroy this structure or retrieve solve information.

2.2.1.9. cusolverDnFunction_t

The cusolverDnFunction_t type indicates which routine needs to be configured by cusolverDnSetAdvOptions(). The value CUSOLVERDN_GETRF corresponds to the routine Getrf.

Value	Meaning
CUSOLVERDN_GETRF	Corresponds to Getrf.

2.2.1.10. cusolverAlgMode_t

The `cusolverAlgMode_t` type indicates which algorithm is selected by `cusolverDnSetAdvOptions()`. The set of algorithms supported for each routine is described in detail along with the routine's documentation.

The default algorithm is `CUSOLVER_ALG_0`. The user can also provide `NULL` to use the default algorithm.

2.2.1.11. cusolverStatus_t

This is the same as `cusolverStatus_t` in the sparse LAPACK section.

2.2.1.12. cusolverDnLoggerCallback_t

`cusolverDnLoggerCallback_t` is a callback function pointer type.

Parameters:

Parameter	Memory	Input / Output	Description
logLevel		Output	See cuSolverDN Logging.
functionName		Output	The name of the API that logged this message.
message		Output	The log message.

Use the below function to set the callback function:

cusolverDnLoggerSetCallback()

2.2.2. cuSolverSP Types

The `float`, `double`, `cuComplex`, and `cuDoubleComplex` data types are supported. The first two are standard C data types, while the last two are exported from `cuComplex.h`.

2.2.2.1. cusolverSpHandle_t

This is a pointer type to an opaque cuSolverSP context, which the user must initialize by calling `cusolverSpCreate()` prior to calling any other library function. An un-initialized Handle object will lead to unexpected behavior, including crashes of cuSolverSP. The handle created and returned by `cusolverSpCreate()` must be passed to every cuSolverSP function.

2.2.2.2. cusparseMatDescr_t

We have chosen to keep the same structure as exists in cuSparse to describe the shape and properties of a matrix. This enables calls to either cuSPARSE or cuSOLVER using the same matrix description.

```
typedef struct {
```

```
    cusparseMatrixType_t MatrixType;
    cusparseFillMode_t FillMode;
    cusparseDiagType_t DiagType;
    cusparseIndexBase_t IndexBase;
} cusparseMatDescr_t;
```

Please read documenation of the cuSPARSE Library to understand each field of `cusparseMatDescr_t`.

2.2.2.3. cusolverStatus_t

This is a status type returned by the library functions and it can have the following values.

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The cuSolver library was not initialized. This is usually caused by the lack of a prior call, an error in the CUDA Runtime API called by the cuSolver routine, or an error in the hardware setup. **To correct:** call `cusolverCreate()` prior to the function call; and check that the hardware, an appropriate version of the driver, and the cuSolver library are correctly installed.
`CUSOLVER_STATUS_ALLOC_FAILED`	Resource allocation failed inside the cuSolver library. This is usually caused by a `cudaMalloc()` failure. **To correct:** prior to the function call, deallocate previously allocated memory as much as possible.
`CUSOLVER_STATUS_INVALID_VALUE`	An unsupported value or parameter was passed to the function (a negative vector size, for example). **To correct:** ensure that all the parameters being passed have valid values.
`CUSOLVER_STATUS_ARCH_MISMATCH`	The function requires a feature absent from the device architecture; usually caused by the lack of support for atomic operations or double precision. **To correct:** compile and run the application on a device with compute capability 2.0 or above.
`CUSOLVER_STATUS_EXECUTION_FAILED`	The GPU program failed to execute. This is often caused by a launch failure of the kernel on the GPU, which can be caused by multiple reasons. **To correct:** check that the hardware, an appropriate version of the driver, and the cuSolver library are correctly installed.
`CUSOLVER_STATUS_INTERNAL_ERROR`	An internal cuSolver operation failed. This error is usually caused by a `cudaMemcpyAsync()` failure.

	To correct: check that the hardware, an appropriate version of the driver, and the cuSolver library are correctly installed. Also, check that the memory passed as a parameter to the routine is not being deallocated prior to the routine's completion.
`CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED`	The matrix type is not supported by this function. This is usually caused by passing an invalid matrix descriptor to the function. **To correct:** check that the fields in `descrA` were set correctly.

2.2.3. cuSolverRF Types

cuSolverRF only supports `double`.

2.2.3.1. cusolverRfHandle_t

The `cusolverRfHandle_t` is a pointer to an opaque data structure that contains the cuSolverRF library handle. The user must initialize the handle by calling `cusolverRfCreate()` prior to any other cuSolverRF library calls. The handle is passed to all other cuSolverRF library calls.

2.2.3.2. cusolverRfMatrixFormat_t

The `cusolverRfMatrixFormat_t` is an enum that indicates the input/output matrix format assumed by the `cusolverRfSetupDevice()`, `cusolverRfSetupHost()`, `cusolverRfResetValues()`, `cusolveRfExtractBundledFactorsHost()` and `cusolverRfExtractSplitFactorsHost()` routines.

Value	Meaning
`CUSOLVER_MATRIX_FORMAT_CSR`	Matrix format CSR is assumed. (default)
`CUSOLVER_MATRIX_FORMAT_CSC`	Matrix format CSC is assumed.

2.2.3.3. cusolverRfNumericBoostReport_t

The `cusolverRfNumericBoostReport_t` is an enum that indicates whether numeric boosting (of the pivot) was used during the `cusolverRfRefactor()` and `cusolverRfSolve()` routines. The numeric boosting is disabled by default.

Value	Meaning
`CUSOLVER_NUMERIC_BOOST_NOT_USED`	Numeric boosting not used. (default)
`CUSOLVER_NUMERIC_BOOST_USED`	Numeric boosting used.

2.2.3.4. cusolverRfResetValuesFastMode_t

The `cusolverRfResetValuesFastMode_t` is an enum that indicates the mode used for the `cusolverRfResetValues()` routine. The fast mode requires extra memory and is recommended only if very fast calls to `cusolverRfResetValues()` are needed.

Value	Meaning
CUSOLVER_RESET_VALUES_FAST_MODE_OFF	Fast mode disabled. (default)
CUSOLVER_RESET_VALUES_FAST_MODE_ON	Ffast mode enabled.

2.2.3.5. cusolverRfFactorization_t

The cusolverRfFactorization_t is an enum that indicates which (internal) algorithm is used for refactorization in the cusolverRfRefactor() routine.

Value	Meaning
CUSOLVER_FACTORIZATION_ALG0	Algorithm 0. (default)
CUSOLVER_FACTORIZATION_ALG1	Algorithm 1.
CUSOLVER_FACTORIZATION_ALG2	Algorithm 2. Domino-based scheme.

2.2.3.6. cusolverRfTriangularSolve_t

The cusolverRfTriangularSolve_t is an enum that indicates which (internal) algorithm is used for triangular solve in the cusolverRfSolve() routine.

Value	Meaning
CUSOLVER_TRIANGULAR_SOLVE_ALG1	Algorithm 1. (default)
CUSOLVER_TRIANGULAR_SOLVE_ALG2	Algorithm 2. Domino-based scheme.
CUSOLVER_TRIANGULAR_SOLVE_ALG3	Aalgorithm 3. Domino-based scheme.

2.2.3.7. cusolverRfUnitDiagonal_t

The cusolverRfUnitDiagonal_t is an enum that indicates whether and where the unit diagonal is stored in the input/output triangular factors in the cusolverRfSetupDevice(), cusolverRfSetupHost() and cusolverRfExtractSplitFactorsHost() routines.

Value	Meaning
CUSOLVER_UNIT_DIAGONAL_STORED_L	Unit diagonal is stored in lower triangular factor. (default)
CUSOLVER_UNIT_DIAGONAL_STORED_U	Unit diagonal is stored in upper triangular factor.
CUSOLVER_UNIT_DIAGONAL_ASSUMED_L	Unit diagonal is assumed in lower triangular factor.
CUSOLVER_UNIT_DIAGONAL_ASSUMED_U	Unit diagonal is assumed in upper triangular factor.

2.2.3.8. cusolverStatus_t

The cusolverStatus_t is an enum that indicates success or failure of the cuSolverRF library call. It is returned by all the cuSolver library routines, and it uses the same enumerated values as the sparse and dense Lapack routines.

2.3. cuSolver Formats Reference

2.3.1. Index Base Format

The CSR or CSC format requires either zero-based or one-based index for a sparse matrix `A`. The GLU library supports only zero-based indexing. Otherwise, both one-based and zero-based indexing are supported in cuSolver.

2.3.2. Vector (Dense) Format

The vectors are assumed to be stored linearly in memory. For example, the vector

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

is represented as

$$\begin{pmatrix} x_1 & x_2 & \cdots & x_n \end{pmatrix}$$

2.3.3. Matrix (Dense) Format

The dense matrices are assumed to be stored in column-major order in memory. The sub-matrix can be accessed using the leading dimension of the original matrix. For examle, the `m*n` (sub-)matrix

$$\begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ a_{2,1} & \cdots & a_{2,n} \\ \vdots & & \\ a_{m,1} & \cdots & a_{m,n} \end{pmatrix}$$

is represented as

$$\begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ a_{2,1} & \cdots & a_{2,n} \\ \vdots & \ddots & \vdots \\ a_{m,1} & \cdots & a_{m,n} \\ \vdots & \ddots & \vdots \\ a_{\mathrm{lda},1} & \cdots & a_{\mathrm{lda},n} \end{pmatrix}$$

with its elements arranged linearly in memory as

$$\begin{pmatrix} a_{1,1} & a_{2,1} & \cdots & a_{m,1} & \cdots & a_{\mathrm{lda},1} & \cdots & a_{1,n} & a_{2,n} & \cdots & a_{m,n} & \cdots & a_{\mathrm{lda},n} \end{pmatrix}$$

where `lda` $\geq$ `m` is the leading dimension of `A`.

2.3.4. Matrix (CSR) Format

In CSR format the matrix is represented by the following parameters:

Parameter	Type	Size	Meaning
`n`	`(int)`		The number of rows (and columns) in the matrix.
`nnz`	`(int)`		The number of non-zero elements in the matrix.
`csrRowPtr`	`(int *)`	`n+1`	The array of offsets corresponding to the start of each row in the arrays `csrColInd` and `csrVal`. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix.
`csrColInd`	`(int *)`	`nnz`	The array of column indices corresponding to the non-zero elements in the matrix. **It is assumed that this array is sorted by row and by column within each row.**
`csrVal`	`(S|D|C|Z)*`	`nnz`	The array of values corresponding to the non-zero elements in the matrix. **It is assumed that this array is sorted by row and by column within each row.**

Note that in our CSR format, sparse matrices are assumed to be stored in row-major order, in other words, the index arrays are first sorted by row indices and then within each row by column indices. Also it is assumed that each pair of row and column indices appears only once.

For example, the `4x4` matrix

$$A = \begin{pmatrix} 1.0 & 3.0 & 0.0 & 0.0 \\ 0.0 & 4.0 & 6.0 & 0.0 \\ 2.0 & 5.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 0.0 & 9.0 \end{pmatrix}$$

is represented as

$$\text{csrRowPtr} = (0 \quad 2 \quad 4 \quad 8 \quad 9)$$

$$\text{csrColInd} = (0 \quad 1 \quad 1 \quad 2 \quad 0 \quad 1 \quad 2 \quad 3 \quad 3)$$

$$\text{csrVal} = (1.0 \quad 3.0 \quad 4.0 \quad 6.0 \quad 2.0 \quad 5.0 \quad 7.0 \quad 8.0 \quad 9.0)$$

2.3.5. Matrix (CSC) Format

In CSC format the matrix is represented by the following parameters:

Parameter	Type	Size	Meaning
n	(int)		The number of rows (and columns) in the matrix.
nnz	(int)		The number of non-zero elements in the matrix.
cscColPtr	(int *)	n+1	The array of offsets corresponding to the start of each column in the arrays cscRowInd and cscVal. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix.
cscRowInd	(int *)	nnz	The array of row indices corresponding to the non-zero elements in the matrix. **It is assumed that this array is sorted by column and by row within each column.**
cscVal	(S|D|C|Z)*	nnz	The array of values corresponding to the non-zero elements in the matrix. **It is assumed that this array is sorted by column and by row within each column.**

Note that in our CSC format, sparse matrices are assumed to be stored in column-major order, in other words, the index arrays are first sorted by column indices and then within each column by row indices. Also it is assumed that each pair of row and column indices appears only once.

For example, the 4x4 matrix

$$A = \begin{pmatrix} 1.0 & 3.0 & 0.0 & 0.0 \\ 0.0 & 4.0 & 6.0 & 0.0 \\ 2.0 & 5.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 0.0 & 9.0 \end{pmatrix}$$

is represented as

$$\text{cscColPtr} = (0 \quad 2 \quad 5 \quad 7 \quad 9)$$

$$\text{cscRowInd} = (0 \quad 2 \quad 0 \quad 1 \quad 2 \quad 1 \quad 2 \quad 2 \quad 3)$$

$$\text{cscVal} = (1.0 \quad 2.0 \quad 3.0 \quad 4.0 \quad 5.0 \quad 6.0 \quad 7.0 \quad 8.0 \quad 9.0)$$

2.4. cuSolverDN: dense LAPACK Function Reference

This section describes the API of cuSolverDN, which provides a subset of dense LAPACK functions.

2.4.1. cuSolverDN Helper Function Reference

The cuSolverDN helper functions are described in this section.

2.4.1.1. cusolverDnCreate()

```
cusolverStatus_t
cusolverDnCreate(cusolverDnHandle_t *handle);
```

This function initializes the cuSolverDN library and creates a handle on the cuSolverDN context. It must be called before any other cuSolverDN API function is invoked. It allocates hardware resources necessary for accessing the GPU.

Parameter	Memory	In/out	Meaning
handle	host	output	The pointer to the handle to the cuSolverDN context.

Status Returned

CUSOLVER_STATUS_SUCCESS	The initialization succeeded.
CUSOLVER_STATUS_NOT_INITIALIZED	The CUDA Runtime initialization failed.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.

2.4.1.2. cusolverDnDestroy()

```
cusolverStatus_t
cusolverDnDestroy(cusolverDnHandle_t handle);
```

This function releases CPU-side resources used by the cuSolverDN library.

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.

Status Returned

CUSOLVER_STATUS_SUCCESS	The shutdown succeeded.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

2.4.1.3. cusolverDnSetStream()

```
cusolverStatus_t
cusolverDnSetStream(cusolverDnHandle_t handle, cudaStream_t streamId)
```

This function sets the stream to be used by the cuSolverDN library to execute its routines.

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
streamId	host	input	The stream to be used by the library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The stream was set successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

2.4.1.4. cusolverDnGetStream()

```
cusolverStatus_t
cusolverDnGetStream(cusolverDnHandle_t handle, cudaStream_t *streamId)
```

This function sets the stream to be used by the cuSolverDN library to execute its routines.

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
streamId	host	output	The stream to be used by the library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The stream was set successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

2.4.1.5. cusolverDnLoggerSetCallback()

```
cusolverStatus_t cusolverDnLoggerSetCallback(cusolverDnLoggerCallback_t callback);
```

This function sets the logging callback function.

Parameters:

Parameter	Memory	Input / Output	Description
callback		Input	Pointer to a callback function. See cusolverDnLoggerCallback_t.

Returns:

Return Value	Description
CUSOLVER_STATUS_SUCCESS	If the callback function was successfully set.

See cusolverStatus_t for a complete list of valid return codes.

2.4.1.6. cusolverDnLoggerSetFile()

```
cusolverStatus_t cusolverDnLoggerSetFile(FILE* file);
```

This function sets the logging output file. Note: once registered using this function call, the provided file handle must not be closed unless the function is called again to switch to a different file handle.

Parameters:

Parameter	Memory	Input / Output	Description
file		Input	Pointer to an open file. File should have write permission.

Returns:

Return Value	Description
CUSOLVER_STATUS_SUCCESS	If logging file was successfully set.

See cusolverStatus_t for a complete list of valid return codes.

2.4.1.7. cusolverDnLoggerOpenFile()

```
cusolverStatus_t cusolverDnLoggerOpenFile(const char* logFile);
```

This function opens a logging output file in the given path.

Parameters:

Parameter	Memory	Input / Output	Description
logFile		Input	Path of the logging output file.

Returns:

Return Value	Description
CUSOLVER_STATUS_SUCCESS	If the logging file was successfully opened.

See cusolverStatus_t for a complete list of valid return codes.

2.4.1.8. cusolverDnLoggerSetLevel()

```
cusolverStatus_t cusolverDnLoggerSetLevel(int level);
```

This function sets the value of the logging level.

Parameters:

Parameter	Memory	Input / Output	Description
level		Input	Value of the logging level. See cuSolverDN Logging.

Returns:

Return Value	Description
CUSOLVER_STATUS_INVALID_VALUE	If the value was not a valid logging level. See cuSolverDN Logging.
CUSOLVER_STATUS_SUCCESS	If the logging level was successfully set.

See cusolverStatus_t for a complete list of valid return codes.

2.4.1.9. cusolverDnLoggerSetMask()

```
cusolverStatus_t cusolverDnLoggerSetMask(int mask);
```

This function sets the value of the logging mask.

Parameters:

Parameter	Memory	Input / Output	Description
mask		Input	Value of the logging mask. See cuSolverDN Logging.

Returns:

Return Value	Description
CUSOLVER_STATUS_SUCCESS	If the logging mask was successfully set.

See cusolverStatus_t for a complete list of valid return codes.

2.4.1.10. cusolverDnLoggerForceDisable()

```
cusolverStatus_t cusolverDnLoggerForceDisable();
```

This function disables logging for the entier run.

Returns:

Return Value	Description
CUSOLVER_STATUS_SUCCESS	If logging was successfully disabled.

See cusolverStatus_t for a complete list of valid return codes.

2.4.1.11. cusolverDnCreateSyevjInfo()

```
cusolverStatus_t
cusolverDnCreateSyevjInfo(
    syevjInfo_t *info);
```

This function creates and initializes the structure of `syevj`, `syevjBatched` and `sygvj` to default values.

Parameter	Memory	In/out	Meaning
info	host	output	The pointer to the structure of syevj.

Status Returned

CUSOLVER_STATUS_SUCCESS	The structure was initialized successfully.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.

2.4.1.12. cusolverDnDestroySyevjInfo()

```
cusolverStatus_t
cusolverDnDestroySyevjInfo(
    syevjInfo_t info);
```

This function destroys and releases any memory required by the structure.

Parameter	Memory	In/out	Meaning
info	host	input	The structure of syevj.

Status Returned

CUSOLVER_STATUS_SUCCESS	The resources are released successfully.

2.4.1.13. cusolverDnXsyevjSetTolerance()

```
cusolverStatus_t
cusolverDnXsyevjSetTolerance(
    syevjInfo_t info,
    double tolerance)
```

This function configures tolerance of syevj.

Parameter	Memory	In/out	Meaning
info	host	in/out	The pointer to the structure of syevj.
tolerance	host	input	accuracy of numerical eigenvalues.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.

2.4.1.14. cusolverDnXsyevjSetMaxSweeps()

```
cusolverStatus_t
cusolverDnXsyevjSetMaxSweeps(
    syevjInfo_t info,
    int max_sweeps)
```

This function configures maximum number of sweeps in syevj. The default value is 100.

Parameter	Memory	In/out	Meaning
info	host	in/out	The pointer to the structure of syevj.
max_sweeps	host	input	Maximum number of sweeps.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.

2.4.1.15. cusolverDnXsyevjSetSortEig()

```
cusolverStatus_t
cusolverDnXsyevjSetSortEig(
    syevjInfo_t info,
    int sort_eig)
```

If sort_eig is zero, the eigenvalues are not sorted. This function only works for syevjBatched. syevj and sygvj always sort eigenvalues in ascending order. By default, eigenvalues are always sorted in ascending order.

Parameter	Memory	In/out	Meaning
info	host	in/out	The pointer to the structure of syevj.
sort_eig	host	input	If sort_eig is zero, the eigenvalues are not sorted.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.

2.4.1.16. cusolverDnXsyevjGetResidual()

```
cusolverStatus_t
cusolverDnXsyevjGetResidual(
    cusolverDnHandle_t handle,
    syevjInfo_t info,
    double *residual)
```

This function reports residual of syevj or sygvj. It does not support syevjBatched. If the user calls this function after syevjBatched, the error CUSOLVER_STATUS_NOT_SUPPORTED is returned.

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
info	host	input	The pointer to the structure of syevj.
residual	host	output	Residual of syevj.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_SUPPORTED	Does not support batched version.

2.4.1.17. cusolverDnXsyevjGetSweeps()

```
cusolverStatus_t
cusolverDnXsyevjGetSweeps(
    cusolverDnHandle_t handle,
    syevjInfo_t info,
    int *executed_sweeps)
```

This function reports number of executed sweeps of `syevj` or `sygvj`. It does not support `syevjBatched`. If the user calls this function after `syevjBatched`, the error `CUSOLVER_STATUS_NOT_SUPPORTED` is returned.

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`info`	`host`	`input`	The pointer to the structure of `syevj`.
`executed_sweeps`	`host`	`output`	Number of executed sweeps.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_SUPPORTED`	Does not support batched version.

2.4.1.18. cusolverDnCreateGesvdjInfo()

```
cusolverStatus_t
cusolverDnCreateGesvdjInfo(
    gesvdjInfo_t *info);
```

This function creates and initializes the structure of `gesvdj` and `gesvdjBatched` to default values.

Parameter	Memory	In/out	Meaning
`info`	`host`	`output`	The pointer to the structure of `gesvdj`.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The structure was initialized successfully.
`CUSOLVER_STATUS_ALLOC_FAILED`	The resources could not be allocated.

2.4.1.19. cusolverDnDestroyGesvdjInfo()

```
cusolverStatus_t
cusolverDnDestroyGesvdjInfo(
    gesvdjInfo_t info);
```

This function destroys and releases any memory required by the structure.

Parameter	Memory	In/out	Meaning
info	host	input	The structure of gesvdj.

Status Returned

CUSOLVER_STATUS_SUCCESS	The resources are released successfully.

2.4.1.20. cusolverDnXgesvdjSetTolerance()

```
cusolverStatus_t
cusolverDnXgesvdjSetTolerance(
    gesvdjInfo_t info,
    double tolerance)
```

This function configures tolerance of gesvdj.

Parameter	Memory	In/out	Meaning
info	host	in/out	The pointer to the structure of gesvdj.
tolerance	host	input	Accuracy of numerical singular values.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.

2.4.1.21. cusolverDnXgesvdjSetMaxSweeps()

```
cusolverStatus_t
cusolverDnXgesvdjSetMaxSweeps(
    gesvdjInfo_t info,
    int max_sweeps)
```

This function configures the maximum number of sweeps in gesvdj. The default value is 100.

Parameter	Memory	In/out	Meaning
info	host	in/out	The pointer to the structure of gesvdj.
max_sweeps	host	input	Maximum number of sweeps.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.

2.4.1.22. cusolverDnXgesvdjSetSortEig()

```
cusolverStatus_t
cusolverDnXgesvdjSetSortEig(
    gesvdjInfo_t info,
    int sort_svd)
```

If `sort_svd` is zero, the singular values are not sorted. This function only works for `gesvdjBatched`. `gesvdj` always sorts singular values in descending order. By default, singular values are always sorted in descending order.

Parameter	Memory	In/out	Meaning
`info`	`host`	`in/out`	The pointer to the structure of `gesvdj`.
`sort_svd`	`host`	`input`	If `sort_svd` is zero, the singular values are not sorted.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.

2.4.1.23. cusolverDnXgesvdjGetResidual()

```
cusolverStatus_t
cusolverDnXgesvdjGetResidual(
    cusolverDnHandle_t handle,
    gesvdjInfo_t info,
    double *residual)
```

This function reports residual of `gesvdj`. It does not support `gesvdjBatched`. If the user calls this function after `gesvdjBatched`, the error `CUSOLVER_STATUS_NOT_SUPPORTED` is returned.

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`info`	`host`	`input`	The pointer to the structure of `gesvdj`.
`residual`	`host`	`output`	Residual of `gesvdj`.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_SUPPORTED`	Does not support batched version

2.4.1.24. cusolverDnXgesvdjGetSweeps()

```
cusolverStatus_t
cusolverDnXgesvdjGetSweeps(
    cusolverDnHandle_t handle,
    gesvdjInfo_t info,
    int *executed_sweeps)
```

This function reports number of executed sweeps of `gesvdj`. It does not support `gesvdjBatched`. If the user calls this function after `gesvdjBatched`, the error `CUSOLVER_STATUS_NOT_SUPPORTED` is returned.

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`info`	`host`	`input`	The pointer to the structure of `gesvdj`.

Parameter	Memory	In/out	Meaning
executed_sweeps	host	output	Number of executed sweeps.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_SUPPORTED	Does not support batched version

2.4.1.25. cusolverDnIRSParamsCreate()

```
cusolverStatus_t
cusolverDnIRSParamsCreate(cusolverDnIRSParams_t *params);
```

This function creates and initializes the structure of parameters for an IRS solver such as the `cusolverDnIRSXgesv()` or the `cusolverDnIRSXgels()` functions to default values. The params structure created by this function can be used by one or more call to the same or to a different IRS solver. Note that in CUDA 10.2, the behavior was different and a new `params` structure was needed to be created per each call to an IRS solver. Also note that the user can also change configurations of the params and then call a new IRS instance, but be careful that the previous call was done because any change to the configuration before the previous call was done could affect it.

Parameter	Memory	In/out	Meaning
params	host	output	Pointer to the `cusolverDnIRSParams_t Params` structure

Status Returned

CUSOLVER_STATUS_SUCCESS	The structure was created and initialized successfully.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.

2.4.1.26. cusolverDnIRSParamsDestroy()

```
cusolverStatus_t
cusolverDnIRSParamsDestroy(cusolverDnIRSParams_t params);
```

This function destroys and releases any memory required by the Params structure.

Parameter	Memory	In/out	Meaning
params	host	input	The `cusolverDnIRSParams_t Params` structure.

Status Returned

CUSOLVER_STATUS_SUCCESS	The resources are released successfully.
CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The `Params` structure was not created.

CUSOLVER_STATUS_IRS_INFOS_NOT_DESTROYED	Not all the Infos structure associated with this Params structure have been destroyed yet.

2.4.1.27. cusolverDnIRSParamsSetSolverPrecisions()

```
cusolverStatus_t
    cusolverDnIRSParamsSetSolverPrecisions(
            cusolverDnIRSParams_t params,
            cusolverPrecType_t solver_main_precision,
            cusolverPrecType_t solver_lowest_precision );
```

This function sets both the main and the lowest precision for the Iterative Refinement Solver (IRS). By main precision, we mean the precision of the Input and Output datatype. By lowest precision, we mean the solver is allowed to use as lowest computational precision during the LU factorization process. Note that the user has to set both the main and lowest precision before the first call to the IRS solver because they are NOT set by default with the params structure creation, as it depends on the Input Output data type and user request. It is a wrapper to both cusolverDnIRSParamsSetSolverMainPrecision() and cusolverDnIRSParamsSetSolverLowestPrecision(). All possible combinations of main/lowest precision are described in the table below. Usually the lowest precision defines the speedup that can be achieved. The ratio of the performance of the lowest precision over the main precision (e.g., Inputs/Outputs datatype) define the upper bound of the speedup that could be obtained. More precisely, it depends on many factors, but for large matrices sizes, it is the ratio of the matrix-matrix rank-k product (e.g., GEMM where K is 256 and M=N=size of the matrix) that define the possible speedup. For instance, if the inout precision is real double precision CUSOLVER_R_64F and the lowest precision is CUSOLVER_R_32F, then we can expect a speedup of at most 2X for large problem sizes. If the lowest precision was CUSOLVER_R_16F, then we can expect 3X-4X. A reasonable strategy should take the number of right-hand sides, the size of the matrix as well as the convergence rate into account.

Parameter	Memory	In/out	Meaning
params	host	in/out	The cusolverDnIRSParams_t Params structure.
solver_main_precision	host	input	Allowed Inputs/Outputs datatype (for example CUSOLVER_R_FP64 for a real double precision data). See the table below for the supported precisions.
solver_lowest_precision	host	input	Allowed lowest compute type (for example CUSOLVER_R_16F for half precision computation). See the table below for the supported precisions.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The Params structure was not created.

Table 2. **Supported Inputs/Outputs data type and lower precision for the IRS solver**

Inputs/Outputs Data Type (e.g., main precision)	Supported values for the lowest precision
CUSOLVER_C_64F	CUSOLVER_C_64F, CUSOLVER_C_32F, CUSOLVER_C_16F, CUSOLVER_C_16BF, CUSOLVER_C_TF32
CUSOLVER_C_32F	CUSOLVER_C_32F, CUSOLVER_C_16F, CUSOLVER_C_16BF, CUSOLVER_C_TF32
CUSOLVER_R_64F	CUSOLVER_R_64F, CUSOLVER_R_32F, CUSOLVER_R_16F, CUSOLVER_R_16BF, CUSOLVER_R_TF32
CUSOLVER_R_32F	CUSOLVER_R_32F, CUSOLVER_R_16F, CUSOLVER_R_16BF, CUSOLVER_R_TF32

2.4.1.28. cusolverDnIRSParamsSetSolverMainPrecision()

```
cusolverStatus_t
cusolverDnIRSParamsSetSolverMainPrecision(
    cusolverDnIRSParams_t params,
    cusolverPrecType_t solver_main_precision);
```

This function sets the main precision for the Iterative Refinement Solver (IRS). By main precision, we mean, the type of the Input and Output data. Note that the user has to set both the main and lowest precision before a first call to the IRS solver because they are NOT set by default with the `params` structure creation, as it depends on the Input Output data type and user request. user can set it by either calling this function or by calling `cusolverDnIRSParamsSetSolverPrecisions()` which set both the main and the lowest precision together. All possible combinations of main/lowest precision are described in the table in the `cusolverDnIRSParamsSetSolverPrecisions()` section above.

Parameter	Memory	In/out	Meaning
params	host	in/out	The `cusolverDnIRSParams_t Params` structure.
solver_main_precision	host	input	Allowed Inputs/Outputs datatype (for example CUSOLVER_R_FP64 for a real double precision data). See the table in the `cusolverDnIRSParamsSetSolverPrecisions()` section above for the supported precisions.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The `Params` structure was not created.

2.4.1.29. cusolverDnIRSParamsSetSolverLowestPrecision()

```
cusolverStatus_t
cusolverDnIRSParamsSetSolverLowestPrecision(
    cusolverDnIRSParams_t params,
    cusolverPrecType_t lowest_precision_type);
```

This function sets the lowest precision that will be used by Iterative Refinement Solver. By lowest precision, we mean the solver is allowed to use as lowest computational precision during the LU factorization process. Note that the user has to set both the main and lowest precision before a first call to the IRS solver because they are NOT set by default with the `params` structure creation, as it depends on the Input Output data type and user request. Usually the lowest precision defines the speedup that can be achieved. The ratio of the performance of the lowest precision over the main precision (e.g., Inputs/Outputs datatype) define somehow the upper bound of the speedup that could be obtained. More precisely, it depends on many factors, but for large matrices sizes, it is the ratio of the matrix-matrix rank-k product (e.g., GEMM where K is 256 and M=N=size of the matrix) that define the possible speedup. For instance, if the inout precision is real double precision CUSOLVER_R_64F and the lowest precision is CUSOLVER_R_32F, then we can expect a speedup of at most 2X for large problem sizes. If the lowest precision was CUSOLVER_R_16F, then we can expect 3X-4X. A reasonable strategy should take the number of right-hand sides, the size of the matrix as well as the convergence rate into account.

Parameter	Memory	In/out	Meaning
`params`	`host`	`in/out`	The `cusolverDnIRSParams_t Params` structure.
`lowest_precision_type`	`host`	`input`	Allowed lowest compute type (for example CUSOLVER_R_16F for half precision computation). See the table in the `cusolverDnIRSParamsSetSolverPrecisions()` section above for the supported precisions.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED`	The Params structure was not created.

2.4.1.30. cusolverDnIRSParamsSetRefinementSolver()

```
cusolverStatus_t
cusolverDnIRSParamsSetRefinementSolver(
    cusolverDnIRSParams_t params,
    cusolverIRSRefinement_t solver);
```

This function sets the refinement solver to be used in the Iterative Refinement Solver functions such as the `cusolverDnIRSXgesv()` or the `cusolverDnIRSXgels()` functions. Note that the

user has to set the refinement algorithm before a first call to the IRS solver because it is NOT set by default with the creating of params. Details about values that can be set to and theirs meaning are described in the table below.

Parameter	Memory	In/out	Meaning
params	host	in/out	The cusolverDnIRSParams_t Params structure
solver	host	input	Type of the refinement solver to be used by the IRS solver such as cusolverDnIRSXgesv() or cusolverDnIRSXgels().

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The Params structure was not created.

CUSOLVER_IRS_REFINE_NOT_SET	Solver is not set, this value is what is set when creating the params structure. IRS solver will return an error.
CUSOLVER_IRS_REFINE_NONE	No refinement solver; the IRS solver performs a factorization followed by a solve without any refinement. For example, if the IRS solver was cusolverDnIRSXgesv(), this is equivalent to a Xgesv routine without refinement and where the factorization is carried out in the lowest precision. If for example the main precision was CUSOLVER_R_64F and the lowest was CUSOLVER_R_64F as well, then this is equivalent to a call to cusolverDnDgesv().
CUSOLVER_IRS_REFINE_CLASSICAL	Classical iterative refinement solver. Similar to the one used in LAPACK routines.
CUSOLVER_IRS_REFINE_GMRES	GMRES (Generalized Minimal Residual) based iterative refinement solver. In recent study, the GMRES method has drawn the scientific community attention for its ability to be used as refinement solver that outperforms the classical iterative refinement method. Based on our experimentation, we recommend this setting.
CUSOLVER_IRS_REFINE_CLASSICAL_GMRES	Classical iterative refinement solver that uses the GMRES (Generalized Minimal Residual) internally to solve the correction equation at each iteration. We call the *classical refinement iteration* the outer iteration while the *GMRES* is called inner iteration. Note that if the tolerance of the inner GMRES is set very low, let say to machine precision, then the outer *classical refinement iteration* will performs only one iteration and thus this option will behaves like CUSOLVER_IRS_REFINE_GMRES.

CUSOLVER_IRS_REFINE_GMRES_GMRES	Similar to CUSOLVER_IRS_REFINE_CLASSICAL_GMRES which consists of classical refinement process that uses GMRES to solve the inner correction system, here it is a GMRES (Generalized Minimal Residual) based iterative refinement solver that uses another GMRES internally to solve the preconditioned system.

2.4.1.31. cusolverDnIRSParamsSetTol()

```
cusolverStatus_t
cusolverDnIRSParamsSetTol(
            cusolverDnIRSParams_t params,
            double val );
```

This function sets the tolerance for the refinement solver. By default it is such that all the RHS satisfy:

`RNRM < SQRT(N)*XNRM*ANRM*EPS*BWDMAX` where

- RNRM is the infinity-norm of the residual
- XNRM is the infinity-norm of the solution
- ANRM is the infinity-operator-norm of the matrix A
- EPS is the machine epsilon for the Inputs/Outputs datatype that matches LAPACK <X>LAMCH('Epsilon')
- BWDMAX, the value BWDMAX is fixed to 1.0

The user can use this function to change the tolerance to a lower or higher value. Our goal is to give the user more control such a way he can investigate and control every detail of the IRS solver. Note that the tolerance value is always in *real double precision* whatever the Inputs/Outputs datatype is.

Parameter	Memory	In/out	Meaning
params	host	in/out	The cusolverDnIRSParams_t Params structure.
val	host	input	Double precision real value to which the refinement tolerance will be set.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The Params structure was not created.

2.4.1.32. cusolverDnIRSParamsSetTolInner()

```
cusolverStatus_t
```

```
    cusolverDnIRSParamsSetTolInner(
            cusolverDnIRSParams_t params,
            double val );
```

This function sets the tolerance for the inner refinement solver when the refinement solver consists of two-levels solver (e.g., CUSOLVER_IRS_REFINE_CLASSICAL_GMRES or CUSOLVER_IRS_REFINE_GMRES_GMRES cases). It is not referenced in case of one level refinement solver such as CUSOLVER_IRS_REFINE_CLASSICAL or CUSOLVER_IRS_REFINE_GMRES. It is set to 1e-4 by default. This function set the tolerance for the inner solver (e.g. the inner GMRES). For example, if the Refinement Solver was set to CUSOLVER_IRS_REFINE_CLASSICAL_GMRES, setting this tolerance mean that the inner GMRES solver will converge to that tolerance at each outer iteration of the classical refinement solver. Our goal is to give the user more control such a way he can investigate and control every detail of the IRS solver. Note the, the tolerance value is always in *real double precision* whatever the Inputs/Outputs datatype is.

Parameter	Memory	In/out	Meaning
params	host	in/out	The cusolverDnIRSParams_t Params structure.
val	host	input	Double precision real value to which the tolerance of the inner refinement solver will be set.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The Params structure was not created.

2.4.1.33. cusolverDnIRSParamsSetMaxIters()

```
cusolverStatus_t
cusolverDnIRSParamsSetMaxIters(
    cusolverDnIRSParams_t params,
    int max_iters);
```

This function sets the total number of allowed refinement iterations after which the solver will stop. Total means any iteration which means the sum of the outer and the inner iterations (inner is meaningful when two-levels refinement solver is set). Default value is set to 50. Our goal is to give the user more control such a way he can investigate and control every detail of the IRS solver.

Parameter	Memory	In/out	Meaning
params	host	in/out	The cusolverDnIRSParams_t Params structure.
max_iters	host	input	Maximum total number of iterations allowed for the refinement solver.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.

CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The Params structure was not created.

2.4.1.34. cusolverDnIRSParamsSetMaxItersInner()

```
cusolverStatus_t
    cusolverDnIRSParamsSetMaxItersInner(
            cusolverDnIRSParams_t params,
            cusolver_int_t maxiters_inner );
```

This function sets the maximal number of iterations allowed for the inner refinement solver. It is not referenced in case of one level refinement solver such as CUSOLVER_IRS_REFINE_CLASSICAL or CUSOLVER_IRS_REFINE_GMRES. The inner refinement solver will stop after reaching either the inner tolerance or the MaxItersInner value. By default, it is set to 50. Note that this value could not be larger than the MaxIters since MaxIters is the total number of allowed iterations. Note that if the user calls `cusolverDnIRSParamsSetMaxIters` after calling this function, `SetMaxIters` has priority and will overwrite `MaxItersInner` to the minimum value of `(MaxIters, MaxItersInner)`.

Parameter	Memory	In/out	Meaning
params	host	in/out	The `cusolverDnIRSParams_t Params` structure
maxiters_inner	host	input	Maximum number of allowed inner iterations for the inner refinement solver. Meaningful when the refinement solver is a two-levels solver such as CUSOLVER_IRS_REFINE_CLASSICAL_GMRES or CUSOLVER_IRS_REFINE_GMRES_GMRES. Value should be less or equal to `MaxIters`.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The Params structure was not created.
CUSOLVER_STATUS_IRS_PARAMS_INVALID	If the value was larger than `MaxIters`.

2.4.1.35. cusolverDnIRSParamsEnableFallback()

```
cusolverStatus_t
    cusolverDnIRSParamsEnableFallback(
            cusolverDnIRSParams_t params );
```

This function enable the fallback to the main precision in case the Iterative Refinement Solver (IRS) failed to converge. In other term, if the IRS solver failed to converge, the solver will return a no convergence code (e.g., `niter` < 0), but can either return the non-convergent solution as it is (e.g., disable fallback) or can fallback (e.g., enable fallback) to the main

precision (which is the precision of the Inputs/Outputs data) and solve the problem from scratch returning the good solution. This is the behavior by default, and it will guarantee that the IRS solver always provide the good solution. This function is provided because we provided `cusolverDnIRSParamsDisableFallback` which allows the user to disable the fallback and thus this function allow the user to re-enable it.

Parameter	Memory	In/out	Meaning
`params`	`host`	`in/out`	The `cusolverDnIRSParams_t Params` structure

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED`	The `Params` structure was not created.

2.4.1.36. cusolverDnIRSParamsDisableFallback()

```
cusolverStatus_t
    cusolverDnIRSParamsDisableFallback(
            cusolverDnIRSParams_t params );
```

This function disables the fallback to the main precision in case the Iterative Refinement Solver (IRS) failed to converge. In other term, if the IRS solver failed to converge, the solver will return a no convergence code (e.g., `niter` < 0), but can either return the non-convergent solution as it is (e.g., disable fallback) or can fallback (e.g., enable fallback) to the main precision (which is the precision of the Inputs/Outputs data) and solve the problem from scratch returning the good solution. This function disables the fallback and the returned solution is whatever the refinement solver was able to reach before it returns. Disabling fallback does not guarantee that the solution is the good one. However, if users want to keep getting the solution of the lower precision in case the IRS did not converge after certain number of iterations, they need to disable the fallback. The user can re-enable it by calling `cusolverDnIRSParamsEnableFallback`.

Parameter	Memory	In/out	Meaning
`params`	`host`	`in/out`	The `cusolverDnIRSParams_t Params` structure

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED`	The `Params` structure was not created.

2.4.1.37. cusolverDnIRSParamsGetMaxIters()

```
cusolverStatus_t
    cusolverDnIRSParamsGetMaxIters(
            cusolverDnIRSParams_t params,
            cusolver_int_t *maxiters );
```

This function returns the current setting in the `params` structure for the maximal allowed number of iterations (e.g., either the default `MaxIters`, or the one set by the user in case he set it using `cusolverDnIRSParamsSetMaxIters`). Note that this function returns the current setting in the `params` configuration and not to be confused with the `cusolverDnIRSInfosGetMaxIters` which return the maximal allowed number of iterations for a particular call to an IRS solver. To be clearer, the `params` structure can be used for many calls to an IRS solver. A user can change the allowed `MaxIters` between calls while the `Infos` structure in `cusolverDnIRSInfosGetMaxIters` contains information about a particular call and cannot be reused for different calls, and thus, `cusolverDnIRSInfosGetMaxIters` returns the allowed `MaxIters` for that call.

Parameter	Memory	In/out	Meaning
`params`	`host`	`in`	The `cusolverDnIRSParams_t Params` structure.
`maxiters`	`host`	`output`	The maximal number of iterations that is currently set.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED`	The `Params` structure was not created.

2.4.1.38. cusolverDnIRSInfosCreate()

```
cusolverStatus_t
cusolverDnIRSInfosCreate(
    cusolverDnIRSInfos_t* infos )
```

This function creates and initializes the `Infos` structure that will hold the refinement information of an Iterative Refinement Solver (IRS) call. Such information includes the total number of iterations that was needed to converge (`Niters`), the outer number of iterations (meaningful when two-levels preconditioner such as CUSOLVER_IRS_REFINE_CLASSICAL_GMRES is used), the maximal number of iterations that was allowed for that call, and a pointer to the matrix of the convergence history residual norms. The `Infos` structure needs to be created before a call to an IRS solver. The `Infos` structure is valid for only one call to an IRS solver, since it holds info about that solve and thus each solve will requires its own `Infos` structure.

Parameter	Memory	In/out	Meaning
`info`	`host`	`output`	Pointer to the `cusolverDnIRSInfos_t Infos` structure.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The structure was initialized successfully.
`CUSOLVER_STATUS_ALLOC_FAILED`	The resources could not be allocated.

2.4.1.39. cusolverDnIRSInfosDestroy()

```
cusolverStatus_t
cusolverDnIRSInfosDestroy(
    cusolverDnIRSInfos_t infos );
```

This function destroys and releases any memory required by the `Infos` structure. This function destroys all the information (e.g., Niters performed, OuterNiters performed, residual history etc.) about a solver call; thus, this function should only be called after the user is finished with the information.

Parameter	Memory	In/out	Meaning
`info`	`host`	`in/out`	The `cusolverDnIRSInfos_t Infos` structure.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The resources are released successfully.
`CUSOLVER_STATUS_IRS_INFOS_NOT_INITIALIZED`	The `Infos` structure was not created.

2.4.1.40. cusolverDnIRSInfosGetMaxIters()

```
cusolverStatus_t
    cusolverDnIRSInfosGetMaxIters(
            cusolverDnIRSInfos_t infos,
            cusolver_int_t *maxiters );
```

This function returns the maximal allowed number of iterations that was set for the corresponding call to the IRS solver. Note that this function returns the setting that was set when that call happened and is not to be confused with the `cusolverDnIRSParamsGetMaxIters` which returns the current setting in the `params` configuration structure. To be clearer, the `params` structure can be used for many calls to an IRS solver. A user can change the allowed `MaxIters` between calls while the `Infos` structure in `cusolverDnIRSInfosGetMaxIters` contains information about a particular call and cannot be reused for different calls, thus `cusolverDnIRSInfosGetMaxIters` returns the allowed `MaxIters` for that call.

Parameter	Memory	In/out	Meaning
`infos`	`host`	`in`	The `cusolverDnIRSInfos_t Infos` structure.
`maxiters`	`host`	`output`	The maximal number of iterations that is currently set.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_IRS_INFOS_NOT_INITIALIZED`	The `Infos` structure was not created.

2.4.1.41. cusolverDnIRSInfosGetNiters()

```
cusolverStatus_t cusolverDnIRSInfosGetNiters(
            cusolverDnIRSInfos_t infos,
            cusolver_int_t *niters );
```

This function returns the total number of iterations performed by the IRS solver. If it was negative, it means that the IRS solver did not converge and if the user did not disable the fallback to full precision, then the fallback to a full precision solution happened and solution is good. Please refer to the description of negative `niters` values in the corresponding IRS linear solver functions such as `cusolverDnXgesv()` or `cusolverDnXgels()`.

Parameter	Memory	In/out	Meaning
`infos`	`host`	`in`	The `cusolverDnIRSInfos_t Infos` structure.
`niters`	`host`	`output`	The total number of iterations performed by the IRS solver.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_IRS_INFOS_NOT_INITIALIZED`	The `Infos` structure was not created.

2.4.1.42. cusolverDnIRSInfosGetOuterNiters()

```
cusolverStatus_t
    cusolverDnIRSInfosGetOuterNiters(
            cusolverDnIRSInfos_t infos,
            cusolver_int_t *outer_niters );
```

This function returns the number of iterations performed by the outer refinement loop of the IRS solver. When the refinement solver consists of a one-level solver such as CUSOLVER_IRS_REFINE_CLASSICAL or CUSOLVER_IRS_REFINE_GMRES, it is the same as `Niters`. When the refinement solver consists of a two-levels solver such as CUSOLVER_IRS_REFINE_CLASSICAL_GMRES or CUSOLVER_IRS_REFINE_GMRES_GMRES, it is the number of iterations of the outer loop. Refer to the description of the cusolverIRSRefinement_t section for more details.

Parameter	Memory	In/out	Meaning
`infos`	`host`	`in`	The `cusolverDnIRSInfos_t Infos` structure.
`outer_niters`	`host`	`output`	The number of iterations of the outer refinement loop of the IRS solver.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_IRS_INFOS_NOT_INITIALIZED`	The `Infos` structure was not created.

2.4.1.43. cusolverDnIRSInfosRequestResidual()

```
cusolverStatus_t cusolverDnIRSInfosRequestResidual(
        cusolverDnIRSInfos_t infos );
```

This function tells the IRS solver to store the convergence history (residual norms) of the refinement phase in a matrix that can be accessed via a pointer returned by the cusolverDnIRSInfosGetResidualHistory() function.

Parameter	Memory	In/out	Meaning
infos	host	in	The cusolverDnIRSInfos_t Infos structure

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_IRS_INFOS_NOT_INITIALIZED	The Infos structure was not created.

2.4.1.44. cusolverDnIRSInfosGetResidualHistory()

```
cusolverStatus_t
cusolverDnIRSInfosGetResidualHistory(
    cusolverDnIRSInfos_t infos,
    void **residual_history );
```

If the user called cusolverDnIRSInfosRequestResidual() before the call to the IRS function, then the IRS solver will store the convergence history (residual norms) of the refinement phase in a matrix that can be accessed via a pointer returned by this function. The datatype of the residual norms depends on the input and output data type. If the Inputs/Outputs datatype is double precision real or complex (CUSOLVER_R_FP64 or CUSOLVER_C_FP64), this residual will be of type real double precision (FP64) *double*, otherwise if the Inputs/Outputs datatype is single precision real or complex (CUSOLVER_R_FP32 or CUSOLVER_C_FP32), this residual will be real single precision FP32 *float*.

The residual history matrix consists of two columns (even for the multiple right-hand side case NRHS) of MaxIters+1 row, thus a matrix of size (MaxIters+1,2). Only the first OuterNiters +1 rows contains the residual norms the other (e.g., OuterNiters+2:Maxiters+1) are garbage. On the first column, each row *"i"* specify the total number of iterations happened till this outer iteration *"i"* and on the second columns the residual norm corresponding to this outer iteration *"i"*. Thus, the first row (e.g., outer iteration *"0"*) consists of the initial residual (e.g., the residual before the refinement loop start) then the consecutive rows are the residual obtained at each outer iteration of the refinement loop. Note, it only consists of the history of the outer loop.

If the refinement solver was CUSOLVER_IRS_REFINE_CLASSICAL or CUSOLVER_IRS_REFINE_GMRES, then OuterNiters=Niters (Niters is the total number of iterations performed) and there is Niters+1 rows of norms that correspond to the Niters outer iterations.

If the refinement solver was CUSOLVER_IRS_REFINE_CLASSICAL_GMRES or CUSOLVER_IRS_REFINE_GMRES_GMRES, then OuterNiters <= Niters corresponds to the outer iterations performed by the outer refinement loop. Thus, there is OuterNiters+1 residual norms where row *"i"* correspond to the outer iteration *"i"* and the first column specify the total number of iterations (outer and inner) that were performed till this step the second columns correspond to the residual norm at this step.

For example, let's say the user specifies CUSOLVER_IRS_REFINE_CLASSICAL_GMRES as a refinement solver and say it needed 3 outer iterations to converge and 4,3,3 inner iterations at each outer, respectively. This consists of 10 total iterations. Row 0 corresponds to the first residual before the refinement start, so it has 0 in its first column. On row 1 which corresponds to the outer iteration 1, it will be 4 (4 is the total number of iterations that were performed till now), on row 2 it will be 7, and on row 3 it will be 10.

In summary, let's define `ldh=Maxiters+1`, the leading dimension of the residual matrix. then `residual_history[i]` shows the total number of iterations performed at the outer iteration *"i"* and `residual_history[i+ldh]` corresponds to the norm of the residual at this outer iteration.

Parameter	Memory	In/out	Meaning
`infos`	`host`	`in`	The `cusolverDnIRSInfos_t Infos` structure.
`residual_history`	`host`	`output`	Returns a void pointer to the matrix of the convergence history residual norms. See the description above for the relation between the residual norm datatype and the inout datatype.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_IRS_INFOS_NOT_INITIALIZED`	The `Infos` structure was not created.
`CUSOLVER_STATUS_INVALID_VALUE`	This function was called without calling `cusolverDnIRSInfosRequestResidual()` in advance.

2.4.1.45. cusolverDnCreateParams()

```
cusolverStatus_t
cusolverDnCreateParams(
    cusolverDnParams_t *params);
```

This function creates and initializes the structure of `64-bit API` to default values.

Parameter	Memory	In/out	Meaning
`params`	`host`	`output`	The pointer to the structure of `64-bit API`.

Status Returned

CUSOLVER_STATUS_SUCCESS	The structure was initialized successfully.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.

2.4.1.46. cusolverDnDestroyParams()

```
cusolverStatus_t
cusolverDnDestroyParams(
    cusolverDnParams_t params);
```

This function destroys and releases any memory required by the structure.

Parameter	Memory	In/out	Meaning
params	host	input	The structure of 64-bit API.

Status Returned

CUSOLVER_STATUS_SUCCESS	The resources were released successfully.

2.4.1.47. cusolverDnSetAdvOptions()

```
cusolverStatus_t
cusolverDnSetAdvOptions (
    cusolverDnParams_t params,
    cusolverDnFunction_t function,
    cusolverAlgMode_t algo   );
```

This function configures algorithm `algo` of `function`, a `64-bit API` routine.

Parameter	Memory	In/out	Meaning
params	host	in/out	The pointer to the structure of 64-bit API.
function	host	input	The routine to be configured.
algo	host	input	The algorithm to be configured.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_INVALID_VALUE	Wrong combination of function and algo.

2.4.2. Dense Linear Solver Reference (legacy)

This section describes linear solver API of cuSolverDN, including Cholesky factorization, LU with partial pivoting, QR factorization and Bunch-Kaufman (LDLT) factorization.

2.4.2.1. cusolverDn<t>potrf()

These helper functions calculate the necessary size of work buffers.

```
cusolverStatus_t
```

```
cusolverDnSpotrf_bufferSize(cusolverDnHandle_t handle,
                  cublasFillMode_t uplo,
                  int n,
                  float *A,
                  int lda,
                  int *Lwork );

cusolverStatus_t
cusolverDnDpotrf_bufferSize(cusolveDnHandle_t handle,
                  cublasFillMode_t uplo,
                  int n,
                  double *A,
                  int lda,
                  int *Lwork );

cusolverStatus_t
cusolverDnCpotrf_bufferSize(cusolverDnHandle_t handle,
                  cublasFillMode_t uplo,
                  int n,
                  cuComplex *A,
                  int lda,
                  int *Lwork );

cusolverStatus_t
cusolverDnZpotrf_bufferSize(cusolverDnHandle_t handle,
                  cublasFillMode_t uplo,
                  int n,
                  cuDoubleComplex *A,
                  int lda,
                  int *Lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSpotrf(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            float *A,
            int lda,
            float *Workspace,
            int Lwork,
            int *devInfo );

cusolverStatus_t
cusolverDnDpotrf(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            double *A,
            int lda,
            double *Workspace,
            int Lwork,
            int *devInfo );
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCpotrf(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
```

```
            cuComplex *A,
            int lda,
            cuComplex *Workspace,
            int Lwork,
            int *devInfo );

cusolverStatus_t
cusolverDnZpotrf(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            cuDoubleComplex *A,
            int lda,
            cuDoubleComplex *Workspace,
            int Lwork,
            int *devInfo );
```

This function computes the Cholesky factorization of a Hermitian positive-definite matrix.

`A` is an `n×n` Hermitian matrix, only the lower or upper part is meaningful. The input parameter `uplo` indicates which part of the matrix is used. The function would leave other parts untouched.

If input parameter `uplo` is `CUBLAS_FILL_MODE_LOWER`, only the lower triangular part of `A` is processed, and replaced by the lower triangular Cholesky factor `L`.

$$A = L * L^H$$

If input parameter `uplo` is `CUBLAS_FILL_MODE_UPPER`, only upper triangular part of `A` is processed, and replaced by upper triangular Cholesky factor `U`.

$$A = U^H * U$$

The user has to provide working space which is pointed by input parameter `Workspace`. The input parameter `Lwork` is size of the working space, and it is returned by `potrf_bufferSize()`.

If Cholesky factorization failed, i.e. some leading minor of `A` is not positive definite, or equivalently some diagonal elements of `L` or `U` is not a real number. The output parameter `devInfo` would indicate smallest leading minor of `A` which is not positive definite.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

API of potrf

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`uplo`	`host`	`input`	Indicates if matrix `A` lower or upper part is stored; the other part is not referenced.
`n`	`host`	`input`	Number of rows and columns of matrix `A`.
`A`	`device`	`in/out`	<type> array of dimension `lda * n` with `lda` is not less than `max(1,n)`.

Parameter	Memory	In/out	Meaning
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
Workspace	device	in/out	Working space, <type> array of size Lwork.
Lwork	host	input	Size of Workspace, returned by potrf_bufferSize.
devInfo	device	output	If devInfo = 0, the Cholesky factorization is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle). if devInfo = i, the leading minor of order i is not positive definite.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.2. cusolverDnPotrf()[DEPRECATED]

[[DEPRECATED]] use cusolverDnXpotrf() instead. The routine will be removed in the next major release.

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnPotrf_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cublasFillMode_t uplo,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType computeType,
    size_t *workspaceInBytes )
```

The routine bellow

```
cusolverStatus_t
cusolverDnPotrf(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cublasFillMode_t uplo,
    int64_t n,
```

```
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    cudaDataType computeType,
    void *pBuffer,
    size_t workspaceInBytes,
    int *info )
```

Computes the Cholesky factorization of a Hermitian positive-definite matrix using the generic API interfacte.

`A` is an `n×n` Hermitian matrix, only lower or upper part is meaningful. The input parameter `uplo` indicates which part of the matrix is used. The function would leave other part untouched.

If input parameter `uplo` is `CUBLAS_FILL_MODE_LOWER`, only lower triangular part of `A` is processed, and replaced by lower triangular Cholesky factor `L`.

$$A = L * L^H$$

If input parameter `uplo` is `CUBLAS_FILL_MODE_UPPER`, only upper triangular part of `A` is processed, and replaced by upper triangular Cholesky factor `U`.

$$A = U^H * U$$

The user has to provide working space which is pointed by input parameter `pBuffer`. The input parameter `workspaceInBytes` is size in bytes of the working space, and it is returned by `cusolverDnPotrf_bufferSize()`.

If Cholesky factorization failed, i.e. some leading minor of `A` is not positive definite, or equivalently some diagonal elements of `L` or `U` is not a real number. The output parameter `info` would indicate smallest leading minor of `A` which is not positive definite.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

Currently, `cusolverDnPotrf` supports only the default algorithm.

Table of algorithms supported by cusolverDnPotrf

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

List of input arguments for `cusolverDnPotrf_bufferSize` and `cusolverDnPotrf`:

API of potrf

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	handle to the cuSolverDN library context.
`params`	`host`	`input`	structure with information collected by `cusolverDnSetAdvOptions`.
`uplo`	`host`	`input`	indicates if matrix `A` lower or upper part is stored, the other part is not referenced.
`n`	`host`	`input`	number of rows and columns of matrix `A`.

Parameter	Memory	In/out	Meaning
dataTypeA	host	in	data type of array A.
A	device	in/out	array of dimension lda * n with lda is not less than max(1,n).
lda	host	input	leading dimension of two-dimensional array used to store matrix A.
computeType	host	in	data type of computation.
pBuffer	device	in/out	Working space. Array of type void of size workspaceInBytes bytes.
workspaceInBytes	host	input	size in bytes of pBuffer, returned by cusolverDnPotrf_bufferSize.
info	device	output	if info = 0, the Cholesky factorization is successful. if info = -i, the i-th parameter is wrong (not counting handle). if info = i, the leading minor of order i is not positive definite.

The generic API has two different types, dataTypeA is data type of the matrix A, computeType is compute type of the operation. cusolverDnPotrf only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	SPOTRF
CUDA_R_64F	CUDA_R_64F	DPOTRF
CUDA_C_32F	CUDA_C_32F	CPOTRF
CUDA_C_64F	CUDA_C_64F	ZPOTRF

Status Returned

CUSOLVER_STATUS_SUCCESS	the operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	invalid parameters were passed (n<0 or lda<max(1,n)).
CUSOLVER_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	an internal operation failed.

2.4.2.3. cusolverDn<t>potrs()

```
cusolverStatus_t
cusolverDnSpotrs(cusolverDnHandle_t handle,
           cublasFillMode_t uplo,
           int n,
           int nrhs,
           const float *A,
           int lda,
```

```
            float *B,
            int ldb,
            int *devInfo);

cusolverStatus_t
cusolverDnDpotrs(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            int nrhs,
            const double *A,
            int lda,
            double *B,
            int ldb,
            int *devInfo);

cusolverStatus_t
cusolverDnCpotrs(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            int nrhs,
            const cuComplex *A,
            int lda,
            cuComplex *B,
            int ldb,
            int *devInfo);

cusolverStatus_t
cusolverDnZpotrs(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            int nrhs,
            const cuDoubleComplex *A,
            int lda,
            cuDoubleComplex *B,
            int ldb,
            int *devInfo);
```

This function solves a system of linear equations

$$A*X=B$$

where `A` is an `n×n` Hermitian matrix, only lower or upper part is meaningful. The input parameter `uplo` indicates which part of the matrix is used. The function would leave other part untouched.

The user has to call `potrf` first to factorize matrix `A`. If input parameter `uplo` is `CUBLAS_FILL_MODE_LOWER`, `A` is lower triangular Cholesky factor `L` correspoding to $A=L*L^H$. If input parameter `uplo` is `CUBLAS_FILL_MODE_UPPER`, `A` is upper triangular Cholesky factor `U` corresponding to $A=U^H*U$.

The operation is in-place, i.e. matrix `X` overwrites matrix `B` with the same leading dimension `ldb`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

API of potrs

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolveDN library context.
uplo	host	input	Indicates if matrix A lower or upper part is stored, the other part is not referenced.
n	host	input	Number of rows and columns of matrix A.
nrhs	host	input	Number of columns of matrix X and B.
A	device	input	<type> array of dimension lda * n with lda is not less than max(1,n). A is either lower cholesky factor L or upper Cholesky factor U.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
B	device	in/out	<type> array of dimension ldb * nrhs. ldb is not less than max(1,n). As an input, B is right hand side matrix. As an output, B is the solution matrix.
devInfo	device	output	If devInfo = 0, the Cholesky factorization is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, nrhs<0, lda<max(1,n) or ldb<max(1,n)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.4. cusolverDnPotrs()[DEPRECATED]

[[DEPRECATED]] use cusolverDnXpotrs() instead. The routine will be removed in the next major release.

```
cusolverStatus_t
cusolverDnPotrs(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cublasFillMode_t uplo,
    int64_t n,
    int64_t nrhs,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType dataTypeB,
```

```
    void *B,
    int64_t ldb,
    int *info)
```

This function solves a system of linear equations

$$A*X=B$$

where A is a n×n Hermitian matrix, only lower or upper part is meaningful using the generic API interface. The input parameter uplo indicates which part of the matrix is used. The function would leave other part untouched.

The user has to call cusolverDnPotrf first to factorize matrix A. If input parameter uplo is CUBLAS_FILL_MODE_LOWER, A is lower triangular Cholesky factor L correspoding to $A=L*L^H$. If input parameter uplo is CUBLAS_FILL_MODE_UPPER, A is upper triangular Cholesky factor U corresponding to $A=U^H*U$.

The operation is in-place, i.e. matrix X overwrites matrix B with the same leading dimension ldb.

If output parameter info = -i (less than zero), the i-th parameter is wrong (not counting handle).

Currently, cusolverDnPotrs supports only the default algorithm.

Table of algorithms supported by cusolverDnPotrs

CUSOLVER_ALG_0 or NULL	Default algorithm.

List of input arguments for cusolverDnPotrs:

API of potrs

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolveDN library context.
params	host	input	Structure with information collected by cusolverDnSetAdvOptions.
uplo	host	input	Indicates if matrix A lower or upper part is stored, the other part is not referenced.
n	host	input	Number of rows and columns of matrix A.
nrhs	host	input	Number of columns of matrix X and B.
dataTypeA	host	in	Data type of array A.
A	device	input	Array of dimension lda * n with lda is not less than max(1,n). A is either lower cholesky factor L or upper Cholesky factor U.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
dataTypeB	host	in	Data type of array B.

Parameter	Memory	In/out	Meaning
B	device	in/out	Array of dimension ldb * nrhs. ldb is not less than max(1,n). As an input, B is right hand side matrix. As an output, B is the solution matrix.
info	device	output	If info = 0, the Cholesky factorization is successful. if info = -i, the i-th parameter is wrong (not counting handle).

The generic API has two different types, dataTypeA is data type of the matrix A, dataTypeB is data type of the matrix B. cusolverDnPotrs only supports the following four combinations.

Valid combination of data type and compute type

dataTypeA	dataTypeB	Meaning
CUDA_R_32F	CUDA_R_32F	SPOTRS
CUDA_R_64F	CUDA_R_64F	DPOTRS
CUDA_C_32F	CUDA_C_32F	CPOTRS
CUDA_C_64F	CUDA_C_64F	ZPOTRS

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, nrhs<0, lda<max(1,n) or ldb<max(1,n)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.5. cusolverDn<t>potri()

These helper functions calculate the necessary size of work buffers.

```
cusolverStatus_t
cusolverDnSpotri_bufferSize(cusolverDnHandle_t handle,
                  cublasFillMode_t uplo,
                  int n,
                  float *A,
                  int lda,
                  int *Lwork );

cusolverStatus_t
cusolverDnDpotri_bufferSize(cusolveDnHandle_t handle,
                  cublasFillMode_t uplo,
                  int n,
                  double *A,
                  int lda,
                  int *Lwork );

cusolverStatus_t
```

```
cusolverDnCpotri_bufferSize(cusolverDnHandle_t handle,
                  cublasFillMode_t uplo,
                  int n,
                  cuComplex *A,
                  int lda,
                  int *Lwork );

cusolverStatus_t
cusolverDnZpotri_bufferSize(cusolverDnHandle_t handle,
                  cublasFillMode_t uplo,
                  int n,
                  cuDoubleComplex *A,
                  int lda,
                  int *Lwork);
```

The S and D data types are real valued single and double precision, respectively.

```

cusolverStatus_t
cusolverDnSpotri(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            float *A,
            int lda,
            float *Workspace,
            int Lwork,
            int *devInfo );

cusolverStatus_t
cusolverDnDpotri(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            double *A,
            int lda,
            double *Workspace,
            int Lwork,
            int *devInfo );
```

The C and Z data types are complex valued single and double precision, respectively.

```

cusolverStatus_t
cusolverDnCpotri(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            cuComplex *A,
            int lda,
            cuComplex *Workspace,
            int Lwork,
            int *devInfo );

cusolverStatus_t
cusolverDnZpotri(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            cuDoubleComplex *A,
            int lda,
            cuDoubleComplex *Workspace,
            int Lwork,
            int *devInfo );
```

This function computes the inverse of a positive-definite matrix `A` using the Cholesky factorization

$$A = L^* L^H = U^H * U$$

computed by `potrf()`.

`A` is a `n×n` matrix containing the triangular factor `L` or `U` computed by the Cholesky factorization. Only lower or upper part is meaningful and the input parameter `uplo` indicates which part of the matrix is used. The function would leave the other part untouched.

If the input parameter `uplo` is `CUBLAS_FILL_MODE_LOWER`, only lower triangular part of `A` is processed, and replaced the by lower triangular part of the inverse of `A`.

If the input parameter `uplo` is `CUBLAS_FILL_MODE_UPPER`, only upper triangular part of `A` is processed, and replaced by the upper triangular part of the inverse of `A`.

The user has to provide the working space which is pointed to by input parameter `Workspace`. The input parameter `Lwork` is the size of the working space, returned by `potri_bufferSize()`.

If the computation of the inverse fails, i.e. some leading minor of `L` or `U`, is null, the output parameter `devInfo` would indicate the smallest leading minor of `L` or `U` which is not positive definite.

If the output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting the handle).

API of potri

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`uplo`	`host`	`input`	Indicates if matrix `A` lower or upper part is stored, the other part is not referenced.
`n`	`host`	`input`	Number of rows and columns of matrix `A`.
`A`	`device`	`in/out`	<type> array of dimension `lda * n` where `lda` is not less than `max(1,n)`.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `A`.
`Workspace`	`device`	`in/out`	Working space, <type> array of size `Lwork`.
`Lwork`	`host`	`input`	Size of `Workspace`, returned by `potri_bufferSize`.
`devInfo`	`device`	`output`	If `devInfo = 0`, the computation of the inverse is successful. if `devInfo = -i`, the `i-th` parameter is wrong (not counting handle). if `devInfo = i`, the leading minor of order `i` is zero.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.6. cusolverDn<t>getrf()

These helper functions calculate the size of work buffers needed.

Please visit cuSOLVER Library Samples - getrf for a code example.

```
cusolverStatus_t
cusolverDnSgetrf_bufferSize(cusolverDnHandle_t handle,
                        int m,
                        int n,
                        float *A,
                        int lda,
                        int *Lwork );

cusolverStatus_t
cusolverDnDgetrf_bufferSize(cusolverDnHandle_t handle,
                        int m,
                        int n,
                        double *A,
                        int lda,
                        int *Lwork );

cusolverStatus_t
cusolverDnCgetrf_bufferSize(cusolverDnHandle_t handle,
                        int m,
                        int n,
                        cuComplex *A,
                        int lda,
                        int *Lwork );

cusolverStatus_t
cusolverDnZgetrf_bufferSize(cusolverDnHandle_t handle,
                        int m,
                        int n,
                        cuDoubleComplex *A,
                        int lda,
                        int *Lwork );
```

The S and D data types are real single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSgetrf(cusolverDnHandle_t handle,
            int m,
            int n,
            float *A,
            int lda,
            float *Workspace,
```

```
            int *devIpiv,
            int *devInfo );

cusolverStatus_t
cusolverDnDgetrf(cusolverDnHandle_t handle,
            int m,
            int n,
            double *A,
            int lda,
            double *Workspace,
            int *devIpiv,
            int *devInfo );
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCgetrf(cusolverDnHandle_t handle,
            int m,
            int n,
            cuComplex *A,
            int lda,
            cuComplex *Workspace,
            int *devIpiv,
            int *devInfo );

cusolverStatus_t
cusolverDnZgetrf(cusolverDnHandle_t handle,
            int m,
            int n,
            cuDoubleComplex *A,
            int lda,
            cuDoubleComplex *Workspace,
            int *devIpiv,
            int *devInfo );
```

This function computes the LU factorization of a `m×n` matrix

$$P*A = L*U$$

where `A` is a `m×n` matrix, `P` is a permutation matrix, `L` is a lower triangular matrix with unit diagonal, and `U` is an upper triangular matrix.

The user has to provide working space which is pointed by input parameter `Workspace`. The input parameter `Lwork` is size of the working space, and it is returned by `getrf_bufferSize()`.

If LU factorization failed, i.e. matrix `A` (`U`) is singular, The output parameter `devInfo=i` indicates `U(i,i) = 0`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

If `devIpiv` is null, no pivoting is performed. The factorization is `A=L*U`, which is not numerically stable.

No matter LU factorization failed or not, the output parameter `devIpiv` contains pivoting sequence, row `i` is interchanged with row `devIpiv(i)`.

The user can combine `getrf` and `getrs` to complete a linear solver.

Remark: `getrf` uses fastest implementation with large workspace of size `m*n`. The user can choose the legacy implementation with minimal workspace by `Getrf` and `cusolverDnSetAdvOptions(params, CUSOLVERDN_GETRF, CUSOLVER_ALG_1)`.

API of getrf

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`m`	`host`	`input`	Number of rows of matrix `A`.
`n`	`host`	`input`	Number of columns of matrix `A`.
`A`	`device`	`in/out`	<type> array of dimension `lda * n` with `lda` is not less than `max(1,m)`.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `A`.
`Workspace`	`device`	`in/out`	Working space, <type> array of size `Lwork`.
`devIpiv`	`device`	`output`	Array of size at least `min(m,n)`, containing pivot indices.
`devInfo`	`device`	`output`	If `devInfo = 0`, the LU factorization is successful. if `devInfo = -i`, the `i-th` parameter is wrong (not counting handle). if `devInfo = i`, the `U(i,i) = 0`.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.
`CUSOLVER_STATUS_INVALID_VALUE`	Invalid parameters were passed (`m,n<0` or `lda<max(1,m)`).
`CUSOLVER_STATUS_ARCH_MISMATCH`	The device only supports compute capability 2.0 and above.
`CUSOLVER_STATUS_INTERNAL_ERROR`	An internal operation failed.

2.4.2.7. cusolverDnGetrf()[DEPRECATED]

[[DEPRECATED]] use `cusolverDnXgetrf()` instead. The routine will be removed in the next major release.

The helper function below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnGetrf_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
```

```
    const void *A,
    int64_t lda,
    cudaDataType computeType,
    size_t *workspaceInBytes )
```

The following function:

```
cusolverStatus_t
cusolverDnGetrf(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    int64_t *ipiv,
    cudaDataType computeType,
    void *pBuffer,
    size_t workspaceInBytes,
    int *info )
```

computes the LU factorization of a `m×n` matrix

$$P*A=L*U$$

where `A` is an `m×n` matrix, `P` is a permutation matrix, `L` is a lower triangular matrix with unit diagonal, and `U` is an upper triangular matrix using the generic API interface.

If LU factorization failed, i.e. matrix `A` (`U`) is singular, The output parameter `info=i` indicates `U(i,i) = 0`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

If `ipiv` is null, no pivoting is performed. The factorization is `A=L*U`, which is not numerically stable.

No matter LU factorization failed or not, the output parameter `ipiv` contains pivoting sequence, row `i` is interchanged with row `ipiv(i)`.

The user has to provide working space which is pointed by input parameter `pBuffer`. The input parameter `workspaceInBytes` is size in bytes of the working space, and it is returned by `cusolverDnGetrf_bufferSize()`.

The user can combine `cusolverDnGetrf` and `cusolverDnGetrs` to complete a linear solver.

Currently, `cusolverDnGetrf` supports two algorithms. To select legacy implementation, the user has to call `cusolverDnSetAdvOptions`.

Table of algorithms supported by cusolverDnGetrf

`CUSOLVER_ALG_0` or `NULL`	Default algorithm. The fastest, requires a large workspace of `m*n` elements.
`CUSOLVER_ALG_1`	Legacy implementation

List of input arguments for `cusolverDnGetrf_bufferSize` and `cusolverDnGetrf`:

API of cusolverDnGetrf

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
params	host	input	Structure with information collected by `cusolverDnSetAdvOptions`.
m	host	input	number of rows of matrix `A`.
n	host	input	number of columns of matrix `A`.
dataTypeA	host	in	data type of array `A`.
A	device	in/out	<type> array of dimension `lda * n` with `lda` is not less than `max(1,m)`.
lda	host	input	leading dimension of two-dimensional array used to store matrix `A`.
ipiv	device	output	array of size at least `min(m,n)`, containing pivot indices.
computeType	host	in	data type of computation.
pBuffer	device	in/out	Working space. Array of type `void` of size `workspaceInBytes` bytes.
workspaceInBytes	host	input	size in bytes of `pBuffer`, returned by `cusolverDnGetrf_bufferSize`.
info	device	output	if `info = 0`, the LU factorization is successful. if `info = -i`, the `i-th` parameter is wrong (not counting handle). if `info = i`, the `U(i,i) = 0`.

The generic API has two different types, `dataTypeA` is data type of the matrix `A`, `computeType` is compute type of the operation. `cusolverDnGetrf` only supports the following four combinations.

valid combination of data type and compute type

DataTypeA	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	SGETRF
CUDA_R_64F	CUDA_R_64F	DGETRF
CUDA_C_32F	CUDA_C_32F	CGETRF
CUDA_C_64F	CUDA_C_64F	ZGETRF

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (`m,n<0` or `lda<max(1,m)`).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.8. cusolverDn<t>getrs()

Please visit cuSOLVER Library Samples - getrf for a code example.

```
cusolverStatus_t
cusolverDnSgetrs(cusolverDnHandle_t handle,
           cublasOperation_t trans,
           int n,
           int nrhs,
           const float *A,
           int lda,
           const int *devIpiv,
           float *B,
           int ldb,
           int *devInfo );

cusolverStatus_t
cusolverDnDgetrs(cusolverDnHandle_t handle,
           cublasOperation_t trans,
           int n,
           int nrhs,
           const double *A,
           int lda,
           const int *devIpiv,
           double *B,
           int ldb,
           int *devInfo );

cusolverStatus_t
cusolverDnCgetrs(cusolverDnHandle_t handle,
           cublasOperation_t trans,
           int n,
           int nrhs,
           const cuComplex *A,
           int lda,
           const int *devIpiv,
           cuComplex *B,
           int ldb,
           int *devInfo );

cusolverStatus_t
cusolverDnZgetrs(cusolverDnHandle_t handle,
           cublasOperation_t trans,
           int n,
           int nrhs,
           const cuDoubleComplex *A,
           int lda,
           const int *devIpiv,
           cuDoubleComplex *B,
           int ldb,
           int *devInfo );
```

This function solves a linear system of multiple right-hand sides

$$\mathrm{op(A)} * X = B$$

where A is an n×n matrix, and was LU-factored by getrf, that is, lower trianular part of A is L, and upper triangular part (including diagonal elements) of A is U. B is a n×nrhs right-hand side matrix.

The input parameter trans is defined by

$$\mathrm{op}(A) = \begin{cases} A & \text{if trans == CUBLAS\_OP\_N} \\ A^T & \text{if trans == CUBLAS\_OP\_T} \\ A^H & \text{if trans == CUBLAS\_OP\_C} \end{cases}$$

The input parameter devIpiv is an output of getrf. It contains pivot indices, which are used to permutate right-hand sides.

If output parameter devInfo = -i (less than zero), the i-th parameter is wrong (not counting handle).

The user can combine getrf and getrs to complete a linear solver.

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
trans	host	input	Operation op(A) that is non- or (conj.) transpose.
n	host	input	Number of rows and columns of matrix A.
nrhs	host	input	Number of right-hand sides.
A	device	input	<type> array of dimension lda * n with lda is not less than max(1,n).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
devIpiv	device	input	Array of size at least n, containing pivot indices.
B	device	output	<type> array of dimension ldb * nrhs with ldb is not less than max(1,n).
ldb	host	input	Leading dimension of two-dimensional array used to store matrix B.
devInfo	device	output	If devInfo = 0, the operation is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n) or ldb<max(1,n)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.9. cusolverDnGetrs()[DEPRECATED]

[[DEPRECATED]] use `cusolverDnXgetrs()` instead. The routine will be removed in the next major release.

```
cusolverStatus_t
cusolverDnGetrs(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cublasOperation_t trans,
    int64_t n,
    int64_t nrhs,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    const int64_t *ipiv,
    cudaDataType dataTypeB,
    void *B,
    int64_t ldb,
    int *info )
```

This function solves a linear system of multiple right-hand sides

$$\mathrm{op(A)} * X = B$$

where `A` is a `n×n` matrix, and was LU-factored by `cusolverDnGetrf`, that is, lower trianular part of A is `L`, and upper triangular part (including diagonal elements) of `A` is `U`. `B` is a `n×nrhs` right-hand side matrix using the generic API interface.

The input parameter `trans` is defined by

$$\mathrm{op}(A) = \begin{cases} A & \text{if trans == CUBLAS\_OP\_N} \\ A^T & \text{if trans == CUBLAS\_OP\_T} \\ A^H & \text{if trans == CUBLAS\_OP\_C} \end{cases}$$

The input parameter `ipiv` is an output of `cusolverDnGetrf`. It contains pivot indices, which are used to permutate right-hand sides.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

The user can combine `cusolverDnGetrf` and `cusolverDnGetrs` to complete a linear solver.

Currently, `cusolverDnGetrs` supports only the default algorithm.

Table of algorithms supported by cusolverDnGetrs

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

List of input arguments for `cusolverDnGetrss`:

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`params`	`host`	`input`	Structure with information collected by `cusolverDnSetAdvOptions`.

Parameter	Memory	In/out	Meaning
trans	host	input	Operation op(A) that is non- or (conj.) transpose.
n	host	input	Number of rows and columns of matrix A.
nrhs	host	input	Number of right-hand sides.
dataTypeA	host	in	Data type of array A.
A	device	input	Array of dimension lda * n with lda is not less than max(1,n).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
ipiv	device	input	Array of size at least n, containing pivot indices.
dataTypeB	host	in	Data type of array B.
B	device	output	<type> array of dimension ldb * nrhs with ldb is not less than max(1,n).
ldb	host	input	Leading dimension of two-dimensional array used to store matrix B.
info	device	output	If info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle).

The generic API has two different types, dataTypeA is data type of the matrix A and dataTypeB is data type of the matrix B. cusolverDnGetrs only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	dataTypeB	Meaning
CUDA_R_32F	CUDA_R_32F	SGETRS
CUDA_R_64F	CUDA_R_64F	DGETRS
CUDA_C_32F	CUDA_C_32F	CGETRS
CUDA_C_64F	CUDA_C_64F	ZGETRS

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n) or ldb<max(1,n)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.10. cusolverDn<t1><t2>gesv()

These functions are modelled after functions DSGESV and ZCGESV from LAPACK. They compute the solution of a system of linear equations with one or multiple right hand sides

using mixed precision iterative refinement techniques based on the LU factorization `Xgesv`. These functions are similar in term of functionalities to the full precision LU solver (`Xgesv`, where X denotes Z,C,D,S) but it uses lower precision internally in order to provide faster time to solution, from here cames the name mixed precision. Mixed precision iterative refinement techniques means that the solver compute an LU factorization in lower precision and then iteratively refine the solution to achieve the accuracy of the Inputs/Outputs datatype precision. The <t1> corresponds to the Inputs/Outputs datatype precision while <t2> represent the internal lower precision at which the factorization will be carried on.

$$A \times X = B$$

Where `A` is `n-by-n` matrix and `X` and `B` are `n-by-nrhs` matrices.

Functions API are designed to be as close as possible to LAPACK API to be considered as a quick and easy drop-in replacement. Parameters and behavior are mostly the same as LAPACK counterparts. Description of these functions and differences from LAPACK is given below. `<t1><t2>gesv()` functions are designated by two floating point precisions The <t1> corresponds to the main precision (e.g., Inputs/Outputs datatype precision) and the <t2> represent the internal lower precision at which the factorization will be carried on. `cusolver<t1><t2>gesv()` first attempts to factorize the matrix in lower precision and use this factorization within an iterative refinement procedure to obtain a solution with same normwise backward error as the main precision <t1>. If the approach fails to converge, then the method fallback to the main precision factorization and solve (Xgesv) such a way that there is always a good solution at the output of these functions. If <t2> is equal to <t1>, then it is not a mixed precision process but rather a full one precision factorisation, solve and refinement within the same main precision.

The iterative refinement process is stopped if

```
ITER > ITERMAX
```

or for all the RHS we have:

```
RNRM < SQRT(N)*XNRM*ANRM*EPS*BWDMAX
```

where

- ITER is the number of the current iteration in the iterative refinement process
- RNRM is the infinity-norm of the residual
- XNRM is the infinity-norm of the solution
- ANRM is the infinity-operator-norm of the matrix A
- EPS is the machine epsilon that matches LAPACK <t1>LAMCH('Epsilon')

The value ITERMAX and BWDMAX are fixed to 50 and 1.0 respectively.

The function returns value describes the results of the solving process. A CUSOLVER_STATUS_SUCCESS indicates that the function finished with success otherwise, it indicates if one of the API arguments is incorrect, or if the function did not finish with success. More details about the error will be in the `niters` and the `dinfo` API parameters. See their

description below for more details. User should provide the required workspace allocated on device memory. The amount of bytes required can be queried by calling the respective function `<t1><t2>gesv_bufferSize()`.

Note that in addition to the two mixed precision functions available in LAPACK (e.g., `dsgesv` and `zcgesv`), we provide a large set of mixed precision functions that include half, bfloat and tensorfloat as a lower precision as well as same precision functions (e.g., main and lowest precision are equal <t2> is equal to <t1>). The following table specifies which precisions will be used for which interface function.

Tensor Float (TF32), introduced with NVIDIA Ampere Architecture GPUs, is the most robust tensor core accelerated compute mode for the iterative refinement solver. It is able to solve the widest range of problems in HPC arising from different applications and provides up to 4X and 5X speedup for real and complex systems, respectively. On Volta and Turing architecture GPUs, half precision tensor core acceleration is recommended. In cases where the iterative refinement solver fails to converge to the desired accuracy (main precision, INOUT data precision), it is recommended to use main precision as internal lowest precision (i.e., `cusolverDn[DD,ZZ]gesv` for the FP64 case).

Table 3. **Supported combinations of floating point precisions for cusolver <t1><t2>gesv() functions**

Interface function	Main precision (matrix, rhs and solution datatype)	Lowest precision allowed to be used internally
cusolverDnZZgesv	cuDoubleComplex	double complex
cusolverDnZCgesv *has LAPACK counterparts	cuDoubleComplex	single complex
cusolverDnZKgesv	cuDoubleComplex	half complex
cusolverDnZEgesv	cuDoubleComplex	bfloat complex
cusolverDnZYgesv	cuDoubleComplex	tensorfloat complex
cusolverDnCCgesv	cuComplex	single complex
cusolverDnCKgesv	cuComplex	half complex
cusolverDnCEgesv	cuComplex	bfloat complex
cusolverDnCYgesv	cuComplex	tensorfloat complex
cusolverDnDDgesv	double	double
cusolverDnDSgesv *has LAPACK counterparts	double	single
cusolverDnDHgesv	double	half
cusolverDnDBgesv	double	bfloat
cusolverDnDXgesv	double	tensorfloat
cusolverDnSSgesv	float	single
cusolverDnSHgesv	float	half
cusolverDnSBgesv	float	bfloat
cusolverDnSXgesv	float	tensorfloat

cusolverDn<t1><t2>gesv_bufferSize() functions will return workspace buffer size in bytes required for the corresponding cusolverDn<t1><t2>gesv() function.

```
cusolverStatus_t
cusolverDnZZgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    cuDoubleComplex             *   dA,
    int                             ldda,
    int                         *   dipiv,
    cuDoubleComplex             *   dB,
    int                             lddb,
    cuDoubleComplex             *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnZCgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    cuDoubleComplex             *   dA,
    int                             ldda,
    int                         *   dipiv,
    cuDoubleComplex             *   dB,
    int                             lddb,
    cuDoubleComplex             *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnZKgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    cuDoubleComplex             *   dA,
    int                             ldda,
    int                         *   dipiv,
    cuDoubleComplex             *   dB,
    int                             lddb,
    cuDoubleComplex             *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnZEgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    cuDoubleComplex             *   dA,
    int                             ldda,
    int                         *   dipiv,
    cuDoubleComplex             *   dB,
    int                             lddb,
    cuDoubleComplex             *   dX,
    int                             lddx,
```

```
    void                          *   dwork,
    size_t                        *   lwork_bytes);

cusolverStatus_t
cusolverDnZYgesv_bufferSize(
    cusolverHandle_t                  handle,
    int                               n,
    int                               nrhs,
    cuDoubleComplex               *   dA,
    int                               ldda,
    int                           *   dipiv,
    cuDoubleComplex               *   dB,
    int                               lddb,
    cuDoubleComplex               *   dX,
    int                               lddx,
    void                          *   dwork,
    size_t                        *   lwork_bytes);

cusolverStatus_t
cusolverDnCCgesv_bufferSize(
    cusolverHandle_t                  handle,
    int                               n,
    int                               nrhs,
    cuComplex                     *   dA,
    int                               ldda,
    int                           *   dipiv,
    cuComplex                     *   dB,
    int                               lddb,
    cuComplex                     *   dX,
    int                               lddx,
    void                          *   dwork,
    size_t                        *   lwork_bytes);

cusolverStatus_t
cusolverDnCKgesv_bufferSize(
    cusolverHandle_t                  handle,
    int                               n,
    int                               nrhs,
    cuComplex                     *   dA,
    int                               ldda,
    int                           *   dipiv,
    cuComplex                     *   dB,
    int                               lddb,
    cuComplex                     *   dX,
    int                               lddx,
    void                          *   dwork,
    size_t                        *   lwork_bytes);

cusolverStatus_t
cusolverDnCEgesv_bufferSize(
    cusolverHandle_t                  handle,
    int                               n,
    int                               nrhs,
    cuComplex                     *   dA,
    int                               ldda,
    int                           *   dipiv,
    cuComplex                     *   dB,
    int                               lddb,
    cuComplex                     *   dX,
    int                               lddx,
    void                          *   dwork,
```

```
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnCYgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    cuComplex                   *   dA,
    int                             ldda,
    int                         *   dipiv,
    cuComplex                   *   dB,
    int                             lddb,
    cuComplex                   *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnDDgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    double                      *   dA,
    int                             ldda,
    int                         *   dipiv,
    double                      *   dB,
    int                             lddb,
    double                      *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnDSgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    double                      *   dA,
    int                             ldda,
    int                         *   dipiv,
    double                      *   dB,
    int                             lddb,
    double                      *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnDHgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    double                      *   dA,
    int                             ldda,
    int                         *   dipiv,
    double                      *   dB,
    int                             lddb,
    double                      *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);
```

```
cusolverStatus_t
cusolverDnDBgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    double                    *     dA,
    int                             ldda,
    int                       *     dipiv,
    double                    *     dB,
    int                             lddb,
    double                    *     dX,
    int                             lddx,
    void                      *     dwork,
    size_t                    *     lwork_bytes);

cusolverStatus_t
cusolverDnDXgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    double                    *     dA,
    int                             ldda,
    int                       *     dipiv,
    double                    *     dB,
    int                             lddb,
    double                    *     dX,
    int                             lddx,
    void                      *     dwork,
    size_t                    *     lwork_bytes);

cusolverStatus_t
cusolverDnSSgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    float                     *     dA,
    int                             ldda,
    int                       *     dipiv,
    float                     *     dB,
    int                             lddb,
    float                     *     dX,
    int                             lddx,
    void                      *     dwork,
    size_t                    *     lwork_bytes);

cusolverStatus_t
cusolverDnSHgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    float                     *     dA,
    int                             ldda,
    int                       *     dipiv,
    float                     *     dB,
    int                             lddb,
    float                     *     dX,
    int                             lddx,
    void                      *     dwork,
    size_t                    *     lwork_bytes);
```

```
cusolverStatus_t
cusolverDnSBgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    float                       *   dA,
    int                             ldda,
    int                         *   dipiv,
    float                       *   dB,
    int                             lddb,
    float                       *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnSXgesv_bufferSize(
    cusolverHandle_t                handle,
    int                             n,
    int                             nrhs,
    float                       *   dA,
    int                             ldda,
    int                         *   dipiv,
    float                       *   dB,
    int                             lddb,
    float                       *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);
```

Parameters of cusolverDn<T1><T2>gesv_bufferSize() functions

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cusolverDN library context.
n	host	input	Number of rows and columns of square matrix A. Should be non-negative.
nrhs	host	input	Number of right hand sides to solve. Should be non-negative.
dA	device	None	Matrix A with size n-by-n. Can be NULL.
ldda	host	input	Leading dimension of two-dimensional array used to store matrix A. lda >= n.
dipiv	device	None	Pivoting sequence. Not used and can be NULL.
dB	device	None	Set of right hand sides B of size n-by-nrhs. Can be NULL.
lddb	host	input	Leading dimension of two-dimensional array used to store matrix of right hand sides B. ldb >= n.
dX	device	None	Set of soultion vectors X of size n-by-nrhs. Can be NULL.

Parameter	Memory	In/out	Meaning
lddx	host	input	Leading dimension of two-dimensional array used to store matrix of solution vectors X. ldx >= n.
dwork	device	none	Pointer to device workspace. Not used and can be NULL.
lwork_bytes	host	output	Pointer to a variable where required size of temporary workspace in bytes will be stored. Can't be NULL.

```
cusolverStatus_t cusolverDnZZgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
        int                     ldda,
        int                 *   dipiv,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnZCgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
        int                     ldda,
        int                 *   dipiv,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnZKgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
        int                     ldda,
        int                 *   dipiv,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);
```

```
cusolverStatus_t cusolverDnZEgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
        int                     ldda,
        int                 *   dipiv,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnZYgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
        int                     ldda,
        int                 *   dipiv,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnCCgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        cuComplex           *   dA,
        int                     ldda,
        int                 *   dipiv,
        cuComplex           *   dB,
        int                     lddb,
        cuComplex           *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnCKgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        cuComplex           *   dA,
        int                     ldda,
        int                 *   dipiv,
        cuComplex           *   dB,
        int                     lddb,
        cuComplex           *   dX,
        int                     lddx,
        void                *   dWorkspace,
```

```
        size_t                  lwork_bytes,
        int                *    niter,
        int                *    dinfo);

cusolverStatus_t cusolverDnCEgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        cuComplex          *    dA,
        int                     ldda,
        int                *    dipiv,
        cuComplex          *    dB,
        int                     lddb,
        cuComplex          *    dX,
        int                     lddx,
        void               *    dWorkspace,
        size_t                  lwork_bytes,
        int                *    niter,
        int                *    dinfo);

cusolverStatus_t cusolverDnCYgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        cuComplex          *    dA,
        int                     ldda,
        int                *    dipiv,
        cuComplex          *    dB,
        int                     lddb,
        cuComplex          *    dX,
        int                     lddx,
        void               *    dWorkspace,
        size_t                  lwork_bytes,
        int                *    niter,
        int                *    dinfo);

cusolverStatus_t cusolverDnDDgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        double             *    dA,
        int                     ldda,
        int                *    dipiv,
        double             *    dB,
        int                     lddb,
        double             *    dX,
        int                     lddx,
        void               *    dWorkspace,
        size_t                  lwork_bytes,
        int                *    niter,
        int                *    dinfo);

cusolverStatus_t cusolverDnDSgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        double             *    dA,
        int                     ldda,
        int                *    dipiv,
        double             *    dB,
        int                     lddb,
```

```
        double                *   dX,
        int                       lddx,
        void                  *   dWorkspace,
        size_t                    lwork_bytes,
        int                   *   niter,
        int                   *   dinfo);

cusolverStatus_t cusolverDnDHgesv(
        cusolverDnHandle_t        handle,
        int                       n,
        int                       nrhs,
        double                *   dA,
        int                       ldda,
        int                   *   dipiv,
        double                *   dB,
        int                       lddb,
        double                *   dX,
        int                       lddx,
        void                  *   dWorkspace,
        size_t                    lwork_bytes,
        int                   *   niter,
        int                   *   dinfo);

cusolverStatus_t cusolverDnDBgesv(
        cusolverDnHandle_t        handle,
        int                       n,
        int                       nrhs,
        double                *   dA,
        int                       ldda,
        int                   *   dipiv,
        double                *   dB,
        int                       lddb,
        double                *   dX,
        int                       lddx,
        void                  *   dWorkspace,
        size_t                    lwork_bytes,
        int                   *   niter,
        int                   *   dinfo);

cusolverStatus_t cusolverDnDXgesv(
        cusolverDnHandle_t        handle,
        int                       n,
        int                       nrhs,
        double                *   dA,
        int                       ldda,
        int                   *   dipiv,
        double                *   dB,
        int                       lddb,
        double                *   dX,
        int                       lddx,
        void                  *   dWorkspace,
        size_t                    lwork_bytes,
        int                   *   niter,
        int                   *   dinfo);

cusolverStatus_t cusolverDnSSgesv(
        cusolverDnHandle_t        handle,
        int                       n,
        int                       nrhs,
        float                 *   dA,
        int                       ldda,
```

```
        int                 *   dipiv,
        float               *   dB,
        int                     lddb,
        float               *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnSHgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        float               *   dA,
        int                     ldda,
        int                 *   dipiv,
        float               *   dB,
        int                     lddb,
        float               *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnSBgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        float               *   dA,
        int                     ldda,
        int                 *   dipiv,
        float               *   dB,
        int                     lddb,
        float               *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnSXgesv(
        cusolverDnHandle_t      handle,
        int                     n,
        int                     nrhs,
        float               *   dA,
        int                     ldda,
        int                 *   dipiv,
        float               *   dB,
        int                     lddb,
        float               *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);
```

Parameters of cusolverDn<T1><T2>gesv() functions

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cusolverDN library context.
n	host	input	Number of rows and columns of square matrix A. Should be non-negative.
nrhs	host	input	Number of right hand sides to solve. Should be non-negative.
dA	device	in/out	Matrix A with size n-by-n. Can't be NULL. On return - unchanged if the iterative refinement process converged. If not - will contains the factorization of the matrix A in the main precision <T1> (A = P * L * U, where P - permutation matrix defined by vector ipiv, L and U - lower and upper triangular matrices).
ldda	host	input	Leading dimension of two-dimensional array used to store matrix A. lda >= n.
dipiv	device	output	Vector that defines permutation for the factorization - row i was interchanged with row ipiv[i]
dB	device	input	Set of right hand sides B of size n-by-nrhs . Can't be NULL.
lddb	host	input	Leading dimension of two-dimensional array used to store matrix of right hand sides B. ldb >= n.
dX	device	output	Set of soultion vectors X of size n-by-nrhs . Can't be NULL.
lddx	host	input	Leading dimension of two-dimensional array used to store matrix of solution vectors X. ldx >= n.
dWorkspace	device	input	Pointer to an allocated workspace in device memory of size lwork_bytes.
lwork_bytes	host	input	Size of the allocated device workspace. Should be at least what was returned by cusolverDn<T1><T2>gesv_bufferSize() function.
niters	host	output	If iter is ▶ <0 : iterative refinement has failed, main precision (Inputs/Outputs precision) factorization has been performed ▶ -1 : taking into account machine parameters, n, nrhs, it is a priori not worth working in lower precision ▶ -2 : overflow of an entry when moving from main to lower precision ▶ -3 : failure during the factorization

Parameter	Memory	In/out	Meaning
			▶ -5 : overflow occured during computation ▶ -50: solver stopped the iterative refinement after reaching maximum allowed iterations ▶ >0 : iter is a number of iterations solver perfromed to reach convergence criteria
dinfo	device	output	Status of the IRS solver on the return. If 0 - solve was successful. If dinfo = -i then i-th argument is not valid. If dinfo = i, then U(i,i) computed in main precision is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed, for example: ▶ n<0 ▶ lda<max(1,n) ▶ ldb<max(1,n) ▶ ldx<max(1,n)
CUSOLVER_STATUS_ARCH_MISMATCH	The IRS solver supports compute capability 7.0 and above. The lowest precision options CUSOLVER_[CR]_16BF and CUSOLVER_[CR]_TF32 are only available on compute capability 8.0 and above.
CUSOLVER_STATUS_INVALID_WORKSPACE	lwork_bytes is smaller than the required workspace.
CUSOLVER_STATUS_IRS_OUT_OF_RANGE	Numerical error related to niters <0, see niters description for more details.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal error occured, check the dinfo and the niters arguments for more details.

2.4.2.11. cusolverDnIRSXgesv()

This function is designed to perform same functionality as cusolverDn<T1><T2>gesv() functions, but wrapped in a more generic and expert interface that gives user more control to parametrize the function as well as it provides more informations on output. cusolverDnIRSXgesv() allows additional control of the solver parameters such as setting:

- the main precision (Inputs/Outputs precision) of the solver
- the lowest precision to be used internally by the solver
- the refinement solver type
- the maximum allowed number of iterations in the refinement phase
- the tolerance of the refinement solver
- the fallback to main precision
- and more

through the configuration parameters structure `gesv_irs_params` and its helper functions. For more details about what configuration can be set and its meaning please refer to all the functions in the cuSolverDN Helper Function Section that start with `cusolverDnIRSParamsxxxx()`. Moreover, `cusolverDnIRSXgesv()` provides additional informations on the output such as the convergence history (e.g., the residual norms) at each iteration and the number of iterations needed to converge. For more details about what informations can be retrieved and its meaning please refer to all the functions in the cuSolverDN Helper Function Section that start with `cusolverDnIRSInfosxxxx()`

The function returns value describes the results of the solving process. A CUSOLVER_STATUS_SUCCESS indicates that the function finished with success otherwise, it indicates if one of the API arguments is incorrect, or if the configurations of params/infos structure is incorrect or if the function did not finish with success. More details about the error can be found by checking the `niters` and the `dinfo` API parameters. See their description below for further details. User should provide the required workspace allocated on device for the `cusolverDnIRSXgesv()` function. The amount of bytes required for the function can be queried by calling the respective function `cusolverDnIRSXgesv_bufferSize()`. Note that, if the user would like a praticular configuration to be set via the params structure, it should be set before the call to `cusolverDnIRSXgesv_bufferSize()` to get the size of the required workspace.

Tensor Float (TF32), introduced with NVIDIA Ampere Architecture GPUs, is the most robust tensor core accelerated compute mode for the iterative refinement solver. It is able to solve the widest range of problems in HPC arising from different applications and provides up to 4X and 5X speedup for real and complex systems, respectively. On Volta and Turing architecture GPUs, half precision tensor core acceleration is recommended. In cases where the iterative refinement solver fails to converge to the desired accuracy (main precision, INOUT data precision), it is recommended to use main precision as internal lowest precision.

The following table provides all possible combinations values for the lowest precision corresponding to the Inputs/Outputs data type. Note that if the lowest precision matches the Inputs/Outputs datatype, then the main precision factorization will be used.

Table 4. **Supported Inputs/Outputs data type and lower precision for the IRS solver**

Inputs/Outputs Data Type (e.g., main precision)	Supported values for the lowest precision
CUSOLVER_C_64F	CUSOLVER_C_64F, CUSOLVER_C_32F, CUSOLVER_C_16F, CUSOLVER_C_16BF, CUSOLVER_C_TF32
CUSOLVER_C_32F	CUSOLVER_C_32F, CUSOLVER_C_16F, CUSOLVER_C_16BF, CUSOLVER_C_TF32
CUSOLVER_R_64F	CUSOLVER_R_64F, CUSOLVER_R_32F, CUSOLVER_R_16F, CUSOLVER_R_16BF, CUSOLVER_R_TF32
CUSOLVER_R_32F	CUSOLVER_R_32F, CUSOLVER_R_16F, CUSOLVER_R_16BF, CUSOLVER_R_TF32

The `cusolverDnIRSXgesv_bufferSize()` function returns the required workspace buffer size in bytes for the corresponding `cusolverDnXgesv()` call with the given `gesv_irs_params` configuration.

```
cusolverStatus_t
cusolverDnIRSXgesv_bufferSize(
        cusolverDnHandle_t           handle,
        cusolverDnIRSParams_t        gesv_irs_params,
        cusolver_int_t               n,
        cusolver_int_t               nrhs,
        size_t                 *     lwork_bytes);
```

Table 5. **Parameters of cusolverDnIRSXgesv_bufferSize() functions**

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cusolverDn library context.
params	host	input	Xgesv configuration parameters
n	host	input	Number of rows and columns of the square matrix A. Should be non-negative.
nrhs	host	input	Number of right hand sides to solve. Should be non-negative. Note that nrhs is limited to 1 if the selected IRS refinement solver is CUSOLVER_IRS_REFINE_GMRES, CUSOLVER_IRS_REFINE_GMRES_GMRES, CUSOLVER_IRS_REFINE_CLASSICAL_GMRES.
lwork_bytes	host	out	Pointer to a variable, where the required size in bytes, of the workspace will be stored after a call to cusolverDnIRSXgesv_bufferSize. Can't be NULL.

```
cusolverStatus_t cusolverDnIRSXgesv(
        cusolverDnHandle_t           handle,
        cusolverDnIRSParams_t        gesv_irs_params,
        cusolverDnIRSInfos_t         gesv_irs_infos,
        int                          n,
        int                          nrhs,
        void                    *    dA,
        int                          ldda,
        void                    *    dB,
        int                          lddb,
        void                    *    dX,
        int                          lddx,
        void                    *    dWorkspace,
        size_t                       lwork_bytes,
        int                     *    dinfo);
```

Table 6. **Parameters of cusolverDnIRSXgesv() functions**

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cusolverDn library context.
gesv_irs_params	host	input	Configuration parameters structure, can serve one or more calls to any IRS solver
gesv_irs_infos	host	in/out	Info structure, where information about a particular solve will be stored. The gesv_irs_infos structure correspond to a particular call. Thus different calls requires different gesv_irs_infos structure otherwise, it will be overwritten.
n	host	input	Number of rows and columns of square matrix A. Should be non-negative.
nrhs	host	input	Number of right hand sides to solve. Should be non-negative. Note that, nrhs is limited to 1 if the selected IRS refinement solver is CUSOLVER_IRS_REFINE_GMRES, CUSOLVER_IRS_REFINE_GMRES_GMRES, CUSOLVER_IRS_REFINE_CLASSICAL_GMRES.
dA	device	in/out	Matrix A with size n-by-n. Can't be NULL. On return - will contain the factorization of the matrix A in the main precision (A = P * L * U, where P - permutation matrix defined by vector ipiv, L and U - lower and upper triangular matrices) if the iterative refinement solver was set to CUSOLVER_IRS_REFINE_NONE and the lowest precision is equal to the main precision (Inputs/Ouputs datatype), or if the iterative refinement solver did not converge and the fallback to main precision was enabled (fallback enabled

Parameter	Memory	In/out	Meaning
			is the default setting); unchanged otherwise.
lddа	host	input	Leading dimension of two-dimensional array used to store matrix A. lda >= n.
dB	device	input	Set of right hand sides B of size n-by-nrhs . Can't be NULL.
lddb	host	input	Leading dimension of two-dimensional array used to store matrix of right hand sides B. ldb >= n.
dX	device	output	Set of soultion vectors X of size n-by-nrhs . Can't be NULL.
lddx	host	input	Leading dimension of two-dimensional array used to store matrix of solution vectors X. ldx >= n.
dWorkspace	device	input	Pointer to an allocated workspace in device memory of size lwork_bytes.
lwork_bytes	host	input	Size of the allocated device workspace. Should be at least what was returned by cusolverDnIRSXgesv_bufferSize() function
niters	host	output	If iter is ▶ <0 : iterative refinement has failed, main precision (Inputs/Outputs precision) factorization has been performed if fallback is enabled. ▶ -1 : taking into account machine parameters, n, nrhs, it is a priori not worth working in lower precision ▶ -2 : overflow of an entry when moving from main to lower precision ▶ -3 : failure during the factorization ▶ -5 : overflow occured during computation ▶ -maxiter: solver stopped the iterative refinement after reaching maximum allowed iterations. ▶ >0 : iter is a number of iterations solver perfromed to reach convergence criteria
dinfo	device	output	Status of the IRS solver on the return. If 0 - solve was successful. If dinfo = -i then i-th argument is not valid. If dinfo = i, then U(i,i) computed in main precision is exactly zero. The factorization has been

Parameter	Memory	In/out	Meaning
			completed, but the factor U is exactly singular, so the solution could not be computed.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed, for example: ▶ n<0 ▶ lda<max(1,n) ▶ ldb<max(1,n) ▶ ldx<max(1,n)
CUSOLVER_STATUS_ARCH_MISMATCH	The IRS solver supports compute capability 7.0 and above. The lowest precision options CUSOLVER_[CR]_16BF and CUSOLVER_[CR]_TF32 are only available on compute capability 8.0 and above.
CUSOLVER_STATUS_INVALID_WORKSPACE	lwork_bytes is smaller than the required workspace. Could happen if the users called cusolverDnIRSXgesv_bufferSize() function, then changed some of the configurations setting such as the lowest precision.
CUSOLVER_STATUS_IRS_OUT_OF_RANGE	Numerical error related to niters <0, see niters description for more details.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal error occured, check the dinfo and the niters arguments for more details.
CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The configuration parameter gesv_irs_params structure was not created.
CUSOLVER_STATUS_IRS_PARAMS_INVALID	One of the configuration parameter in the gesv_irs_params structure is not valid.
CUSOLVER_STATUS_IRS_PARAMS_INVALID_PREC	The main and/or the lowest precision configuration parameter in the gesv_irs_params structure is not valid, check the table above for the supported combinations.
CUSOLVER_STATUS_IRS_PARAMS_INVALID_MAXITER	The maxiter configuration parameter in the gesv_irs_params structure is not valid.
CUSOLVER_STATUS_IRS_PARAMS_INVALID_REFINE	The refinement solver configuration parameter in the gesv_irs_params structure is not valid.
CUSOLVER_STATUS_IRS_NOT_SUPPORTED	One of the configuration parameter in the gesv_irs_params structure is not supported. For example if nrhs >1, and refinement solver was set to CUSOLVER_IRS_REFINE_GMRES.

CUSOLVER_STATUS_IRS_INFOS_NOT_INITIALIZED	The information structure gesv_irs_infos was not created.
CUSOLVER_STATUS_ALLOC_FAILED	CPU memory allocation failed, most likely during the allocation of the residual array that store the residual norms.

2.4.2.12. cusolverDn<t>geqrf()

These helper functions calculate the size of work buffers needed.

```
cusolverStatus_t
cusolverDnSgeqrf_bufferSize(cusolverDnHandle_t handle,
                        int m,
                        int n,
                        float *A,
                        int lda,
                        int *Lwork );

cusolverStatus_t
cusolverDnDgeqrf_bufferSize(cusolverDnHandle_t handle,
                        int m,
                        int n,
                        double *A,
                        int lda,
                        int *Lwork );

cusolverStatus_t
cusolverDnCgeqrf_bufferSize(cusolverDnHandle_t handle,
                        int m,
                        int n,
                        cuComplex *A,
                        int lda,
                        int *Lwork );

cusolverStatus_t
cusolverDnZgeqrf_bufferSize(cusolverDnHandle_t handle,
                        int m,
                        int n,
                        cuDoubleComplex *A,
                        int lda,
                        int *Lwork );
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSgeqrf(cusolverDnHandle_t handle,
            int m,
            int n,
            float *A,
            int lda,
            float *TAU,
            float *Workspace,
            int Lwork,
            int *devInfo );

cusolverStatus_t
cusolverDnDgeqrf(cusolverDnHandle_t handle,
            int m,
            int n,
```

```
            double *A,
            int lda,
            double *TAU,
            double *Workspace,
            int Lwork,
            int *devInfo );
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCgeqrf(cusolverDnHandle_t handle,
            int m,
            int n,
            cuComplex *A,
            int lda,
            cuComplex *TAU,
            cuComplex *Workspace,
            int Lwork,
            int *devInfo );

cusolverStatus_t
cusolverDnZgeqrf(cusolverDnHandle_t handle,
            int m,
            int n,
            cuDoubleComplex *A,
            int lda,
            cuDoubleComplex *TAU,
            cuDoubleComplex *Workspace,
            int Lwork,
            int *devInfo );
```

This function computes the QR factorization of a `m×n` matrix

$$A = Q * R$$

where `A` is an `m×n` matrix, `Q` is an `m×n` matrix, and `R` is a `n×n` upper triangular matrix.

The user has to provide working space which is pointed by input parameter `Workspace`. The input parameter `Lwork` is size of the working space, and it is returned by `geqrf_bufferSize()`.

The matrix `R` is overwritten in upper triangular part of `A`, including diagonal elements.

The matrix `Q` is not formed explicitly, instead, a sequence of householder vectors are stored in lower triangular part of `A`. The leading nonzero element of householder vector is assumed to be 1 such that output parameter `TAU` contains the scaling factor `τ`. If `v` is original householder vector, `q` is the new householder vector corresponding to `τ`, satisying the following relation

$$I - 2 * v * v^H = I - \tau * q * q^H$$

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

API of geqrf

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.

Parameter	Memory	In/out	Meaning
m	host	input	Number of rows of matrix A.
n	host	input	Number of columns of matrix A.
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,m).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
TAU	device	output	<type> array of dimension at least min(m,n).
Workspace	device	in/out	Working space, <type> array of size Lwork.
Lwork	host	input	Size of working array Workspace.
devInfo	device	output	If devInfo = 0, the LU factorization is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or lda<max(1,m)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.13. cusolverDnGeqrf()[DEPRECATED]

[[DEPRECATED]] use cusolverDnXgeqrf() instead. The routine will be removed in the next major release.

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnGeqrf_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType dataTypeTau,
    const void *tau,
    cudaDataType computeType,
    size_t *workspaceInBytes )
```

The following routine:

```
cusolverStatus_t
cusolverDnGeqrf(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    cudaDataType dataTypeTau,
    void *tau,
    cudaDataType computeType,
    void *pBuffer,
    size_t workspaceInBytes,
    int *info )
```

computes the QR factorization of an `m×n` matrix

$$A = Q * R$$

where `A` is a `m×n` matrix, `Q` is an `m×n` matrix, and `R` is an `n×n` upper triangular matrix using the generic API interface.

The user has to provide working space which is pointed by input parameter `pBuffer`. The input parameter `workspaceInBytes` is size in bytes of the working space, and it is returned by `cusolverDnGeqrf_bufferSize()`.

The matrix `R` is overwritten in upper triangular part of `A`, including diagonal elements.

The matrix `Q` is not formed explicitly, instead, a sequence of householder vectors are stored in lower triangular part of `A`. The leading nonzero element of householder vector is assumed to be 1 such that output parameter `TAU` contains the scaling factor `τ`. If `v` is original householder vector, `q` is the new householder vector corresponding to `τ`, satisying the following relation

$$I - 2 * v * v^H = I - \tau * q * q^H$$

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

Currently, `cusolverDnGeqrf` supports only the default algorithm.

Table of algorithms supported by cusolverDnGeqrf

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

List of input arguments for `cusolverDnGeqrf_bufferSize` and `cusolverDnGeqrf`:

API of geqrf

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`params`	`host`	`input`	Structure with information collected by `cusolverDnSetAdvOptions`.
`m`	`host`	`input`	Number of rows of matrix `A`.

Parameter	Memory	In/out	Meaning
n	host	input	Number of columns of matrix A.
dataTypeA	host	in	Data type of array A.
A	device	in/out	Array of dimension lda * n with lda is not less than max(1,m).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
TAU	device	output	Array of dimension at least min(m,n).
computeType	host	in	Data type of computation.
pBuffer	device	in/out	Working space. Array of type void of size workspaceInBytes bytes.
workspaceInBytes	host	input	Size in bytes of working array pBuffer.
info	device	output	If info = 0, the LU factorization is successful. if info = -i, the i-th parameter is wrong (not counting handle).

The generic API has two different types, dataTypeA is data type of the matrix A and array tau and computeType is compute type of the operation. cusolverDnGeqrf only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	SGEQRF
CUDA_R_64F	CUDA_R_64F	DGEQRF
CUDA_C_32F	CUDA_C_32F	CGEQRF
CUDA_C_64F	CUDA_C_64F	ZGEQRF

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or lda<max(1,m)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.14. cusolverDn<t1><t2>gels()

These functions compute the solution of a system of linear equations with one or multiple right hand sides using mixed precision iterative refinement techniques based on the QR factorization Xgels. These functions are similar in term of functionalities to the full precision LAPACK QR (least squares) solver (Xgels, where X denotes Z,C,D,S) but it uses lower precision internally in order to provide faster time to solution, from here cames the name mixed precision. Mixed precision iterative refinement techniques means that the solver compute

an QR factorization in lower precision and then iteratively refine the solution to achieve the accuracy of the Inputs/Outputs datatype precision. The <t1> corresponds to the Inputs/Outputs datatype precision while <t2> represent the internal lower precision at which the factorization will be carried on.

$$A \times X = B$$

Where `A` is `m-by-n` matrix and `X` is `n-by-nrhs` and `B` is `m-by-nrhs` matrices.

Functions API are designed to be as close as possible to LAPACK API to be considered as a quick and easy drop-in replacement. Description of these functions is given below. `<t1><t2>gels()` functions are designated by two floating point precisions The `<t1>` corresponds to the main precision (e.g., Inputs/Outputs datatype precision) and the `<t2>` represent the internal lower precision at which the factorization will be carried on. `cusolver<t1><t2>gels()` first attempts to factorize the matrix in lower precision and use this factorization within an iterative refinement procedure to obtain a solution with same normwise backward error as the main precision `<t1>`. If the approach fails to converge, then the method fallback to the main precision factorization and solve (Xgels) such a way that there is always a good solution at the output of these functions. If `<t2>` is equal to `<t1>`, then it is not a mixed precision process but rather a full one precision factorisation, solve and refinement within the same main precision.

The iterative refinement process is stopped if:

```
ITER > ITERMAX
```

or for all the RHS we have:

```
RNRM < SQRT(N)*XNRM*ANRM*EPS*BWDMAX
```

where

- ITER is the number of the current iteration in the iterative refinement process
- RNRM is the infinity-norm of the residual
- XNRM is the infinity-norm of the solution
- ANRM is the infinity-operator-norm of the matrix A
- EPS is the machine epsilon that matches `LAPACK <t1>LAMCH('Epsilon')`

The values ITERMAX and BWDMAX are fixed to 50 and 1.0 respectively.

The function returns value describes the results of the solving process. A CUSOLVER_STATUS_SUCCESS indicates that the function finished with success otherwise, it indicates if one of the API arguments is incorrect, or if the function did not finish with success. More details about the error will be in the `niters` and the `dinfo` API parameters. See their description below for more details. User should provide the required workspace allocated on device memory. The amount of bytes required can be queried by calling the respective function `<t1><t2>gels_bufferSize()`.

We provide a large set of mixed precision functions that include half, bfloat and tensorfloat as a lower precision as well as same precision functions (e.g., main and lowest precision are equal <t2> is equal to <t1>). The following table specifies which precisions will be used for which interface function:

Tensor Float (TF32), introduced with NVIDIA Ampere Architecture GPUs, is the most robust tensor core accelerated compute mode for the iterative refinement solver. It is able to solve the widest range of problems in HPC arising from different applications and provides up to 4X and 5X speedup for real and complex systems, respectively. On Volta and Turing architecture GPUs, half precision tensor core acceleration is recommended. In cases where the iterative refinement solver fails to converge to the desired accuracy (main precision, INOUT data precision), it is recommended to use main precision as internal lowest precision (i.e., `cusolverDn[DD,ZZ]gels` for the FP64 case).

Table 7. **Supported combinations of floating point precisions for cusolver <t1><t2>gels() functions**

Interface function	Main precision (matrix, rhs and solution datatype)	Lowest precision allowed to be used internally
cusolverDnZZgels	cuDoubleComplex	double complex
cusolverDnZCgels	cuDoubleComplex	single complex
cusolverDnZKgels	cuDoubleComplex	half complex
cusolverDnZEgels	cuDoubleComplex	bfloat complex
cusolverDnZYgels	cuDoubleComplex	tensorfloat complex
cusolverDnCCgels	cuComplex	single complex
cusolverDnCKgels	cuComplex	half complex
cusolverDnCEgels	cuComplex	bfloat complex
cusolverDnCYgels	cuComplex	tensorfloat complex
cusolverDnDDgels	double	double
cusolverDnDSgels	double	single
cusolverDnDHgels	double	half
cusolverDnDBgels	double	bfloat
cusolverDnDXgels	double	tensorfloat
cusolverDnSSgels	float	single
cusolverDnSHgels	float	half
cusolverDnSBgels	float	bfloat
cusolverDnSXgels	float	tensorfloat

`cusolverDn<t1><t2>gels_bufferSize()` functions will return workspace buffer size in bytes required for the corresponding `cusolverDn<t1><t2>gels()` function.

```
cusolverStatus_t
cusolverDnZZgels_bufferSize(
    cusolverHandle_t                handle,
```

```
    int                              m,
    int                              n,
    int                              nrhs,
    cuDoubleComplex            *     dA,
    int                              ldda,
    cuDoubleComplex            *     dB,
    int                              lddb,
    cuDoubleComplex            *     dX,
    int                              lddx,
    void                       *     dwork,
    size_t                     *     lwork_bytes);

cusolverStatus_t
cusolverDnZCgels_bufferSize(
    cusolverHandle_t                 handle,
    int                              m,
    int                              n,
    int                              nrhs,
    cuDoubleComplex            *     dA,
    int                              ldda,
    cuDoubleComplex            *     dB,
    int                              lddb,
    cuDoubleComplex            *     dX,
    int                              lddx,
    void                       *     dwork,
    size_t                     *     lwork_bytes);

cusolverStatus_t
cusolverDnZKgels_bufferSize(
    cusolverHandle_t                 handle,
    int                              m,
    int                              n,
    int                              nrhs,
    cuDoubleComplex            *     dA,
    int                              ldda,
    cuDoubleComplex            *     dB,
    int                              lddb,
    cuDoubleComplex            *     dX,
    int                              lddx,
    void                       *     dwork,
    size_t                     *     lwork_bytes);

cusolverStatus_t
cusolverDnZEgels_bufferSize(
    cusolverHandle_t                 handle,
    int                              m,
    int                              n,
    int                              nrhs,
    cuDoubleComplex            *     dA,
    int                              ldda,
    cuDoubleComplex            *     dB,
    int                              lddb,
    cuDoubleComplex            *     dX,
    int                              lddx,
    void                       *     dwork,
    size_t                     *     lwork_bytes);

cusolverStatus_t
cusolverDnZYgels_bufferSize(
    cusolverHandle_t                 handle,
    int                              m,
```

```
    int                                 n,
    int                                 nrhs,
    cuDoubleComplex             *       dA,
    int                                 ldda,
    cuDoubleComplex             *       dB,
    int                                 lddb,
    cuDoubleComplex             *       dX,
    int                                 lddx,
    void                        *       dwork,
    size_t                      *       lwork_bytes);

cusolverStatus_t
cusolverDnCCgels_bufferSize(
    cusolverHandle_t                    handle,
    int                                 m,
    int                                 n,
    int                                 nrhs,
    cuComplex                   *       dA,
    int                                 ldda,
    cuComplex                   *       dB,
    int                                 lddb,
    cuComplex                   *       dX,
    int                                 lddx,
    void                        *       dwork,
    size_t                      *       lwork_bytes);

cusolverStatus_t
cusolverDnCKgels_bufferSize(
    cusolverHandle_t                    handle,
    int                                 m,
    int                                 n,
    int                                 nrhs,
    cuComplex                   *       dA,
    int                                 ldda,
    cuComplex                   *       dB,
    int                                 lddb,
    cuComplex                   *       dX,
    int                                 lddx,
    void                        *       dwork,
    size_t                      *       lwork_bytes);

cusolverStatus_t
cusolverDnCEgels_bufferSize(
    cusolverHandle_t                    handle,
    int                                 m,
    int                                 n,
    int                                 nrhs,
    cuComplex                   *       dA,
    int                                 ldda,
    cuComplex                   *       dB,
    int                                 lddb,
    cuComplex                   *       dX,
    int                                 lddx,
    void                        *       dwork,
    size_t                      *       lwork_bytes);

cusolverStatus_t
cusolverDnCYgels_bufferSize(
    cusolverHandle_t                    handle,
    int                                 m,
    int                                 n,
```

```
    int                              nrhs,
    cuComplex                  *     dA,
    int                              ldda,
    cuComplex                  *     dB,
    int                              lddb,
    cuComplex                  *     dX,
    int                              lddx,
    void                       *     dwork,
    size_t                     *     lwork_bytes);

cusolverStatus_t
cusolverDnDDgels_bufferSize(
    cusolverHandle_t                 handle,
    int                              m,
    int                              n,
    int                              nrhs,
    double                     *     dA,
    int                              ldda,
    double                     *     dB,
    int                              lddb,
    double                     *     dX,
    int                              lddx,
    void                       *     dwork,
    size_t                     *     lwork_bytes);

cusolverStatus_t
cusolverDnDSgels_bufferSize(
    cusolverHandle_t                 handle,
    int                              m,
    int                              n,
    int                              nrhs,
    double                     *     dA,
    int                              ldda,
    double                     *     dB,
    int                              lddb,
    double                     *     dX,
    int                              lddx,
    void                       *     dwork,
    size_t                     *     lwork_bytes);

cusolverStatus_t
cusolverDnDHgels_bufferSize(
    cusolverHandle_t                 handle,
    int                              m,
    int                              n,
    int                              nrhs,
    double                     *     dA,
    int                              ldda,
    double                     *     dB,
    int                              lddb,
    double                     *     dX,
    int                              lddx,
    void                       *     dwork,
    size_t                     *     lwork_bytes);

cusolverStatus_t
cusolverDnDBgels_bufferSize(
    cusolverHandle_t                 handle,
    int                              m,
    int                              n,
    int                              nrhs,
```

```
    double                      *   dA,
    int                             ldda,
    double                      *   dB,
    int                             lddb,
    double                      *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnDXgels_bufferSize(
    cusolverHandle_t                handle,
    int                             m,
    int                             n,
    int                             nrhs,
    double                      *   dA,
    int                             ldda,
    double                      *   dB,
    int                             lddb,
    double                      *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnSSgels_bufferSize(
    cusolverHandle_t                handle,
    int                             m,
    int                             n,
    int                             nrhs,
    float                       *   dA,
    int                             ldda,
    float                       *   dB,
    int                             lddb,
    float                       *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnSHgels_bufferSize(
    cusolverHandle_t                handle,
    int                             m,
    int                             n,
    int                             nrhs,
    float                       *   dA,
    int                             ldda,
    float                       *   dB,
    int                             lddb,
    float                       *   dX,
    int                             lddx,
    void                        *   dwork,
    size_t                      *   lwork_bytes);

cusolverStatus_t
cusolverDnSBgels_bufferSize(
    cusolverHandle_t                handle,
    int                             m,
    int                             n,
    int                             nrhs,
    float                       *   dA,
```

```
    int                                 lddа,
    float                         *     dB,
    int                                 lddb,
    float                         *     dX,
    int                                 lddx,
    void                          *     dwork,
    size_t                        *     lwork_bytes);

cusolverStatus_t
cusolverDnSXgels_bufferSize(
    cusolverHandle_t                    handle,
    int                                 m,
    int                                 n,
    int                                 nrhs,
    float                         *     dA,
    int                                 lddа,
    float                         *     dB,
    int                                 lddb,
    float                         *     dX,
    int                                 lddx,
    void                          *     dwork,
    size_t                        *     lwork_bytes);
```

Table 8. **Parameters of cusolverDn<T1><T2>gels_bufferSize() functions**

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cusolverDN library context.
m	host	input	Number of rows of the matrix A. Should be non-negative and n<=m
n	host	input	Number of columns of the matrix A. Should be non-negative and n<=m.
nrhs	host	input	Number of right hand sides to solve. Should be non-negative.
dA	device	None	Matrix A with size m-by-n. Can be NULL.
lddа	host	input	Leading dimension of two-dimensional array used to store matrix A. lddа >= m.
dB	device	None	Set of right hand sides B of size m-by-nrhs. Can be NULL.
lddb	host	input	Leading dimension of two-dimensional array used to store matrix of right hand sides B. lddb >= max(1,m).
dX	device	None	Set of soultion vectors X of size n-by-nrhs. Can be NULL.
lddx	host	input	Leading dimension of two-dimensional array used to store matrix of solution vectors X. lddx >= max(1,n).
dwork	device	none	Pointer to device workspace. Not used and can be NULL.

Parameter	Memory	In/out	Meaning
lwork_bytes	host	output	Pointer to a variable where required size of temporary workspace in bytes will be stored. Can't be NULL.

```
cusolverStatus_t cusolverDnZZgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
        int                     ldda,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnZCgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
        int                     ldda,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnZKgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
        int                     ldda,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnZEgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
```

```
        int                     lddа,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnZYgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        cuDoubleComplex     *   dA,
        int                     lddа,
        cuDoubleComplex     *   dB,
        int                     lddb,
        cuDoubleComplex     *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnCCgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        cuComplex           *   dA,
        int                     lddа,
        cuComplex           *   dB,
        int                     lddb,
        cuComplex           *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnCKgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        cuComplex           *   dA,
        int                     lddа,
        cuComplex           *   dB,
        int                     lddb,
        cuComplex           *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnCEgels(
        cusolverDnHandle_t      handle,
        int                     m,
```

```
        int                     n,
        int                     nrhs,
        cuComplex           *   dA,
        int                     ldda,
        cuComplex           *   dB,
        int                     lddb,
        cuComplex           *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnCYgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        cuComplex           *   dA,
        int                     ldda,
        cuComplex           *   dB,
        int                     lddb,
        cuComplex           *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnDDgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        double              *   dA,
        int                     ldda,
        double              *   dB,
        int                     lddb,
        double              *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnDSgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        double              *   dA,
        int                     ldda,
        double              *   dB,
        int                     lddb,
        double              *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);
```

```
cusolverStatus_t cusolverDnDHgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        double              *   dA,
        int                     ldda,
        double              *   dB,
        int                     lddb,
        double              *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnDBgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        double              *   dA,
        int                     ldda,
        double              *   dB,
        int                     lddb,
        double              *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnDXgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        double              *   dA,
        int                     ldda,
        double              *   dB,
        int                     lddb,
        double              *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnSSgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        float               *   dA,
        int                     ldda,
        float               *   dB,
        int                     lddb,
        float               *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
```

```
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnSHgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        float               *   dA,
        int                     ldda,
        float               *   dB,
        int                     lddb,
        float               *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnSBgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        float               *   dA,
        int                     ldda,
        float               *   dB,
        int                     lddb,
        float               *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);

cusolverStatus_t cusolverDnSXgels(
        cusolverDnHandle_t      handle,
        int                     m,
        int                     n,
        int                     nrhs,
        float               *   dA,
        int                     ldda,
        float               *   dB,
        int                     lddb,
        float               *   dX,
        int                     lddx,
        void                *   dWorkspace,
        size_t                  lwork_bytes,
        int                 *   niter,
        int                 *   dinfo);
```

Table 9. **Parameters of cusolverDn<T1><T2>gels() functions**

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cusolverDN library context.
m	host	input	Number of rows of the matrix A. Should be non-negative and n<=m

Parameter	Memory	In/out	Meaning
n	host	input	Number of columns of the matrix A. Should be non-negative and n<=m.
nrhs	host	input	Number of right hand sides to solve. Should be non-negative.
dA	device	in/out	Matrix A with size m-by-n. Can't be NULL. On return - unchanged if the lowest precision is not equal to the main precision and the iterative refinement solver converged, - garbage otherwise.
ldda	host	input	Leading dimension of two-dimensional array used to store matrix A. ldda >= m.
dB	device	input	Set of right hand sides B of size m-by-nrhs . Can't be NULL.
lddb	host	input	Leading dimension of two-dimensional array used to store matrix of right hand sides B. lddb >= max(1,m).
dX	device	output	Set of soultion vectors X of size n-by-nrhs . Can't be NULL.
lddx	host	input	Leading dimension of two-dimensional array used to store matrix of solution vectors X. lddx >= max(1,n).
dWorkspace	device	input	Pointer to an allocated workspace in device memory of size lwork_bytes.
lwork_bytes	host	input	Size of the allocated device workspace. Should be at least what was returned by cusolverDn<T1><T2>gels_bufferSize() function
niters	host	output	If iter is ▶ <0 : iterative refinement has failed, main precision (Inputs/Outputs precision) factorization has been performed. ▶ -1 : taking into account machine parameters, n, nrhs, it is a priori not worth working in lower precision ▶ -2 : overflow of an entry when moving from main to lower precision ▶ -3 : failure during the factorization ▶ -5 : overflow occured during computation ▶ -50: solver stopped the iterative refinement after reaching maximum allowed iterations.

Parameter	Memory	In/out	Meaning
			▶ >0 : iter is a number of iterations solver perfromed to reach convergence criteria
dinfo	device	output	Status of the IRS solver on the return. If 0 - solve was successful. If dinfo = -i then i-th argument is not valid.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed, for example: ▶ n<0 ▶ ldda<max(1,m) ▶ lddb<max(1,m) ▶ lddx<max(1,n)
CUSOLVER_STATUS_ARCH_MISMATCH	The IRS solver supports compute capability 7.0 and above. The lowest precision options CUSOLVER_[CR]_16BF and CUSOLVER_[CR]_TF32 are only available on compute capability 8.0 and above.
CUSOLVER_STATUS_INVALID_WORKSPACE	lwork_bytes is smaller than the required workspace.
CUSOLVER_STATUS_IRS_OUT_OF_RANGE	Numerical error related to niters <0, see niters description for more details.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal error occurred; check the dinfo and the niters arguments for more details.

2.4.2.15. cusolverDnIRSXgels()

This function is designed to perform same functionality as cusolverDn<T1><T2>gels() functions, but wrapped in a more generic and expert interface that gives user more control to parametrize the function as well as it provides more informations on output. cusolverDnIRSXgels() allows additional control of the solver parameters such as setting:

- the main precision (Inputs/Outputs precision) of the solver,
- the lowest precision to be used internally by the solver,
- the refinement solver type
- the maximum allowed number of iterations in the refinement phase
- the tolerance of the refinement solver

- the fallback to main precision
- and others

through the configuration parameters structure `gels_irs_params` and its helper functions. For more details about what configuration can be set and its meaning please refer to all the functions in the cuSolverDN Helper Function Section that start with `cusolverDnIRSParamsxxxx()`. Moreover, `cusolverDnIRSXgels()` provides additional informations on the output such as the convergence history (e.g., the residual norms) at each iteration and the number of iterations needed to converge. For more details about what informations can be retrieved and its meaning please refer to all the functions in the cuSolverDN Helper Function Section that start with `cusolverDnIRSInfosxxxx()`.

The function returns value describes the results of the solving process. A CUSOLVER_STATUS_SUCCESS indicates that the function finished with success otherwise, it indicates if one of the API arguments is incorrect, or if the configurations of params/infos structure is incorrect or if the function did not finish with success. More details about the error can be found by checking the `niters` and the `dinfo` API parameters. See their description below for further details. Users should provide the required workspace allocated on device for the `cusolverDnIRSXgels()` function. The amount of bytes required for the function can be queried by calling the respective function `cusolverDnIRSXgels_bufferSize()`. Note that, if the user would like a praticular configuration to be set via the params structure, it should be set before the call to `cusolverDnIRSXgels_bufferSize()` to get the size of the required workspace.

The following table provides all possible combinations values for the lowest precision corresponding to the Inputs/Outputs data type. Note that if the lowest precision matches the Inputs/Outputs datatype, then main precision factorization will be used

Tensor Float (TF32), introduced with NVIDIA Ampere Architecture GPUs, is the most robust tensor core accelerated compute mode for the iterative refinement solver. It is able to solve the widest range of problems in HPC arising from different applications and provides up to 4X and 5X speedup for real and complex systems, respectively. On Volta and Turing architecture GPUs, half precision tensor core acceleration is recommended. In cases where the iterative refinement solver fails to converge to the desired accuracy (main precision, INOUT data precision), it is recommended to use main precision as internal lowest precision.

Table 10. **Supported Inputs/Outputs data type and lower precision for the IRS solver**

Inputs/Outputs Data Type (e.g., main precision)	Supported values for the lowest precision
`CUSOLVER_C_64F`	`CUSOLVER_C_64F, CUSOLVER_C_32F, CUSOLVER_C_16F, CUSOLVER_C_16BF, CUSOLVER_C_TF32`
`CUSOLVER_C_32F`	`CUSOLVER_C_32F, CUSOLVER_C_16F, CUSOLVER_C_16BF, CUSOLVER_C_TF32`
`CUSOLVER_R_64F`	`CUSOLVER_R_64F, CUSOLVER_R_32F, CUSOLVER_R_16F, CUSOLVER_R_16BF, CUSOLVER_R_TF32`

Inputs/Outputs Data Type (e.g., main precision)	Supported values for the lowest precision
CUSOLVER_R_32F	CUSOLVER_R_32F, CUSOLVER_R_16F, CUSOLVER_R_16BF, CUSOLVER_R_TF32

The cusolverDnIRSXgels_bufferSize() function return the required workspace buffer size in bytes for the corresponding cusolverDnXgels() call with given gels_irs_params configuration.

```
cusolverStatus_t
cusolverDnIRSXgels_bufferSize(
        cusolverDnHandle_t           handle,
        cusolverDnIRSParams_t        gels_irs_params,
        cusolver_int_t               m,
        cusolver_int_t               n,
        cusolver_int_t               nrhs,
        size_t                 *     lwork_bytes);
```

Parameters of cusolverDnIRSXgels_bufferSize() functions

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cusolverDn library context.
params	host	input	Xgels configuration parameters
m	host	input	Number of rows of the matrix A. Should be non-negative and n<=m
n	host	input	Number of columns of the matrix A. Should be non-negative and n<=m.
nrhs	host	input	Number of right hand sides to solve. Should be non-negative. Note that, nrhs is limited to 1 if the selected IRS refinement solver is CUSOLVER_IRS_REFINE_GMRES, CUSOLVER_IRS_REFINE_GMRES_GMRES, CUSOLVER_IRS_REFINE_CLASSICAL_GMRES.
lwork_bytes	host	out	Pointer to a variable, where the required size in bytes, of the workspace will be stored after a call to cusolverDnIRSXgels_bufferSize. Can't be NULL.

```
cusolverStatus_t cusolverDnIRSXgels(
        cusolverDnHandle_t           handle,
        cusolverDnIRSParams_t        gels_irs_params,
        cusolverDnIRSInfos_t         gels_irs_infos,
        int                          m,
        int                          n,
        int                          nrhs,
        void                   *     dA,
        int                          ldda,
        void                   *     dB,
```

```
        int                         lddb,
        void                *       dX,
        int                         lddx,
        void                *       dWorkspace,
        size_t                      lwork_bytes,
        int                 *       dinfo);
```

Table 11. **Parameters of cusolverDnIRSXgels() functions**

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cusolverDn library context.
gels_irs_params	host	input	Configuration parameters structure, can serve one or more calls to any IRS solver
gels_irs_infos	host	in/out	Info structure, where information about a particular solve will be stored. The gels_irs_infos struture correspond to a particular call. Thus different calls requires different gels_irs_infos structure otherwise, it will be overwritten.
m	host	input	Number of rows of the matrix A. Should be non-negative and n<=m
n	host	input	Number of columns of the matrix A. Should be non-negative and n<=m.
nrhs	host	input	Number of right hand sides to solve. Should be non-negative. Note that, nrhs is limited to 1 if the selected IRS refinement solver is CUSOLVER_IRS_REFINE_GMRES, CUSOLVER_IRS_REFINE_GMRES_GMRES, CUSOLVER_IRS_REFINE_CLASSICAL_GMRES.
dA	device	in/out	Matrix A with size m-by-n. Can't be NULL. On return - unchanged if the lowest precision is not equal to the main precision and the iterative refinement solver converged, - garbage otherwise.
ldda	host	input	Leading dimension of two-dimensional array used to store matrix A. ldda >= m.
dB	device	input	Set of right hand sides B of size m-by-nrhs . Can't be NULL.
lddb	host	input	Leading dimension of two-dimensional array used to store matrix of right hand sides B. lddb >= max(1,m).
dX	device	output	Set of soultion vectors X of size n-by-nrhs . Can't be NULL.

Parameter	Memory	In/out	Meaning
lddx	host	input	Leading dimension of two-dimensional array used to store matrix of solution vectors X. lddx >= max(1,n).
dWorkspace	device	input	Pointer to an allocated workspace in device memory of size lwork_bytes.
lwork_bytes	host	input	Size of the allocated device workspace. Should be at least what was returned by cusolverDnIRSXgels_bufferSize() function.
niters	host	output	If iter is ▶ <0 : iterative refinement has failed, main precision (Inputs/Outputs precision) factorization has been performed if fallback is enabled ▶ -1 : taking into account machine parameters, n, nrhs, it is a priori not worth working in lower precision ▶ -2 : overflow of an entry when moving from main to lower precision ▶ -3 : failure during the factorization ▶ -5 : overflow occured during computation ▶ -maxiter: solver stopped the iterative refinement after reaching maximum allowed iterations ▶ >0 : iter is a number of iterations solver performed to reach convergence criteria
dinfo	device	output	Status of the IRS solver on the return. If 0 - solve was successful. If dinfo = -i then i-th argument is not valid.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed, for example: ▶ n<0 ▶ ldda<max(1,m) ▶ lddb<max(1,m) ▶ lddx<max(1,n)

CUSOLVER_STATUS_ARCH_MISMATCH	The IRS solver supports compute capability 7.0 and above. The lowest precision options CUSOLVER_[CR]_16BF and CUSOLVER_[CR]_TF32 are only available on compute capability 8.0 and above.
CUSOLVER_STATUS_INVALID_WORKSPACE	`lwork_bytes` is smaller than the required workspace. Could happen if the users called `cusolverDnIRSXgels_bufferSize()` function, then changed some of the configurations setting such as the lowest precision.
CUSOLVER_STATUS_IRS_OUT_OF_RANGE	Numerical error related to `niters` <0; see `niters` description for more details.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal error occured, check the `dinfo` and the `niters` arguments for more details.
CUSOLVER_STATUS_IRS_PARAMS_NOT_INITIALIZED	The configuration parameter `gels_irs_params` structure was not created.
CUSOLVER_STATUS_IRS_PARAMS_INVALID	One of the configuration parameter in the `gels_irs_params` structure is not valid.
CUSOLVER_STATUS_IRS_PARAMS_INVALID_PREC	The main and/or the lowest precision configuration parameter in the `gels_irs_params` structure is not valid, check the table above for the supported combinations.
CUSOLVER_STATUS_IRS_PARAMS_INVALID_MAXITER	The maxiter configuration parameter in the `gels_irs_params` structure is not valid.
CUSOLVER_STATUS_IRS_PARAMS_INVALID_REFINE	The refinement solver configuration parameter in the gels_irs_params structure is not valid.
CUSOLVER_STATUS_IRS_NOT_SUPPORTED	One of the configuration parameter in the `gels_irs_params` structure is not supported. For example if nrhs >1, and refinement solver was set to CUSOLVER_IRS_REFINE_GMRES.
CUSOLVER_STATUS_IRS_INFOS_NOT_INITIALIZED	The information structure `gels_irs_infos` was not created.
CUSOLVER_STATUS_ALLOC_FAILED	CPU memory allocation failed, most likely during the allocation of the residual array that store the residual norms.

2.4.2.16. cusolverDn<t>ormqr()

These helper functions calculate the size of work buffers needed. Please visit cuSOLVER Library Samples - ormqr for a code example.

```
cusolverStatus_t
cusolverDnSormqr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasOperation_t trans,
    int m,
    int n,
    int k,
```

```
    const float *A,
    int lda,
    const float *tau,
    const float *C,
    int ldc,
    int *lwork);

cusolverStatus_t
cusolverDnDormqr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasOperation_t trans,
    int m,
    int n,
    int k,
    const double *A,
    int lda,
    const double *tau,
    const double *C,
    int ldc,
    int *lwork);

cusolverStatus_t
cusolverDnCunmqr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasOperation_t trans,
    int m,
    int n,
    int k,
    const cuComplex *A,
    int lda,
    const cuComplex *tau,
    const cuComplex *C,
    int ldc,
    int *lwork);

cusolverStatus_t
cusolverDnZunmqr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasOperation_t trans,
    int m,
    int n,
    int k,
    const cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *tau,
    const cuDoubleComplex *C,
    int ldc,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSormqr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasOperation_t trans,
```

```
    int m,
    int n,
    int k,
    const float *A,
    int lda,
    const float *tau,
    float *C,
    int ldc,
    float *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDormqr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasOperation_t trans,
    int m,
    int n,
    int k,
    const double *A,
    int lda,
    const double *tau,
    double *C,
    int ldc,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCunmqr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasOperation_t trans,
    int m,
    int n,
    int k,
    const cuComplex *A,
    int lda,
    const cuComplex *tau,
    cuComplex *C,
    int ldc,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZunmqr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasOperation_t trans,
    int m,
    int n,
    int k,
    const cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *tau,
```

```
    cuDoubleComplex *C,
    int ldc,
    cuDoubleComplex *work,
    int lwork,
    int *devInfo);
```

This function overwrites `m×n` matrix `C` by

$$C = \begin{cases} \text{op}(Q)^* C & \text{if side == CUBLAS\_SIDE\_LEFT} \\ C^* \text{op}(Q) & \text{if side == CUBLAS\_SIDE\_RIGHT} \end{cases}$$

The operation of `Q` is defined by

$$\text{op}(Q) = \begin{cases} Q & \text{if transa == CUBLAS\_OP\_N} \\ Q^T & \text{if transa == CUBLAS\_OP\_T} \\ Q^H & \text{if transa == CUBLAS\_OP\_C} \end{cases}$$

`Q` is a unitary matrix formed by a sequence of elementary reflection vectors from QR factorization (`geqrf`) of `A`.

`Q=H(1) H(2) ... H(k)`

`Q` is of order `m` if `side` = `CUBLAS_SIDE_LEFT` and of order `n` if `side` = `CUBLAS_SIDE_RIGHT`.

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is size of the working space, and it is returned by `geqrf_bufferSize()` or `ormqr_bufferSize()`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

The user can combine `geqrf`, `ormqr` and `trsm` to complete a linear solver or a least-square solver.

API of ormqr

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDn library context.
`side`	`host`	`input`	Indicates if matrix `Q` is on the left or right of `C`.
`trans`	`host`	`input`	Operation `op(Q)` that is non- or (conj.) transpose.
`m`	`host`	`input`	Number of rows of matrix `C`.
`n`	`host`	`input`	Number of columns of matrix `C`.
`k`	`host`	`input`	Number of elementary relfections whose product defines the matrix Q.
`A`	`device`	`in/out`	<type> array of dimension `lda * k` with `lda` is not less than `max(1,m)`. The matrix `A` is from `geqrf`, so i-th column contains elementary reflection vector.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `A`. if `side` is

Parameter	Memory	In/out	Meaning
			CUBLAS_SIDE_LEFT, lda >= max(1,m); if side is CUBLAS_SIDE_RIGHT, lda >= max(1,n).
tau	device	input	<type> array of dimension at least min(m,n). The vector tau is from geqrf, so tau(i) is the scalar of i-th elementary reflection vector.
C	device	in/out	<type> array of size ldc * n. On exit, C is overwritten by op(Q)*C.
ldc	host	input	Leading dimension of two-dimensional array of matrix C. ldc >= max(1,m).
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of working array work.
devInfo	device	output	If devInfo = 0, the ormqr is successful. If devInfo = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or wrong lda or ldc).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.17. cusolverDn<t>orgqr()

These helper functions calculate the size of work buffers needed. Please visit cuSOLVER Library Samples - orgqr for a code example.

```
cusolverStatus_t
cusolverDnSorgqr_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int k,
    const float *A,
    int lda,
    const float *tau,
    int *lwork);

cusolverStatus_t
cusolverDnDorgqr_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
```

```
    int k,
    const double *A,
    int lda,
    const double *tau,
    int *lwork);

cusolverStatus_t
cusolverDnCungqr_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int k,
    const cuComplex *A,
    int lda,
    const cuComplex *tau,
    int *lwork);

cusolverStatus_t
cusolverDnZungqr_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int k,
    const cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *tau,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSorgqr(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int k,
    float *A,
    int lda,
    const float *tau,
    float *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDorgqr(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int k,
    double *A,
    int lda,
    const double *tau,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCungqr(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int k,
    cuComplex *A,
    int lda,
    const cuComplex *tau,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZungqr(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int k,
    cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *tau,
    cuDoubleComplex *work,
    int lwork,
    int *devInfo);
```

This function overwrites `m×n` matrix `A` by

$$Q = \mathrm{H}(1) * \mathrm{H}(2) * \ldots * \mathrm{H}(\mathrm{k})$$

where `Q` is a unitary matrix formed by a sequence of elementary reflection vectors stored in `A`.

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is size of the working space, and it is returned by `orgqr_bufferSize()`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

The user can combine `geqrf`, `orgqr` to complete orthogonalization.

API of ormqr

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`m`	`host`	`input`	Number of rows of matrix `Q`. m >= 0;
`n`	`host`	`input`	Number of columns of matrix `Q`. m >= n >= 0;
`k`	`host`	`input`	Number of elementary relfections whose product defines the matrix `Q`. n >= k >= 0;
`A`	`device`	`in/out`	<type> array of dimension `lda * n` with `lda` is not less than `max(1,m)`. i-th column of `A` contains elementary reflection vector.

Parameter	Memory	In/out	Meaning
lda	host	input	Leading dimension of two-dimensional array used to store matrix A. lda >= max(1,m).
tau	device	input	<type> array of dimension k. tau(i) is the scalar of i-th elementary reflection vector.
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of working array work.
devInfo	device	output	If info = 0, the orgqr is successful. if info = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n,k<0, n>m, k>n or lda<m).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.18. cusolverDn<t>sytrf()

These helper functions calculate the size of the needed buffers.

```
cusolverStatus_t
cusolverDnSsytrf_bufferSize(cusolverDnHandle_t handle,
                        int n,
                        float *A,
                        int lda,
                        int *Lwork );

cusolverStatus_t
cusolverDnDsytrf_bufferSize(cusolverDnHandle_t handle,
                        int n,
                        double *A,
                        int lda,
                        int *Lwork );

cusolverStatus_t
cusolverDnCsytrf_bufferSize(cusolverDnHandle_t handle,
                        int n,
                        cuComplex *A,
                        int lda,
                        int *Lwork );

cusolverStatus_t
cusolverDnZsytrf_bufferSize(cusolverDnHandle_t handle,
                        int n,
```

```
                        cuDoubleComplex *A,
                        int lda,
                        int *Lwork );
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSsytrf(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            float *A,
            int lda,
            int *ipiv,
            float *work,
            int lwork,
            int *devInfo );

cusolverStatus_t
cusolverDnDsytrf(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            double *A,
            int lda,
            int *ipiv,
            double *work,
            int lwork,
            int *devInfo );
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCsytrf(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            cuComplex *A,
            int lda,
            int *ipiv,
            cuComplex *work,
            int lwork,
            int *devInfo );

cusolverStatus_t
cusolverDnZsytrf(cusolverDnHandle_t handle,
            cublasFillMode_t uplo,
            int n,
            cuDoubleComplex *A,
            int lda,
            int *ipiv,
            cuDoubleComplex *work,
            int lwork,
            int *devInfo );
```

This function computes the Bunch-Kaufman factorization of a `n×n` symmetric indefinite matrix

`A` is a `n×n` symmetric matrix, only lower or upper part is meaningful. The input parameter `uplo` which part of the matrix is used. The function would leave other part untouched.

If input parameter uplo is CUBLAS_FILL_MODE_LOWER, only lower triangular part of A is processed, and replaced by lower triangular factor L and block diagonal matrix D. Each block of D is either 1x1 or 2x2 block, depending on pivoting.

$$P*A*P^T = L*D*L^T$$

If input parameter uplo is CUBLAS_FILL_MODE_UPPER, only upper triangular part of A is processed, and replaced by upper triangular factor U and block diagonal matrix D.

$$P*A*P^T = U*D*U^T$$

The user has to provide working space which is pointed by input parameter work. The input parameter lwork is size of the working space, and it is returned by sytrf_bufferSize().

If Bunch-Kaufman factorization failed, i.e. A is singular. The output parameter devInfo = i would indicate D(i,i)=0.

If output parameter devInfo = -i (less than zero), the i-th parameter is wrong (not counting handle).

The output parameter devIpiv contains pivoting sequence. If devIpiv(i) = k > 0, D(i,i) is 1x1 block, and i-th row/column of A is interchanged with k-th row/column of A. If uplo is CUBLAS_FILL_MODE_UPPER and devIpiv(i-1) = devIpiv(i) = -m < 0, D(i-1:i,i-1:i) is a 2x2 block, and (i-1)-th row/column is interchanged with m-th row/column. If uplo is CUBLAS_FILL_MODE_LOWER and devIpiv(i+1) = devIpiv(i) = -m < 0, D(i:i+1,i:i+1) is a 2x2 block, and (i+1)-th row/column is interchanged with m-th row/column.

API of sytrf

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
uplo	host	input	Indicates if matrix A lower or upper part is stored, the other part is not referenced.
n	host	input	Number of rows and columns of matrix A.
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,n).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
ipiv	device	output	Array of size at least n, containing pivot indices.
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of working space work.
devInfo	device	output	if devInfo = 0, the LU factorization is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle). if devInfo = i, the D(i,i) = 0.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.19. cusolverDn<t>potrfBatched()

The S and D data types are real valued single and double precision, respectively. Please visit cuSOLVER Library Samples - potrfBatched for a code example.

```
cusolverStatus_t
cusolverDnSpotrfBatched(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    float *Aarray[],
    int lda,
    int *infoArray,
    int batchSize);

cusolverStatus_t
cusolverDnDpotrfBatched(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    double *Aarray[],
    int lda,
    int *infoArray,
    int batchSize);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCpotrfBatched(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    cuComplex *Aarray[],
    int lda,
    int *infoArray,
    int batchSize);

cusolverStatus_t
cusolverDnZpotrfBatched(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    cuDoubleComplex *Aarray[],
    int lda,
    int *infoArray,
```

```
    int batchSize);
```

This function computes the Cholesky factorization of a squence of Hermitian positive-definite matrices.

Each `Aarray[i] for i=0,1,..., batchSize-1` is a `n×n` Hermitian matrix, only lower or upper part is meaningful. The input parameter `uplo` indicates which part of the matrix is used.

If input parameter `uplo` is `CUBLAS_FILL_MODE_LOWER`, only lower triangular part of `A` is processed, and replaced by lower triangular Cholesky factor `L`.

$$A = L * L^H$$

If input parameter `uplo` is `CUBLAS_FILL_MODE_UPPER`, only upper triangular part of `A` is processed, and replaced by upper triangular Cholesky factor `U`.

$$A = U^H * U$$

If Cholesky factorization failed, i.e. some leading minor of `A` is not positive definite, or equivalently some diagonal elements of `L` or `U` is not a real number. The output parameter `infoArray` would indicate smallest leading minor of `A` which is not positive definite.

`infoArray` is an integer array of size `batchsize`. If `potrfBatched` returns `CUSOLVER_STATUS_INVALID_VALUE`, `infoArray[0] = -i` (less than zero), meaning that the `i-th` parameter is wrong (not counting handle). If `potrfBatched` returns `CUSOLVER_STATUS_SUCCESS` but `infoArray[i] = k` is positive, then `i-th` matrix is not positive definite and the Cholesky factorization failed at row `k`.

Remark: the other part of `A` is used as a workspace. For example, if `uplo` is `CUBLAS_FILL_MODE_UPPER`, upper triangle of `A` contains cholesky factor `U` and lower triangle of `A` is destroyed after `potrfBatched`.

Table 12. API of potrfBatched

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`uplo`	`host`	`input`	Indicates if lower or upper part is stored; the other part is used as a workspace.
`n`	`host`	`input`	Number of rows and columns of matrix `A`.
`Aarray`	`device`	`in/out`	Array of pointers to <type> array of dimension `lda * n` with `lda` is not less than `max(1,n)`.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store each matrix `Aarray[i]`.
`infoArray`	`device`	`output`	Array of size `batchSize`. `infoArray[i]` contains information of factorization of `Aarray[i]`. if `potrfBatched` returns `CUSOLVER_STATUS_INVALID_VALUE`,

Parameter	Memory	In/out	Meaning
			infoArray[0] = -i (less than zero) means the i-th parameter is wrong (not counting handle). if potrfBatched returns CUSOLVER_STATUS_SUCCESS, infoArray[i] = 0 means the Cholesky factorization of i-th matrix is successful, and infoArray[i] = k means the leading submatrix of order k of i-th matrix is not positive definite.
batchSize	host	input	Number of pointers in Aarray.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n) or batchSize<1).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.2.20. cusolverDn<t>potrsBatched()

```
cusolverStatus_t
cusolverDnSpotrsBatched(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    int nrhs,
    float *Aarray[],
    int lda,
    float *Barray[],
    int ldb,
    int *info,
    int batchSize);

cusolverStatus_t
cusolverDnDpotrsBatched(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    int nrhs,
    double *Aarray[],
    int lda,
    double *Barray[],
    int ldb,
    int *info,
    int batchSize);

cusolverStatus_t
cusolverDnCpotrsBatched(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    int nrhs,
```

```
    cuComplex *Aarray[],
    int lda,
    cuComplex *Barray[],
    int ldb,
    int *info,
    int batchSize);

cusolverStatus_t
cusolverDnZpotrsBatched(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    int nrhs,
    cuDoubleComplex *Aarray[],
    int lda,
    cuDoubleComplex *Barray[],
    int ldb,
    int *info,
    int batchSize);
```

This function solves a squence of linear systems

$$A[i]*X[i]=B[i]$$

where each `Aarray[i] for i=0,1,..., batchSize-1` is a `n×n` Hermitian matrix, only lower or upper part is meaningful. The input parameter `uplo` indicates which part of the matrix is used.

The user has to call `potrfBatched` first to factorize matrix `Aarray[i]`. If input parameter `uplo` is `CUBLAS_FILL_MODE_LOWER`, `A` is lower triangular Cholesky factor `L` correspoding to $A=L*L^H$. If input parameter `uplo` is `CUBLAS_FILL_MODE_UPPER`, `A` is upper triangular Cholesky factor `U` corresponding to $A=U^H*U$.

The operation is in-place, i.e. matrix `X` overwrites matrix `B` with the same leading dimension `ldb`.

The output parameter `info` is a scalar. If `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

Remark 1: only `nrhs=1` is supported.

Remark 2: `infoArray` from `potrfBatched` indicates if the matrix is positive definite. `info` from `potrsBatched` only shows which input parameter is wrong (not counting handle).

Remark 3: the other part of `A` is used as a workspace. For example, if `uplo` is `CUBLAS_FILL_MODE_UPPER`, upper triangle of `A` contains cholesky factor `U` and lower triangle of `A` is destroyed after `potrsBatched`.

Please visit cuSOLVER Library Samples - potrfBatched for a code example.

API of potrsBatched

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolveDN library context.
`uplo`	`host`	`input`	Indicates if matrix `A` lower or upper part is stored.

Parameter	Memory	In/out	Meaning
n	host	input	Number of rows and columns of matrix A.
nrhs	host	input	Number of columns of matrix X and B.
Aarray	device	in/out	Array of pointers to <type> array of dimension lda * n with lda is not less than max(1,n). Aarray[i] is either lower cholesky factor L or upper Cholesky factor U.
lda	host	input	Leading dimension of two-dimensional array used to store each matrix Aarray[i].
Barray	device	in/out	Array of pointers to <type> array of dimension ldb * nrhs. ldb is not less than max(1,n). As an input, Barray[i] is right hand side matrix. As an output, Barray[i] is the solution matrix.
ldb	host	input	Leading dimension of two-dimensional array used to store each matrix Barray[i].
info	device	output	If info = 0, all parameters are correct. if info = -i, the i-th parameter is wrong (not counting handle).
batchSize	host	input	Number of pointers in Aarray.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, nrhs<0, lda<max(1,n), ldb<max(1,n) or batchSize<0).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3. Dense Eigenvalue Solver Reference (legacy)

This chapter describes eigenvalue solver API of cuSolverDN, including bidiagonalization and SVD.

2.4.3.1. cusolverDn<t>gebrd()

These helper functions calculate the size of work buffers needed.

```
cusolverStatus_t
cusolverDnSgebrd_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int *Lwork );
```

```
cusolverStatus_t
cusolverDnDgebrd_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int *Lwork );

cusolverStatus_t
cusolverDnCgebrd_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int *Lwork );

cusolverStatus_t
cusolverDnZgebrd_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int *Lwork );
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSgebrd(cusolverDnHandle_t handle,
           int m,
           int n,
           float *A,
           int lda,
           float *D,
           float *E,
           float *TAUQ,
           float *TAUP,
           float *Work,
           int Lwork,
           int *devInfo );

cusolverStatus_t
cusolverDnDgebrd(cusolverDnHandle_t handle,
           int m,
           int n,
           double *A,
           int lda,
           double *D,
           double *E,
           double *TAUQ,
           double *TAUP,
           double *Work,
           int Lwork,
           int *devInfo );
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCgebrd(cusolverDnHandle_t handle,
           int m,
           int n,
           cuComplex *A,
```

```
            int lda,
            float *D,
            float *E,
            cuComplex *TAUQ,
            cuComplex *TAUP,
            cuComplex *Work,
            int Lwork,
            int *devInfo );

cusolverStatus_t
cusolverDnZgebrd(cusolverDnHandle_t handle,
            int m,
            int n,
            cuDoubleComplex *A,
            int lda,
            double *D,
            double *E,
            cuDoubleComplex *TAUQ,
            cuDoubleComplex *TAUP,
            cuDoubleComplex *Work,
            int Lwork,
            int *devInfo );
```

This function reduces a general `m×n` matrix `A` to a real upper or lower bidiagonal form `B` by an orthogonal transformation: $Q^H * A * P = B$

If `m>=n`, `B` is upper bidiagonal; if `m<n`, `B` is lower bidiagonal.

The matrix `Q` and `P` are overwritten into matrix `A` in the following sense:

- if `m>=n`, the diagonal and the first superdiagonal are overwritten with the upper bidiagonal matrix `B`; the elements below the diagonal, with the array `TAUQ`, represent the orthogonal matrix `Q` as a product of elementary reflectors, and the elements above the first superdiagonal, with the array `TAUP`, represent the orthogonal matrix `P` as a product of elementary reflectors.
- if `m<n`, the diagonal and the first subdiagonal are overwritten with the lower bidiagonal matrix `B`; the elements below the first subdiagonal, with the array `TAUQ`, represent the orthogonal matrix `Q` as a product of elementary reflectors, and the elements above the diagonal, with the array `TAUP`, represent the orthogonal matrix `P` as a product of elementary reflectors.

The user has to provide working space which is pointed by input parameter `Work`. The input parameter `Lwork` is size of the working space, and it is returned by `gebrd_bufferSize()`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

Remark: `gebrd` only supports `m>=n`.

API of gebrd

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`m`	`host`	`input`	Number of rows of matrix `A`.
`n`	`host`	`input`	Number of columns of matrix `A`.

Parameter	Memory	In/out	Meaning
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,n).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
D	device	output	Real array of dimension min(m,n). The diagonal elements of the bidiagonal matrix B: D(i) = A(i,i).
E	device	output	Real array of dimension min(m,n). The off-diagonal elements of the bidiagonal matrix B: if m>=n, E(i) = A(i,i+1) for i = 1,2,...,n-1; if m<n, E(i) = A(i+1,i) for i = 1,2,...,m-1.
TAUQ	device	output	<type> array of dimension min(m,n). The scalar factors of the elementary reflectors which represent the orthogonal matrix Q.
TAUP	device	output	<type> array of dimension min(m,n). The scalar factors of the elementary reflectors which represent the orthogonal matrix P.
Work	device	in/out	Working space, <type> array of size Lwork.
Lwork	host	input	Size of Work, returned by gebrd_bufferSize.
devInfo	device	output	If devInfo = 0, the operation is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0, or lda<max(1,m)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.2. cusolverDn<t>orgbr()

These helper functions calculate the size of work buffers needed.

```
cusolverStatus_t
cusolverDnSorgbr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
```

```
    int m,
    int n,
    int k,
    const float *A,
    int lda,
    const float *tau,
    int *lwork);

cusolverStatus_t
cusolverDnDorgbr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    int m,
    int n,
    int k,
    const double *A,
    int lda,
    const double *tau,
    int *lwork);

cusolverStatus_t
cusolverDnCungbr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    int m,
    int n,
    int k,
    const cuComplex *A,
    int lda,
    const cuComplex *tau,
    int *lwork);

cusolverStatus_t
cusolverDnZungbr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    int m,
    int n,
    int k,
    const cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *tau,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSorgbr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    int m,
    int n,
    int k,
    float *A,
    int lda,
    const float *tau,
    float *work,
    int lwork,
    int *devInfo);
```

```
cusolverStatus_t
cusolverDnDorgbr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    int m,
    int n,
    int k,
    double *A,
    int lda,
    const double *tau,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCungbr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    int m,
    int n,
    int k,
    cuComplex *A,
    int lda,
    const cuComplex *tau,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZungbr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    int m,
    int n,
    int k,
    cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *tau,
    cuDoubleComplex *work,
    int lwork,
    int *devInfo);
```

This function generates one of the unitary matrices `Q` or `P**H` determined by `gebrd` when reducing a matrix A to bidiagonal form: $Q^H * A * P = B$

`Q` and `P**H` are defined as products of elementary reflectors H(i) or G(i) respectively.

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is size of the working space, and it is returned by `orgbr_bufferSize()`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

API of orgbr

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
side	host	input	If side = CUBLAS_SIDE_LEFT, generate Q. if side = CUBLAS_SIDE_RIGHT, generate P**T.
m	host	input	Number of rows of matrix Q or P**T.
n	host	input	If side = CUBLAS_SIDE_LEFT, m>= n>= min(m,k). if side = CUBLAS_SIDE_RIGHT, n>= m>= min(n,k).
k	host	input	If side = CUBLAS_SIDE_LEFT, the number of columns in the original m-by-k matrix reduced by gebrd. if side = CUBLAS_SIDE_RIGHT, the number of rows in the original k-by-n matrix reduced by gebrd.
A	device	in/out	<type> array of dimension lda * n On entry, the vectors which define the elementary reflectors, as returned by gebrd. On exit, the m-by-n matrix Q or P**T.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A. lda >= max(1,m);
tau	device	input	<type> array of dimension min(m,k) if side is CUBLAS_SIDE_LEFT; of dimension min(n,k) if side is CUBLAS_SIDE_RIGHT; tau(i) must contain the scalar factor of the elementary reflector H(i) or G(i), which determines Q or P**T, as returned by gebrd in its array argument TAUQ or TAUP.
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of working array work.
devInfo	device	output	If info = 0, the ormqr is successful. if info = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	the operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	invalid parameters were passed (m,n<0 or wrong lda).
CUSOLVER_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	an internal operation failed.

2.4.3.3. cusolverDn<t>sytrd()

These helper functions calculate the size of work buffers needed.

```
cusolverStatus_t
cusolverDnSsytrd_bufferSize(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    const float *A,
    int lda,
    const float *d,
    const float *e,
    const float *tau,
    int *lwork);

cusolverStatus_t
cusolverDnDsytrd_bufferSize(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    const double *A,
    int lda,
    const double *d,
    const double *e,
    const double *tau,
    int *lwork);

cusolverStatus_t
cusolverDnChetrd_bufferSize(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    const cuComplex *A,
    int lda,
    const float *d,
    const float *e,
    const cuComplex *tau,
    int *lwork);

cusolverStatus_t
cusolverDnZhetrd_bufferSize(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const double *d,
    const double *e,
    const cuDoubleComplex *tau,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSsytrd(
```

```
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    float *A,
    int lda,
    float *d,
    float *e,
    float *tau,
    float *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDsytrd(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    double *A,
    int lda,
    double *d,
    double *e,
    double *tau,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnChetrd(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    cuComplex *A,
    int lda,
    float *d,
    float *e,
    cuComplex *tau,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t CUDENSEAPI cusolverDnZhetrd(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    cuDoubleComplex *A,
    int lda,
    double *d,
    double *e,
    cuDoubleComplex *tau,
    cuDoubleComplex *work,
    int lwork,
    int *devInfo);
```

This function reduces a general symmetric (Hermitian) `n×n` matrix `A` to real symmetric tridiagonal form `T` by an orthogonal transformation: $Q^H * A * Q = T$

As an output, A contains T and householder reflection vectors. If uplo = CUBLAS_FILL_MODE_UPPER, the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array tau, represent the orthogonal matrix Q as a product of elementary reflectors; If uplo = CUBLAS_FILL_MODE_LOWER, the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array tau, represent the orthogonal matrix Q as a product of elementary reflectors.

The user has to provide working space which is pointed by input parameter work. The input parameter lwork is size of the working space, and it is returned by sytrd_bufferSize().

If output parameter devInfo = -i (less than zero), the i-th parameter is wrong (not counting handle).

API of sytrd

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
uplo	host	input	Specifies which part of A is stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of A is stored. uplo = CUBLAS_FILL_MODE_UPPER: Upper triangle of A is stored.
n	host	input	Number of rows (columns) of matrix A.
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced. On exit, A is overwritten by T and householder reflection vectors.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A. lda >= max(1,n).
D	device	output	Real array of dimension n. The diagonal elements of the tridiagonal matrix T: D(i) = A(i,i).
E	device	output	Real array of dimension (n-1). The off-diagonal elements of the tridiagonal matrix T: if uplo = CUBLAS_FILL_MODE_UPPER, E(i) = A(i,i+1). if uplo = CUBLAS_FILL_MODE_LOWER E(i) = A(i+1,i).

Parameter	Memory	In/out	Meaning
tau	device	output	<type> array of dimension (n-1). The scalar factors of the elementary reflectors which represent the orthogonal matrix Q.
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of work, returned by sytrd_bufferSize.
devInfo	device	output	If devInfo = 0, the operation is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, or lda<max(1,n), or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.4. cusolverDn<t>ormtr()

These helper functions calculate the size of work buffers needed.

```
cusolverStatus_t
cusolverDnSormtr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasFillMode_t uplo,
    cublasOperation_t trans,
    int m,
    int n,
    const float *A,
    int lda,
    const float *tau,
    const float *C,
    int ldc,
    int *lwork);

cusolverStatus_t
cusolverDnDormtr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasFillMode_t uplo,
    cublasOperation_t trans,
    int m,
```

```
    int n,
    const double *A,
    int lda,
    const double *tau,
    const double *C,
    int ldc,
    int *lwork);

cusolverStatus_t
cusolverDnCunmtr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasFillMode_t uplo,
    cublasOperation_t trans,
    int m,
    int n,
    const cuComplex *A,
    int lda,
    const cuComplex *tau,
    const cuComplex *C,
    int ldc,
    int *lwork);

cusolverStatus_t
cusolverDnZunmtr_bufferSize(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasFillMode_t uplo,
    cublasOperation_t trans,
    int m,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *tau,
    const cuDoubleComplex *C,
    int ldc,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSormtr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasFillMode_t uplo,
    cublasOperation_t trans,
    int m,
    int n,
    float *A,
    int lda,
    float *tau,
    float *C,
    int ldc,
    float *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDormtr(
```

```
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasFillMode_t uplo,
    cublasOperation_t trans,
    int m,
    int n,
    double *A,
    int lda,
    double *tau,
    double *C,
    int ldc,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCunmtr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasFillMode_t uplo,
    cublasOperation_t trans,
    int m,
    int n,
    cuComplex *A,
    int lda,
    cuComplex *tau,
    cuComplex *C,
    int ldc,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZunmtr(
    cusolverDnHandle_t handle,
    cublasSideMode_t side,
    cublasFillMode_t uplo,
    cublasOperation_t trans,
    int m,
    int n,
    cuDoubleComplex *A,
    int lda,
    cuDoubleComplex *tau,
    cuDoubleComplex *C,
    int ldc,
    cuDoubleComplex *work,
    int lwork,
    int *devInfo);
```

This function overwrites `m×n` matrix `C` by

$$C = \begin{cases} \text{op}(Q)*C & \text{if side == CUBLAS\_SIDE\_LEFT} \\ C*\text{op}(Q) & \text{if side == CUBLAS\_SIDE\_RIGHT} \end{cases}$$

where `Q` is a unitary matrix formed by a sequence of elementary reflection vectors from `sytrd`.

The operation on `Q` is defined by

$$\mathrm{op}(Q) = \begin{cases} Q & \text{if transa == CUBLAS\_OP\_N} \\ Q^T & \text{if transa == CUBLAS\_OP\_T} \\ Q^H & \text{if transa == CUBLAS\_OP\_C} \end{cases}$$

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is size of the working space, and it is returned by `ormtr_bufferSize()`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

API of ormtr

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`side`	`host`	`input`	`side` = `CUBLAS_SIDE_LEFT`, apply Q or Q**T from the Left; `side` = `CUBLAS_SIDE_RIGHT`, apply Q or Q**T from the Right.
`uplo`	`host`	`input`	`uplo` = `CUBLAS_FILL_MODE_LOWER`: Lower triangle of `A` contains elementary reflectors from `sytrd`. `uplo` = `CUBLAS_FILL_MODE_UPPER`: Upper triangle of `A` contains elementary reflectors from `sytrd`.
`trans`	`host`	`input`	Operation `op(Q)` that is non- or (conj.) transpose.
`m`	`host`	`input`	Number of rows of matrix `C`.
`n`	`host`	`input`	Number of columns of matrix `C`.
`A`	`device`	`in/out`	<type> array of dimension `lda * m` if `side` = `CUBLAS_SIDE_LEFT`; `lda * n` if `side` = `CUBLAS_SIDE_RIGHT`. The matrix `A` from `sytrd` contains the elementary reflectors.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `A`. if `side` is `CUBLAS_SIDE_LEFT`, lda >= max(1,m); if `side` is `CUBLAS_SIDE_RIGHT`, lda >= max(1,n).
`tau`	`device`	`output`	<type> array of dimension `(m-1)` if `side` is `CUBLAS_SIDE_LEFT`; of dimension `(n-1)` if `side` is `CUBLAS_SIDE_RIGHT`; The vector `tau` is from `sytrd`, so `tau(i)` is the scalar of i-th elementary reflection vector.
`C`	`device`	`in/out`	<type> array of size `ldc * n`. On exit, `C` is overwritten by `op(Q)*C` or `C*op(Q)`.
`ldc`	`host`	`input`	Leading dimension of two-dimensional array of matrix `C`. ldc >= max(1,m).

Parameter	Memory	In/out	Meaning
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of working array work.
devInfo	device	output	If devInfo = 0, the ormqr is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or wrong lda or ldc).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.5. cusolverDn<t>orgtr()

These helper functions calculate the size of work buffers needed.

```
cusolverStatus_t
cusolverDnSorgtr_bufferSize(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    const float *A,
    int lda,
    const float *tau,
    int *lwork);

cusolverStatus_t
cusolverDnDorgtr_bufferSize(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    const double *A,
    int lda,
    const double *tau,
    int *lwork);

cusolverStatus_t
cusolverDnCungtr_bufferSize(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    const cuComplex *A,
    int lda,
    const cuComplex *tau,
    int *lwork);

cusolverStatus_t
cusolverDnZungtr_bufferSize(
```

```
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *tau,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSorgtr(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    float *A,
    int lda,
    const float *tau,
    float *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDorgtr(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    double *A,
    int lda,
    const double *tau,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCungtr(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    cuComplex *A,
    int lda,
    const cuComplex *tau,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZungtr(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int n,
    cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *tau,
    cuDoubleComplex *work,
```

```
    int lwork,
    int *devInfo);
```

This function generates a unitary matrix `Q` which is defined as the product of n-1 elementary reflectors of order n, as returned by `sytrd`:

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is size of the working space, and it is returned by `orgtr_bufferSize()`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

API of orgtr

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`uplo`	`host`	`input`	`uplo` = `CUBLAS_FILL_MODE_LOWER`: Lower triangle of `A` contains elementary reflectors from `sytrd`. `uplo` = `CUBLAS_FILL_MODE_UPPER`: Upper triangle of `A` contains elementary reflectors from `sytrd`.
`n`	`host`	`input`	Number of rows (columns) of matrix `Q`.
`A`	`device`	`in/out`	<type> array of dimension `lda * n` On entry, matrix `A` from `sytrd` contains the elementary reflectors. On exit, matrix `A` contains the n-by-n orthogonal matrix `Q`.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `A`. lda >= max(1,n).
`tau`	`device`	`input`	<type> array of dimension `(n-1) tau(i)` is the scalar of i-th elementary reflection vector.
`work`	`device`	`in/out`	Working space, <type> array of size `lwork`.
`lwork`	`host`	`input`	Size of working array `work`.
`devInfo`	`device`	`output`	If `devInfo = 0`, the orgtr is successful. if `devInfo = -i`, the `i-th` parameter is wrong (not counting handle).

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.
`CUSOLVER_STATUS_INVALID_VALUE`	Invalid parameters were passed (`n<0` or wrong `lda`).
`CUSOLVER_STATUS_ARCH_MISMATCH`	The device only supports compute capability 2.0 and above.
`CUSOLVER_STATUS_INTERNAL_ERROR`	An internal operation failed.

2.4.3.6. cusolverDn<t>gesvd()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSgesvd_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int *lwork );

cusolverStatus_t
cusolverDnDgesvd_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int *lwork );

cusolverStatus_t
cusolverDnCgesvd_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int *lwork );

cusolverStatus_t
cusolverDnZgesvd_bufferSize(
    cusolverDnHandle_t handle,
    int m,
    int n,
    int *lwork );
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSgesvd (
    cusolverDnHandle_t handle,
    signed char jobu,
    signed char jobvt,
    int m,
    int n,
    float *A,
    int lda,
    float *S,
    float *U,
    int ldu,
    float *VT,
    int ldvt,
    float *work,
    int lwork,
    float *rwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDgesvd (
    cusolverDnHandle_t handle,
    signed char jobu,
    signed char jobvt,
    int m,
    int n,
    double *A,
    int lda,
    double *S,
```

```
    double *U,
    int ldu,
    double *VT,
    int ldvt,
    double *work,
    int lwork,
    double *rwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCgesvd (
    cusolverDnHandle_t handle,
    signed char jobu,
    signed char jobvt,
    int m,
    int n,
    cuComplex *A,
    int lda,
    float *S,
    cuComplex *U,
    int ldu,
    cuComplex *VT,
    int ldvt,
    cuComplex *work,
    int lwork,
    float *rwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZgesvd (
    cusolverDnHandle_t handle,
    signed char jobu,
    signed char jobvt,
    int m,
    int n,
    cuDoubleComplex *A,
    int lda,
    double *S,
    cuDoubleComplex *U,
    int ldu,
    cuDoubleComplex *VT,
    int ldvt,
    cuDoubleComplex *work,
    int lwork,
    double *rwork,
    int *devInfo);
```

This function computes the singular value decomposition (SVD) of a `m×n` matrix `A` and corresponding the left and/or right singular vectors. The SVD is written

$$A = U * \Sigma * V^H$$

where Σ is an `m×n` matrix which is zero except for its `min(m,n)` diagonal elements, `U` is an `m×m` unitary matrix, and `V` is an `n×n` unitary matrix. The diagonal elements of Σ are the singular values of `A`; they are real and non-negative, and are returned in descending order. The first `min(m,n)` columns of `U` and `V` are the left and right singular vectors of `A`.

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is size of the working space, and it is returned by `gesvd_bufferSize()`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). if `bdsqr` did not converge, `devInfo` specifies how many superdiagonals of an intermediate bidiagonal form did not converge to zero.

The `rwork` is real array of dimension (min(m,n)-1). If `devInfo`>0 and `rwork` is not nil, `rwork` contains the unconverged superdiagonal elements of an upper bidiagonal matrix. This is slightly different from LAPACK which puts unconverged superdiagonal elements in `work` if type is `real`; in `rwork` if type is `complex`. `rwork` can be a NULL pointer if the user does not want the information from supperdiagonal.

Please visit cuSOLVER Library Samples - gesvd for a code example.

Remark 1: `gesvd` only supports `m>=n`.

Remark 2: the routine returns V^H , not `V`.

API of gesvd

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`jobu`	`host`	`input`	Specifies options for computing all or part of the matrix `U`: = 'A': all m columns of U are returned in array U: = 'S': the first min(m,n) columns of U (the left singular vectors) are returned in the array U; = 'O': the first min(m,n) columns of U (the left singular vectors) are overwritten on the array A; = 'N': no columns of U (no left singular vectors) are computed.
`jobvt`	`host`	`input`	Specifies options for computing all or part of the matrix V**T: = 'A': all N rows of V**T are returned in the array VT; = 'S': the first min(m,n) rows of V**T (the right singular vectors) are returned in the array VT; = 'O': the first min(m,n) rows of V**T (the right singular vectors) are overwritten on the array A; = 'N': no rows of V**T (no right singular vectors) are computed.
`m`	`host`	`input`	Number of rows of matrix `A`.
`n`	`host`	`input`	Number of columns of matrix `A`.
`A`	`device`	`in/out`	<type> array of dimension `lda * n` with `lda` is not less than `max(1,m)`. On exit, the contents of `A` are destroyed.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `A`.

Parameter	Memory	In/out	Meaning
S	device	output	Real array of dimension min(m,n). The singular values of A, sorted so that S(i) >= S(i+1).
U	device	output	<type> array of dimension ldu * m with ldu is not less than max(1,m). U contains the m×m unitary matrix U.
ldu	host	input	Leading dimension of two-dimensional array used to store matrix U.
VT	device	output	<type> array of dimension ldvt * n with ldvt is not less than max(1,n). VT contains the n×n unitary matrix V**T.
ldvt	host	input	Leading dimension of two-dimensional array used to store matrix Vt.
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of work, returned by gesvd_bufferSize.
rwork	device	input	Real array of dimension min(m,n)-1. It contains the unconverged superdiagonal elements of an upper bidiagonal matrix if devInfo > 0.
devInfo	device	output	If devInfo = 0, the operation is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle). If devInfo > 0, devInfo indicates how many superdiagonals of an intermediate bidiagonal form did not converge to zero.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or lda<max(1,m) or ldu<max(1,m) or ldvt<max(1,n)).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.7. cusolverDnGesvd()[DEPRECATED]

[[DEPRECATED]] use cusolverDnXgesvd() instead. The routine will be removed in the next major release.

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t cusolverDnGesvd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    signed char jobu,
    signed char jobvt,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType dataTypeS,
    const void *S,
    cudaDataType dataTypeU,
    const void *U,
    int64_t ldu,
    cudaDataType dataTypeVT,
    const void *VT,
    int64_t ldvt,
    cudaDataType computeType,
    size_t *workspaceInBytes);
```

The routine below:

```
cusolverStatus_t CUSOLVERAPI cusolverDnGesvd(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    signed char jobu,
    signed char jobvt,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    cudaDataType dataTypeS,
    void *S,
    cudaDataType dataTypeU,
    void *U,
    int64_t ldu,
    cudaDataType dataTypeVT,
    void *VT,
    int64_t ldvt,
    cudaDataType computeType,
    void *pBuffer,
    size_t workspaceInBytes,
    int *info);
```

This function computes the singular value decomposition (SVD) of a `m×n` matrix `A` and corresponding the left and/or right singular vectors. The SVD is written

$$A = U * \Sigma * V^H$$

where `Σ` is an `m×n` matrix which is zero except for its `min(m,n)` diagonal elements, `U` is an `m×m` unitary matrix, and `V` is an `n×n` unitary matrix. The diagonal elements of `Σ` are the singular values of `A`; they are real and non-negative, and are returned in descending order. The first `min(m,n)` columns of `U` and `V` are the left and right singular vectors of `A`.

The user has to provide working space which is pointed by input parameter `pBuffer`. The input parameter `workspaceInBytes` is size in bytes of the working space, and it is returned by `cusolverDnGesvd_bufferSize()`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). if `bdsqr` did not converge, `info` specifies how many superdiagonals of an intermediate bidiagonal form did not converge to zero.

Currently, `cusolverDnGesvd` supports only the default algorithm.

Table of algorithms supported by cusolverDnGesvd

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

Appendix G.5 provides a simple example of `cusolverDnGesvd`.

Remark 1: `gesvd` only supports `m>=n`.

Remark 2: the routine returns V^H , not `V`.

List of input arguments for `cusolverDnGesvd_bufferSize` and `cusolverDnGesvd`:

API of cusolverDnGesvd

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	handle to the cuSolverDN library context.
`params`	`host`	`input`	structure with information collected by `cusolverDnSetAdvOptions`.
`jobu`	`host`	`input`	specifies options for computing all or part of the matrix `U`: = 'A': all m columns of U are returned in array U: = 'S': the first min(m,n) columns of U (the left singular vectors) are returned in the array U; = 'O': the first min(m,n) columns of U (the left singular vectors) are overwritten on the array A; = 'N': no columns of U (no left singular vectors) are computed.
`jobvt`	`host`	`input`	specifies options for computing all or part of the matrix V**T: = 'A': all N rows of V**T are returned in the array VT; = 'S': the first min(m,n) rows of V**T (the right singular vectors) are returned in the array VT; = 'O': the first min(m,n) rows of V**T (the right singular vectors) are overwritten on the array A; = 'N': no rows of V**T (no right singular vectors) are computed.
`m`	`host`	`input`	number of rows of matrix `A`.
`n`	`host`	`input`	number of columns of matrix `A`.
`dataTypeA`	`host`	`input`	data type of array `A`.
`A`	`device`	`in/out`	array of dimension `lda * n` with `lda` is not less than `max(1,m)`. On exit, the contents of `A` are destroyed.
`lda`	`host`	`input`	leading dimension of two-dimensional array used to store matrix `A`.
`dataTypeS`	`host`	`input`	data type of array `S`.

Parameter	Memory	In/out	Meaning
S	device	output	real array of dimension min(m,n). The singular values of A, sorted so that S(i) >= S(i+1).
dataTypeU	host	input	data type of array U.
U	device	output	array of dimension ldu * m with ldu is not less than max(1,m). U contains the m×m unitary matrix U.
ldu	host	input	leading dimension of two-dimensional array used to store matrix U.
dataTypeVT	host	input	data type of array VT.
VT	device	output	array of dimension ldvt * n with ldvt is not less than max(1,n). VT contains the n×n unitary matrix V**T.
ldvt	host	input	leading dimension of two-dimensional array used to store matrix Vt.
computeType	host	input	data type of computation.
pBuffer	device	in/out	Working space. Array of type void of size workspaceInBytes bytes.
workspaceInBytes	host	input	Size in bytes of pBuffer, returned by cusolverDnGesvd_bufferSize.
info	device	output	if info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle). if info > 0, info indicates how many superdiagonals of an intermediate bidiagonal form did not converge to zero.

The generic API has three different types, dataTypeA is data type of the matrix A, dataTypeS is data type of the vector S and dataTypeU is data type of the matrix U, dataTypeVT is data type of the matrix VT, computeType is compute type of the operation. cusolverDnGesvd only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	DataTypeS	DataTypeU	DataTypeVT	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SGESVD
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DGESVD
CUDA_C_32F	CUDA_R_32F	CUDA_C_32F	CUDA_C_32F	CUDA_C_32F	CGESVD
CUDA_C_64F	CUDA_R_64F	CUDA_C_64F	CUDA_C_64F	CUDA_C_64F	ZGESVD

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or lda<max(1,m) or ldu<max(1,m) or ldvt<max(1,n)).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.8. cusolverDn<t>gesvdj()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSgesvdj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int econ,
    int m,
    int n,
    const float *A,
    int lda,
    const float *S,
    const float *U,
    int ldu,
    const float *V,
    int ldv,
    int *lwork,
    gesvdjInfo_t params);

cusolverStatus_t
cusolverDnDgesvdj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int econ,
    int m,
    int n,
    const double *A,
    int lda,
    const double *S,
    const double *U,
    int ldu,
    const double *V,
    int ldv,
    int *lwork,
    gesvdjInfo_t params);

cusolverStatus_t
cusolverDnCgesvdj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int econ,
    int m,
    int n,
    const cuComplex *A,
    int lda,
    const float *S,
    const cuComplex *U,
    int ldu,
    const cuComplex *V,
    int ldv,
    int *lwork,
```

```
    gesvdjInfo_t params);

cusolverStatus_t
cusolverDnZgesvdj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int econ,
    int m,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const double *S,
    const cuDoubleComplex *U,
    int ldu,
    const cuDoubleComplex *V,
    int ldv,
    int *lwork,
    gesvdjInfo_t params);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSgesvdj(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int econ,
    int m,
    int n,
    float *A,
    int lda,
    float *S,
    float *U,
    int ldu,
    float *V,
    int ldv,
    float *work,
    int lwork,
    int *info,
    gesvdjInfo_t params);

cusolverStatus_t
cusolverDnDgesvdj(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int econ,
    int m,
    int n,
    double *A,
    int lda,
    double *S,
    double *U,
    int ldu,
    double *V,
    int ldv,
    double *work,
    int lwork,
    int *info,
    gesvdjInfo_t params);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCgesvdj(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int econ,
    int m,
    int n,
    cuComplex *A,
    int lda,
    float *S,
    cuComplex *U,
    int ldu,
    cuComplex *V,
    int ldv,
    cuComplex *work,
    int lwork,
    int *info,
    gesvdjInfo_t params);

cusolverStatus_t
cusolverDnZgesvdj(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int econ,
    int m,
    int n,
    cuDoubleComplex *A,
    int lda,
    double *S,
    cuDoubleComplex *U,
    int ldu,
    cuDoubleComplex *V,
    int ldv,
    cuDoubleComplex *work,
    int lwork,
    int *info,
    gesvdjInfo_t params);
```

This function computes the singular value decomposition (SVD) of a `m×n` matrix `A` and corresponding the left and/or right singular vectors. The SVD is written:

$$A = U * \Sigma * V^H$$

where `Σ` is an `m×n` matrix which is zero except for its `min(m,n)` diagonal elements, `U` is an `m×m` unitary matrix, and `V` is an `n×n` unitary matrix. The diagonal elements of `Σ` are the singular values of `A`; they are real and non-negative, and are returned in descending order. The first `min(m,n)` columns of `U` and `V` are the left and right singular vectors of `A`.

`gesvdj` has the same functionality as `gesvd`. The difference is that `gesvd` uses QR algorithm and `gesvdj` uses Jacobi method. The parallelism of Jacobi method gives GPU better

performance on small and medium size matrices. Moreover the user can configure `gesvdj` to perform approximation up to certain accuracy.

`gesvdj` iteratively generates a sequence of unitary matrices to transform matrix `A` to the following form

$$U^H * A * V = S + E$$

where `S` is diagonal and diagonal of `E` is zero.

During the iterations, the Frobenius norm of `E` decreases monotonically. As `E` goes down to zero, `S` is the set of singular values. In practice, Jacobi method stops if

$$||\mathrm{E}||_F \leq \mathrm{eps} * ||\mathrm{A}||_F$$

where `eps` is given tolerance.

`gesvdj` has two parameters to control the accuracy. First parameter is tolerance (`eps`). The default value is machine accuracy but The user can use function `cusolverDnXgesvdjSetTolerance` to set a priori tolerance. The second parameter is maximum number of sweeps which controls number of iterations of Jacobi method. The default value is 100 but the user can use function `cusolverDnXgesvdjSetMaxSweeps` to set a proper bound. The experimentis show 15 sweeps are good enough to converge to machine accuracy. `gesvdj` stops either tolerance is met or maximum number of sweeps is met.

Jacobi method has quadratic convergence, so the accuracy is not proportional to number of sweeps. To guarantee certain accuracy, the user should configure tolerance only.

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is the size of the working space, and it is returned by `gesvdj_bufferSize()`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). If `info = min(m,n)+1`, `gesvdj` does not converge under given tolerance and maximum sweeps.

If the user sets an improper tolerance, `gesvdj` may not converge. For example, tolerance should not be smaller than machine accuracy.

Please visit cuSOLVER Library Samples - gesvdj for a code example.

Remark 1: `gesvdj` supports any combination of `m` and `n`.

Remark 2: the routine returns `V`, not V^H . This is different from `gesvd`.

API of gesvdj

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`jobz`	`host`	`input`	Specifies options to either compute singular value only or singular vectors as well: `jobz` = `CUSOLVER_EIG_MODE_NOVECTOR` : Compute singular values only; `jobz` = `CUSOLVER_EIG_MODE_VECTOR` : Compute singular values and singular vectors.

Parameter	Memory	In/out	Meaning
econ	host	input	econ = 1 for economy size for U and V.
m	host	input	Number of rows of matrix A.
n	host	input	Number of columns of matrix A.
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,m). On exit, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
S	device	output	Real array of dimension min(m,n). The singular values of A, sorted so that S(i) >= S(i+1).
U	device	output	<type> array of dimension ldu * m if econ is zero. If econ is nonzero, the dimension is ldu * min(m,n). U contains the left singular vectors.
ldu	host	input	Leading dimension of two-dimensional array used to store matrix U. ldu is not less than max(1,m).
V	device	output	<type> array of dimension ldv * n if econ is zero. If econ is nonzero, the dimension is ldv * min(m,n). V contains the right singular vectors.
ldv	host	input	Leading dimension of two-dimensional array used to store matrix V. ldv is not less than max(1,n).
work	device	in/out	<type> array of size lwork, working space.
lwork	host	input	Size of work, returned by gesvdj_bufferSize.
info	device	output	If info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle). if info = min(m,n)+1, gesvdj dose not converge under given tolerance and maximum sweeps.
params	host	in/out	Structure filled with parameters of Jacobi algorithm and results of gesvdj.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or lda<max(1,m) or ldu<max(1,m) or ldv<max(1,n) or jobz is not

	CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.9. cusolverDn<t>gesvdjBatched()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSgesvdjBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int m,
    int n,
    const float *A,
    int lda,
    const float *S,
    const float *U,
    int ldu,
    const float *V,
    int ldv,
    int *lwork,
    gesvdjInfo_t params,
    int batchSize);

cusolverStatus_t
cusolverDnDgesvdjBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int m,
    int n,
    const double *A,
    int lda,
    const double *S,
    const double *U,
    int ldu,
    const double *V,
    int ldv,
    int *lwork,
    gesvdjInfo_t params,
    int batchSize);

cusolverStatus_t
cusolverDnCgesvdjBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int m,
    int n,
    const cuComplex *A,
    int lda,
    const float *S,
    const cuComplex *U,
    int ldu,
    const cuComplex *V,
    int ldv,
    int *lwork,
    gesvdjInfo_t params,
```

```
    int batchSize);

cusolverStatus_t
cusolverDnZgesvdjBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int m,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const double *S,
    const cuDoubleComplex *U,
    int ldu,
    const cuDoubleComplex *V,
    int ldv,
    int *lwork,
    gesvdjInfo_t params,
    int batchSize);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSgesvdjBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int m,
    int n,
    float *A,
    int lda,
    float *S,
    float *U,
    int ldu,
    float *V,
    int ldv,
    float *work,
    int lwork,
    int *info,
    gesvdjInfo_t params,
    int batchSize);

cusolverStatus_t
cusolverDnDgesvdjBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int m,
    int n,
    double *A,
    int lda,
    double *S,
    double *U,
    int ldu,
    double *V,
    int ldv,
    double *work,
    int lwork,
    int *info,
    gesvdjInfo_t params,
    int batchSize);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCgesvdjBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int m,
    int n,
    cuComplex *A,
    int lda,
    float *S,
    cuComplex *U,
    int ldu,
    cuComplex *V,
    int ldv,
    cuComplex *work,
    int lwork,
    int *info,
    gesvdjInfo_t params,
    int batchSize);

cusolverStatus_t
cusolverDnZgesvdjBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int m,
    int n,
    cuDoubleComplex *A,
    int lda,
    double *S,
    cuDoubleComplex *U,
    int ldu,
    cuDoubleComplex *V,
    int ldv,
    cuDoubleComplex *work,
    int lwork,
    int *info,
    gesvdjInfo_t params,
    int batchSize);
```

This function computes singular values and singular vectors of a squence of general `m×n` matrices

$$A_j = U_j * \Sigma_j * V_j^H$$

where Σ_j is a real `m×n` diagonal matrix which is zero except for its `min(m,n)` diagonal elements. U_j (left singular vectors) is a `m×m` unitary matrix and V_j (right singular vectors) is a `n×n` unitary matrix. The diagonal elements of Σ_j are the singular values of A_j in either descending order or non-sorting order.

`gesvdjBatched` performs `gesvdj` on each matrix. It requires that all matrices are of the same size `m,n` no greater than 32 and are packed in contiguous way,

$$A = (\mathrm{A0} \quad \mathrm{A1} \quad \cdots)$$

Each matrix is column-major with leading dimension `lda`, so the formula for random access is $A_k(i,j) = A[\,i + lda*j + lda*n*k\,]$.

The parameter `S` also contains singular values of each matrix in contiguous way,

$$S = (S0 \quad S1 \quad \cdots)$$

The formula for random access of `S` is $S_k(j) = S[\,j + \min(m,n)*k\,]$.

Except for tolerance and maximum sweeps, `gesvdjBatched` can either sort the singular values in descending order (default) or chose as-is (without sorting) by the function `cusolverDnXgesvdjSetSortEig`. If the user packs several tiny matrices into diagonal blocks of one matrix, non-sorting option can separate singular values of those tiny matrices.

`gesvdjBatched` cannot report residual and executed sweeps by function `cusolverDnXgesvdjGetResidual` and `cusolverDnXgesvdjGetSweeps`. Any call of the above two returns `CUSOLVER_STATUS_NOT_SUPPORTED`. The user needs to compute residual explicitly.

The user has to provide working space pointed by input parameter `work`. The input parameter `lwork` is the size of the working space, and it is returned by `gesvdjBatched_bufferSize()`.

The output parameter `info` is an integer array of size `batchSize`. If the function returns `CUSOLVER_STATUS_INVALID_VALUE`, the first element `info[0] = -i` (less than zero) indicates `i-th` parameter is wrong (not counting handle). Otherwise, if `info[i] = min(m,n)+1`, `gesvdjBatched` does not converge on `i-th` matrix under given tolerance and maximum sweeps.

Please visit cuSOLVER Library Samples - gesvdjBatched for a code example.

API of syevjBatched

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`jobz`	`host`	`input`	Specifies options to either compute singular value only or singular vectors as well: `jobz` = `CUSOLVER_EIG_MODE_NOVECTOR` : Compute singular values only; `jobz` = `CUSOLVER_EIG_MODE_VECTOR` : Compute singular values and singular vectors.
`m`	`host`	`input`	Number of rows of matrix `Aj`. `m` is no greater than 32.
`n`	`host`	`input`	Number of columns of matrix `Aj`. `n` is no greater than 32.
`A`	`device`	`in/out`	<type> array of dimension `lda * n * batchSize` with `lda` is not less than `max(1,n)`. on Exit: the contents of `Aj` are destroyed.
`lda`	`host`	`input`	Lading dimension of two-dimensional array used to store matrix `Aj`.
`S`	`device`	`output`	Areal array of dimension `min(m,n)*batchSize`. It stores the

Parameter	Memory	In/out	Meaning
			singular values of `Aj` in descending order or non-sorting order.
`U`	`device`	`output`	<type> array of dimension `ldu * m * batchSize`. `Uj` contains the left singular vectors of `Aj`.
`ldu`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `Uj`. `ldu` is not less than `max(1,m)`.
`V`	`device`	`output`	<type> array of dimension `ldv * n * batchSize`. `Vj` contains the right singular vectors of `Aj`.
`ldv`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `Vj`. `ldv` is not less than `max(1,n)`.
`work`	`device`	`in/out`	<type> array of size `lwork`, working space.
`lwork`	`host`	`input`	Size of `work`, returned by `gesvdjBatched_bufferSize`.
`info`	`device`	`output`	An integer array of dimension `batchSize`. If `CUSOLVER_STATUS_INVALID_VALUE` is returned, `info[0] = -i` (less than zero) indicates `i-th` parameter is wrong (not counting handle). Otherwise, if `info[i] = 0`, the operation is successful. if `info[i] = min(m,n)+1`, `gesvdjBatched` dose not converge on `i-th` matrix under given tolerance and maximum sweeps.
`params`	`host`	`in/out`	Structure filled with parameters of Jacobi algorithm.
`batchSize`	`host`	`input`	Number of matrices.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.
`CUSOLVER_STATUS_INVALID_VALUE`	Invalid parameters were passed (`m,n<0` or `lda<max(1,m)` or `ldu<max(1,m)` or `ldv<max(1,n)` or `jobz` is not `CUSOLVER_EIG_MODE_NOVECTOR` or `CUSOLVER_EIG_MODE_VECTOR` , or `batchSize<0`).
`CUSOLVER_STATUS_INTERNAL_ERROR`	An internal operation failed.

2.4.3.10. cusolverDn<t>gesvdaStridedBatched()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSgesvdaStridedBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int rank,
    int m,
    int n,
    const float *A,
    int lda,
    long long int strideA,
    const float *S,
    long long int strideS,
    const float *U,
    int ldu,
    long long int strideU,
    const float *V,
    int ldv,
    long long int strideV,
    int *lwork,
    int batchSize);

cusolverStatus_t
cusolverDnDgesvdaStridedBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int rank,
    int m,
    int n,
    const double *A,
    int lda,
    long long int strideA,
    const double *S,
    long long int strideS,
    const double *U,
    int ldu,
    long long int strideU,
    const double *V,
    int ldv,
    long long int strideV,
    int *lwork,
    int batchSize);

cusolverStatus_t
cusolverDnCgesvdaStridedBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int rank,
    int m,
    int n,
    const cuComplex *A,
    int lda,
    long long int strideA,
    const float *S,
```

```
    long long int strideS,
    const cuComplex *U,
    int ldu,
    long long int strideU,
    const cuComplex *V,
    int ldv,
    long long int strideV,
    int *lwork,
    int batchSize);

cusolverStatus_t
cusolverDnZgesvdaStridedBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int rank,
    int m,
    int n,
    const cuDoubleComplex *A,
    int lda,
    long long int strideA,
    const double *S,
    long long int strideS,
    const cuDoubleComplex *U,
    int ldu,
    long long int strideU,
    const cuDoubleComplex *V,
    int ldv,
    long long int strideV,
    int *lwork,
    int batchSize);
```

The S and D data types are real valued single and double precision, respectively.

```


cusolverStatus_t
cusolverDnSgesvdaStridedBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int rank,
    int m,
    int n,
    const float *A,
    int lda,
    long long int strideA,
    float *S,
    long long int strideS,
    float *U,
    int ldu,
    long long int strideU,
    float *V,
    int ldv,
    long long int strideV,
    float *work,
    int lwork,
    int *info,
    double *h_R_nrmF,
    int batchSize);

cusolverStatus_t
```

```
cusolverDnDgesvdaStridedBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int rank,
    int m,
    int n,
    const double *A,
    int lda,
    long long int strideA,
    double *S,
    long long int strideS,
    double *U,
    int ldu,
    long long int strideU,
    double *V,
    int ldv,
    long long int strideV,
    double *work,
    int lwork,
    int *info,
    double *h_R_nrmF,
    int batchSize);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCgesvdaStridedBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int rank,
    int m,
    int n,
    const cuComplex *A,
    int lda,
    long long int strideA,
    float *S,
    long long int strideS,
    cuComplex *U,
    int ldu,
    long long int strideU,
    cuComplex *V,
    int ldv,
    long long int strideV,
    cuComplex *work,
    int lwork,
    int *info,
    double *h_R_nrmF,
    int batchSize);

cusolverStatus_t
cusolverDnZgesvdaStridedBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    int rank,
    int m,
    int n,
    const cuDoubleComplex *A,
    int lda,
```

```
    long long int strideA,
    double *S,
    long long int strideS,
    cuDoubleComplex *U,
    int ldu,
    long long int strideU,
    cuDoubleComplex *V,
    int ldv,
    long long int strideV,
    cuDoubleComplex *work,
    int lwork,
    int *info,
    double *h_R_nrmF,
    int batchSize);
```

This function `gesvda` (a stands for approximate) approximates the singular value decomposition of a tall skinny `m×n` matrix `A` and corresponding the left and right singular vectors. The economy form of SVD is written by

$$A = U * \Sigma * V^H$$

where `Σ` is an `n×n` matrix. `U` is an `m×n` unitary matrix, and `V` is an `n×n` unitary matrix. The diagonal elements of `Σ` are the singular values of `A`; they are real and non-negative, and are returned in descending order. `U` and `V` are the left and right singular vectors of `A`.

`gesvda` computes eigenvalues of `A**T*A` to approximate singular values and singular vectors. It generates matrices `U` and `V` and transforms the matrix `A` to the following form

$$U^H * A * V = S + E$$

where `S` is diagonal and `E` depends on rounding errors. To certain conditions, `U`, `V` and `S` approximate singular values and singular vectors up to machine zero of single precision. In general, `V` is unitary, `S` is more accurate than `U`. If singular value is far from zero, then left singular vector `U` is accurate. In other words, the accuracy of singular values and left singular vectors depend on the distance between singular value and zero.

The input parameter `rank` decides the number of singualr values and singular vectors are computed in parameter `S`, `U` and `V`.

The output parameter `h_RnrmF` computes Frobenius norm of residual.

$$A - \mathrm{U} * S * V^H$$

if the parameter `rank` is equal `n`. Otherwise, `h_RnrmF` reports

$$||\mathrm{U} * S * V^H|| - ||\mathrm{S}||$$

in Frobenius norm sense, that is, how far `U` is from unitary.

`gesvdaStridedBatched` performs `gesvda` on each matrix. It requires that all matrices are of the same size `m,n` and are packed in a contiguous way,

$$A = (\mathrm{A0} \quad \mathrm{A1} \quad \cdots)$$

Each matrix is column-major with leading dimension `lda`, so the formula for random access is $A_k(i,j) = A[i + lda*j + strideA*k]$. Similarly, the formula for random access of `S` is $S_k(j) = S[j + strideS*k]$, the formula for random access of `U` is $U_k(i,j) = U[i + ldu*j + strideU*k]$ and the formula for random access of `V` is $V_k(i,j) = V[i + ldv*j + strideV*k]$.

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is the size of the working space, and it is returned by `gesvdaStridedBatched_bufferSize()`.

The output parameter `info` is an integer array of size `batchSize`. If the function returns `CUSOLVER_STATUS_INVALID_VALUE`, the first element `info[0] = -i` (less than zero) indicates `i-th` parameter is wrong (not counting handle). Otherwise, if `info[i] = min(m,n)+1`, `gesvdaStridedBatched` does not converge on `i-th` matrix under given tolerance.

Please visit cuSOLVER Library Samples - gesvdaStridedBatched for a code example.

Remark 1: the routine returns `V`, not V^H. This is different from `gesvd`.

Remark 2: if the user is confident on the accuracy of singular values and singular vectors, for example, certain conditions hold (required singular value is far from zero), then the performance can be improved by passing a null pointer to `h_RnrmF`, i.e. no computation of residual norm.

API of gesvda

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`jobz`	`host`	`input`	Specifies options to either compute singular value only or singular vectors as well: `jobz` = `CUSOLVER_EIG_MODE_NOVECTOR` : Compute singular values only; `jobz` = `CUSOLVER_EIG_MODE_VECTOR` : Compute singular values and singular vectors.
`rank`	`host`	`input`	Number of singular values (from largest to smallest).
`m`	`host`	`input`	Number of rows of matrix `Aj`.
`n`	`host`	`input`	Number of columns of matrix `Aj`.
`A`	`device`	`input`	<type> array of dimension `strideA * batchSize` with `lda` is not less than `max(1,m)`. `Aj` is of dimension `m * n`.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `Aj`.
`strideA`	`host`	`input`	Value of type long long int that gives the address offset between `A[i]` and `A[i+1]`. `strideA` is not less than `lda*n`.
`S`	`device`	`output`	A real array of dimension `strideS*batchSize`. It stores the

Parameter	Memory	In/out	Meaning
			singular values of `Aj` in descending order. `Sj` is of dimension `rank * 1`
`strideS`	`host`	`input`	Value of type long long int that gives the address offset between `S[i]` and `S[i+1]`. `strideS` is not less than `rank`.
`U`	`device`	`output`	<type> array of dimension `strideU * batchSize`. `Uj` contains the left singular vectors of `Aj`. `Uj` is of dimension `m * rank`.
`ldu`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `Uj`. `ldu` is not less than `max(1,m)`.
`strideU`	`host`	`input`	Value of type long long int that gives the address offset between `U[i]` and `U[i+1]`. `strideU` is not less than `ldu*rank`.
`V`	`device`	`output`	<type> array of dimension `strideV * batchSize`. `Vj` contains the right singular vectors of `Aj`. `Vj` is of dimension `n * rank`.
`ldv`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `Vj`. `ldv` is not less than `max(1,n)`.
`strideV`	`host`	`input`	Value of type long long int that gives the address offset between `V[i]` and `V[i+1]`. `strideV` is not less than `ldv*rank`.
`work`	`device`	`in/out`	<type> array of size `lwork`, working space.
`lwork`	`host`	`input`	Size of `work`, returned by `gesvdaStridedBatched_bufferSize`.
`info`	`device`	`output`	An integer array of dimension `batchSize`. If `CUSOLVER_STATUS_INVALID_VALUE` is returned, `info[0] = -i` (less than zero) indicates `i-th` parameter is wrong (not counting handle). Otherwise, if `info[i] = 0`, the operation is successful. if `info[i] = min(m,n)+1`, `gesvdaStridedBatched` dose not converge on `i-th` matrix.
`h_RnrmF`	`host`	`output`	<double> array of size `batchSize`. `h_RnrmF[i]` is norm of residual of `i-th` matrix.
`batchSize`	`host`	`input`	Number of matrices. `batchSize` is not less than 1.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.

CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or lda<max(1,m) or ldu<max(1,m) or ldv<max(1,n) or strideA<lda*n or strideS<rank or strideU<ldu*rank or strideV<ldv*rank or batchSize<1 or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.11. cusolverDn<t>syevd()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSsyevd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const float *A,
    int lda,
    const float *W,
    int *lwork);

cusolverStatus_t
cusolverDnDsyevd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const double *A,
    int lda,
    const double *W,
    int *lwork);

cusolverStatus_t
cusolverDnCheevd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuComplex *A,
    int lda,
    const float *W,
    int *lwork);

cusolverStatus_t
cusolverDnZheevd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const double *W,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSsyevd(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    float *A,
    int lda,
    float *W,
    float *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDsyevd(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    double *A,
    int lda,
    double *W,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCheevd(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuComplex *A,
    int lda,
    float *W,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZheevd(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuDoubleComplex *A,
    int lda,
    double *W,
    cuDoubleComplex *work,
    int lwork,
    int *devInfo);
```

This function computes eigenvalues and eigenvectors of a symmetric (Hermitian) n×n matrix A. The standard symmetric eigenvalue problem is

$$A*V = V*\Lambda$$

where Λ is a real n×n diagonal matrix. V is an n×n unitary matrix. The diagonal elements of Λ are the eigenvalues of A in ascending order.

The user has to provide working space which is pointed by input parameter work. The input parameter lwork is size of the working space, and it is returned by syevd_bufferSize().

If output parameter devInfo = -i (less than zero), the i-th parameter is wrong (not counting handle). If devInfo = i (greater than zero), i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

If jobz = CUSOLVER_EIG_MODE_VECTOR, A contains the orthonormal eigenvectors of the matrix A. The eigenvectors are computed by a divide and conquer algorithm.

Please visit cuSOLVER Library Samples - syevd for a code example.

API of syevd

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
jobz	host	input	Specifies options to either compute eigenvalue only or compute eigen-pair: jobz = CUSOLVER_EIG_MODE_NOVECTOR : Compute eigenvalues only; jobz = CUSOLVER_EIG_MODE_VECTOR : Compute eigenvalues and eigenvectors.
uplo	host	input	Specifies which part of A is stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of A is stored. uplo = CUBLAS_FILL_MODE_UPPER: Upper triangle of A is stored.
n	host	input	Number of rows (or columns) of matrix A.
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A. On exit, if jobz = CUSOLVER_EIG_MODE_VECTOR, and devInfo = 0, A contains the orthonormal eigenvectors of the matrix A. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.

Parameter	Memory	In/out	Meaning
W	device	output	A real array of dimension n. The eigenvalue values of A, in ascending order ie, sorted so that W(i) <= W(i+1).
work	device	in/out	Working space, <type> array of size lwork.
Lwork	host	input	Size of work, returned by syevd_bufferSize.
devInfo	device	output	If devInfo = 0, the operation is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle). if devInfo = i (> 0), devInfo indicates i off-diagonal elements of an intermediate tridiagonal form did not converge to zero;

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, or lda<max(1,n), or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.12. cusolverDnSyevd()[DEPRECATED]

[[DEPRECATED]] use cusolverDnXsyevd() instead. The routine will be removed in the next major release.

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
 cusolverDnSyevd_bufferSize(
 cusolverDnHandle_t handle,
 cusolverParams_t params,
 cusolverEigMode_t jobz,
 cublasFillMode_t uplo,
 int n,
 cudaDataType dataTypeA,
 const void *A,
 int64_t lda,
 cudaDataType dataTypeW,
 const void *W,
 cudaDataType computeType,
 size_t *workspaceInBytes);
```

The routine bellow

```
cusolverStatus_t
 cusolverDnSyevd(
 cusolverDnHandle_t handle,
 cusolverParams_t params,
 cusolverEigMode_t jobz,
 cublasFillMode_t uplo,
 int n,
 cudaDataType dataTypeA,
 const void *A,
 int64_t lda,
 cudaDataType dataTypeW,
 const void *W,
 cudaDataType computeType,
 void *pBuffer,
 size_t workspaceInBytes,
 int *info);
```

computes eigenvalues and eigenvectors of a symmetric (Hermitian) `n×n` matrix `A` using the generic API interface. The standard symmetric eigenvalue problem is

$$A*V = V*\Lambda$$

where `Λ` is a real `n×n` diagonal matrix. `V` is an `n×n` unitary matrix. The diagonal elements of `Λ` are the eigenvalues of `A` in ascending order.

The user has to provide working space which is pointed by input parameter `pBuffer`. The input parameter `workspaceInBytes` is size in bytes of the working space, and it is returned by `cusolverDnSyevd_bufferSize()`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). If `info = i` (greater than zero), `i` off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

if `jobz` = CUSOLVER_EIG_MODE_VECTOR, `A` contains the orthonormal eigenvectors of the matrix `A`. The eigenvectors are computed by a divide and conquer algorithm.

Currently, `cusolverDnSyevd` supports only the default algorithm.

Table of algorithms supported by cusolverDnSyevd

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

List of input arguments for `cusolverDnSyevd_bufferSize` and `cusolverDnSyevd`:

API of cusolverDnSyevd

parameter	Memory	In/out	Meaning
handle	host	input	handle to the cuSolverDN library context.
params	host	input	structure with information collected by `cusolverDnSetAdvOptions`.
jobz	host	input	specifies options to either compute eigenvalue only or compute eigen-pair: `jobz` = `CUSOLVER_EIG_MODE_NOVECTOR` : Compute eigenvalues only; `jobz` = `CUSOLVER_EIG_MODE_VECTOR` : Compute eigenvalues and eigenvectors.

uplo	host	input	specifies which part of A is stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of A is stored. uplo = CUBLAS_FILL_MODE_UPPER: Upper triangle of A is stored.
n	host	input	number of rows (or columns) of matrix A.
dataTypeA	host	in	data type of array A.
A	device	in/out	array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A. On exit, if jobz = CUSOLVER_EIG_MODE_VECTOR, and info = 0, A contains the orthonormal eigenvectors of the matrix A. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
lda	host	input	leading dimension of two-dimensional array used to store matrix A.
dataTypeW	host	in	data type of array W.
W	device	output	a real array of dimension n. The eigenvalue values of A, in ascending order ie, sorted so that W(i) <= W(i+1).
computeType	host	in	data type of computation.
pBuffer	device	in/out	Working space. Array of type void of size workspaceInBytes bytes.
workspaceInBytes	host	input	Size in bytes of pBuffer, returned by cusolverDnSyevd_bufferSize.
info	device	output	if info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle). if info = i (> 0), info indicates i off-diagonal elements of an intermediate tridiagonal form did not converge to zero;

The generic API has three different types, dataTypeA is data type of the matrix A, dataTypeW is data type of the matrix W and computeType is compute type of the operation. cusolverDnSyevd only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	DataTypeW	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SSYEVD
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DSYEVD

CUDA_C_32F	CUDA_R_32F	CUDA_C_32F	CHEEVD
CUDA_C_64F	CUDA_R_64F	CUDA_C_64F	ZHEEVD

Status Returned

CUSOLVER_STATUS_SUCCESS	the operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	invalid parameters were passed (n<0, or lda<max(1,n), or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	an internal operation failed.

2.4.3.13. cusolverDn<t>syevdx()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSsyevdx_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    const float *A,
    int lda,
    float vl,
    float vu,
    int il,
    int iu,
    int *h_meig,
    const float *W,
    int *lwork);

cusolverStatus_t
cusolverDnDsyevdx_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    const double *A,
    int lda,
    double vl,
    double vu,
    int il,
    int iu,
    int *h_meig,
    const double *W,
    int *lwork);
```

```
cusolverStatus_t
cusolverDnCheevdx_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    const cuComplex *A,
    int lda,
    float vl,
    float vu,
    int il,
    int iu,
    int *h_meig,
    const float *W,
    int *lwork);

cusolverStatus_t
cusolverDnZheevdx_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    const cuDoubleComplex *A,
    int lda,
    double vl,
    double vu,
    int il,
    int iu,
    int *h_meig,
    const double *W,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSsyevdx(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    float *A,
    int lda,
    float vl,
    float vu,
    int il,
    int iu,
    int *h_meig,
    float *W,
    float *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDsyevdx(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
```

```
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    double *A,
    int lda,
    double vl,
    double vu,
    int il,
    int iu,
    int *h_meig,
    double *W,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCheevdx(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    cuComplex *A,
    int lda,
    float vl,
    float vu,
    int il,
    int iu,
    int *h_meig,
    float *W,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZheevdx(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    cuDoubleComplex *A,
    int lda,
    double vl,
    double vu,
    int il,
    int iu,
    int *h_meig,
    double *W,
    cuDoubleComplex *work,
    int lwork,
    int *devInfo);
```

This function computes all or selection of the eigenvalues and optionally eigenvectors of a symmetric (Hermitian) `n×n` matrix `A`. The standard symmetric eigenvalue problem is:

$$A*V = V*\Lambda$$

where Λ is a real n×h_meig diagonal matrix. V is an n×h_meig unitary matrix. h_meig is the number of eigenvalues/eigenvectors computed by the routine, h_meig is equal to n when the whole spectrum (e.g., range = CUSOLVER_EIG_RANGE_ALL) is requested. The diagonal elements of Λ are the eigenvalues of A in ascending order.

The user has to provide working space which is pointed by input parameter work. The input parameter lwork is size of the working space, and it is returned by syevdx_bufferSize().

If output parameter devInfo = -i (less than zero), the i-th parameter is wrong (not counting handle). If devInfo = i (greater than zero), i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

If jobz = CUSOLVER_EIG_MODE_VECTOR, A contains the orthonormal eigenvectors of the matrix A. The eigenvectors are computed by a divide and conquer algorithm.

Please visit cuSOLVER Library Samples - syevdx for a code example.

API of syevdx

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
jobz	host	input	Specifies options to either compute eigenvalue only or compute eigen-pair: jobz = CUSOLVER_EIG_MODE_NOVECTOR : Compute eigenvalues only; jobz = CUSOLVER_EIG_MODE_VECTOR : Compute eigenvalues and eigenvectors.
range	host	input	Specifies options to which selection of eigenvalues and optionally eigenvectors that need to be computed: range = CUSOLVER_EIG_RANGE_ALL : all eigenvalues/eigenvectors will be found, will becomes the classical syevd/heevd routine; range = CUSOLVER_EIG_RANGE_V : all eigenvalues/eigenvectors in the half-open interval (vl,vu] will be found; range = CUSOLVER_EIG_RANGE_I : the il-th through iu-th eigenvalues/eigenvectors will be found;
uplo	host	input	Specifies which part of A is stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of A is stored. uplo = CUBLAS_FILL_MODE_UPPER: Upper triangle of A is stored.
n	host	input	Number of rows (or columns) of matrix A.
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular

Parameter	Memory	In/out	Meaning
			part of the matrix A. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A. On exit, if jobz = CUSOLVER_EIG_MODE_VECTOR, and devInfo = 0, A contains the orthonormal eigenvectors of the matrix A. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.lda is not less than max(1,n).
vl,vu	host	input	Real values float or double for (C, S) or (Z, D) precision respectively. If range = CUSOLVER_EIG_RANGE_V, the lower and upper bounds of the interval to be searched for eigenvalues. vl > vu. Not referenced if range = CUSOLVER_EIG_RANGE_ALL or range = CUSOLVER_EIG_RANGE_I. Note that, if eigenvalues are very close to each other, it is well known that two different eigenvalues routines might find slightly different number of eigenvalues inside the same interval. This is due to the fact that different eigenvalue algorithms, or even same algorithm but different run might find eigenvalues within some rounding error close to the machine precision. Thus, if the user wants to be sure not to miss any eigenvalue within the interval bound, we suggest that the user substract/add epsilon (machine precision) to the interval bound such as (vl=vl-eps, vu=vu+eps]. This suggestion is valid for any selective routine from cuSolver or LAPACK.
il,iu	host	input	Integer. If range = CUSOLVER_EIG_RANGE_I, the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= il <= iu <= n, if n > 0; il = 1 and iu = 0 if n = 0. Not referenced if range = CUSOLVER_EIG_RANGE_ALL or range = CUSOLVER_EIG_RANGE_V.
h_meig	host	output	Integer. The total number of eigenvalues found. 0 <= h_meig <= n. If range = CUSOLVER_EIG_RANGE_ALL, h_meig = n,

Parameter	Memory	In/out	Meaning
			and if range = CUSOLVER_EIG_RANGE_I, h_meig = iu-il+1.
W	device	output	A real array of dimension n. The eigenvalue values of A, in ascending order ie, sorted so that W(i) <= W(i+1).
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of work, returned by syevdx_bufferSize.
devInfo	device	output	If devInfo = 0, the operation is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle). if devInfo = i (> 0), devInfo indicates i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, or lda<max(1,n), or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or range is not CUSOLVER_EIG_RANGE_ALL or CUSOLVER_EIG_RANGE_V or CUSOLVER_EIG_RANGE_I, or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.14. cusolverDnSyevdx()[DEPRECATED]

[[DEPRECATED]] use cusolverDnXsyevdx() instead. The routine will be removed in the next major release.

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
 cusolverDnSyevdx_bufferSize(
 cusolverDnHandle_t handle,
 cusolverParams_t params,
 cusolverEigMode_t jobz,
 cusolverEigRange_t range,
 cublasFillMode_t uplo,
 int n,
 cudaDataType dataTypeA,
 const void *A,
```

```
 int64_t lda,
 void *vl,
 void *vu,
 int64_t il,
 int64_t iu,
 int64_t *h_meig,
 cudaDataType dataTypeW,
 const void *W,
 cudaDataType computeType,
 size_t *workspaceInBytes);
```

The routine bellow

```
cusolverStatus_t
 cusolverDnSyevdx (
 cusolverDnHandle_t handle,
 cusolverParams_t params,
 cusolverEigMode_t jobz,
 cusolverEigRange_t range,
 cublasFillMode_t uplo,
 int n,
 cudaDataType dataTypeA,
 const void *A,
 int64_t lda,
 void *vl,
 void *vu,
 int64_t il,
 int64_t iu,
 int64_t *h_meig,
 cudaDataType dataTypeW,
 const void *W,
 cudaDataType computeType,
 void *pBuffer,
 size_t workspaceInBytes,
 int *info);
```

computes all or selection of the eigenvalues and optionally eigenvectors of a symmetric (Hermitian) `n×n` matrix `A` using the generic API interface. The standard symmetric eigenvalue problem is

$$A*V = V*\Lambda$$

where Λ is a real `n×h_meig` diagonal matrix. `V` is an `n×h_meig` unitary matrix. `h_meig` is the number of eigenvalues/eigenvectors computed by the routine, `h_meig` is equal to `n` when the whole spectrum (e.g., `range` = `CUSOLVER_EIG_RANGE_ALL`) is requested. The diagonal elements of Λ are the eigenvalues of `A` in ascending order.

The user has to provide working space which is pointed by input parameter `pBuffer`. The input parameter `workspaceInBytes` is size in bytesr of the working space, and it is returned by `cusolverDnSyevdx_bufferSize()`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). If `info = i` (greater than zero), `i` off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

if jobz = CUSOLVER_EIG_MODE_VECTOR, A contains the orthonormal eigenvectors of the matrix A. The eigenvectors are computed by a divide and conquer algorithm.

Currently, cusolverDnSyevdx supports only the default algorithm.

Table of algorithms supported by cusolverDnSyevdx

CUSOLVER_ALG_0 or NULL	Default algorithm.

List of input arguments for cusolverDnSyevdx_bufferSize and cusolverDnSyevdx:

API of cusolverDnSyevdx

parameter	Memory	In/out	Meaning
handle	host	input	handle to the cuSolverDN library context.
params	host	input	structure with information collected by cusolverDnSetAdvOptions.
jobz	host	input	specifies options to either compute eigenvalue only or compute eigen-pair: jobz = CUSOLVER_EIG_MODE_NOVECTOR : Compute eigenvalues only; jobz = CUSOLVER_EIG_MODE_VECTOR : Compute eigenvalues and eigenvectors.
range	host	input	specifies options to which selection of eigenvalues and optionally eigenvectors that need to be computed: range = CUSOLVER_EIG_RANGE_ALL : all eigenvalues/eigenvectors will be found, will becomes the classical syevd/heevd routine; range = CUSOLVER_EIG_RANGE_V : all eigenvalues/eigenvectors in the half-open interval (vl,vu] will be found; range = CUSOLVER_EIG_RANGE_I : the il-th through iu-th eigenvalues/eigenvectors will be found;
uplo	host	input	specifies which part of A is stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of A is stored. uplo = CUBLAS_FILL_MODE_UPPER: Upper triangle of A is stored.
n	host	input	number of rows (or columns) of matrix A.
dataTypeA	host	in	data type of array A.
A	device	in/out	array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A. On exit, if jobz

			= CUSOLVER_EIG_MODE_VECTOR, and info = 0, A contains the orthonormal eigenvectors of the matrix A. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
lda	host	input	leading dimension of two-dimensional array used to store matrix A.lda is not less than max(1,n).
vl,vu	host	input	If range = CUSOLVER_EIG_RANGE_V, the lower and upper bounds of the interval to be searched for eigenvalues. vl > vu. Not referenced if range = CUSOLVER_EIG_RANGE_ALL or range = CUSOLVER_EIG_RANGE_I. Note that, if eigenvalues are very close to each other, it is well known that two different eigenvalues routines might find slightly different number of eigenvalues inside the same interval. This is due to the fact that different eigenvalue algorithms, or even same algorithm but different run might find eigenvalues within some rounding error close to the machine precision. Thus, if the user want to be sure not to miss any eigenvalue within the interval bound, we suggest that, the user substract/add epsilon (machine precision) to the interval bound such as (vl=vl-eps, vu=vu+eps]. this suggestion is valid for any selective routine from cuSolver or LAPACK.
il,iu	host	input	integer. If range = CUSOLVER_EIG_RANGE_I, the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= il <= iu <= n, if n > 0; il = 1 and iu = 0 if n = 0. Not referenced if range = CUSOLVER_EIG_RANGE_ALL or range = CUSOLVER_EIG_RANGE_V.
h_meig	host	output	integer. The total number of eigenvalues found. 0 <= h_meig <= n. If range = CUSOLVER_EIG_RANGE_ALL, h_meig = n, and if range = CUSOLVER_EIG_RANGE_I, h_meig = iu-il+1.
dataTypeW	host	in	data type of array W.
W	device	output	a real array of dimension n. The eigenvalue values of A, in ascending order ie, sorted so that W(i) <= W(i+1).
computeType	host	in	data type of computation.

pBuffer	device	in/out	Working space. Array of type void of size workspaceInBytes bytes.
workspaceInBytes	host	input	Size in bytes of pBuffer, returned by cusolverDnSyevdx_bufferSize.
info	device	output	if info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle). if info = i (> 0), info indicates i off-diagonal elements of an intermediate tridiagonal form did not converge to zero;

The generic API has three different types, dataTypeA is data type of the matrix A, dataTypeW is data type of the matrix W and computeType is compute type of the operation. cusolverDnSyevdx only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	DataTypeW	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SSYEVDX
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DSYEVDX
CUDA_C_32F	CUDA_R_32F	CUDA_C_32F	CHEEVDX
CUDA_C_64F	CUDA_R_64F	CUDA_C_64F	ZHEEVDX

Status Returned

CUSOLVER_STATUS_SUCCESS	the operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	invalid parameters were passed (n<0, or lda<max(1,n), or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or range is not CUSOLVER_EIG_RANGE_ALL or CUSOLVER_EIG_RANGE_V or CUSOLVER_EIG_RANGE_I, or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	an internal operation failed.

2.4.3.15. cusolverDn<t>sygvd()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSsygvd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
```

```
    int n,
    const float *A,
    int lda,
    const float *B,
    int ldb,
    const float *W,
    int *lwork);

cusolverStatus_t
cusolverDnDsygvd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const double *A,
    int lda,
    const double *B,
    int ldb,
    const double *W,
    int *lwork);

cusolverStatus_t
cusolverDnChegvd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuComplex *A,
    int lda,
    const cuComplex *B,
    int ldb,
    const float *W,
    int *lwork);

cusolverStatus_t
cusolverDnZhegvd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *B,
    int ldb,
    const double *W,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSsygvd(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
```

```
    float *A,
    int lda,
    float *B,
    int ldb,
    float *W,
    float *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDsygvd(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    double *A,
    int lda,
    double *B,
    int ldb,
    double *W,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnChegvd(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuComplex *A,
    int lda,
    cuComplex *B,
    int ldb,
    float *W,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZhegvd(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuDoubleComplex *A,
    int lda,
    cuDoubleComplex *B,
    int ldb,
    double *W,
    cuDoubleComplex *work,
    int lwork,
    int *devInfo);
```

This function computes eigenvalues and eigenvectors of a symmetric (Hermitian) n×n matrix-pair (A,B). The generalized symmetric-definite eigenvalue problem is

$$\text{eig(A,B)} = \begin{cases} A*V = B*V*\Lambda & \text{if itype = CUSOLVER\_EIG\_TYPE\_1} \\ A*B*V = V*\Lambda & \text{if itype = CUSOLVER\_EIG\_TYPE\_2} \\ B*A*V = V*\Lambda & \text{if itype = CUSOLVER\_EIG\_TYPE\_3} \end{cases}$$

where the matrix B is positive definite. Λ is a real n×n diagonal matrix. The diagonal elements of Λ are the eigenvalues of (A, B) in ascending order. V is an n×n orthogonal matrix. The eigenvectors are normalized as follows:

$$\begin{cases} V^H * B * V = I & \text{if itype = CUSOLVER\_EIG\_TYPE\_1, CUSOLVER\_EIG\_TYPE\_2} \\ V^H * \text{inv(B)} * V = I & \text{if itype = CUSOLVER\_EIG\_TYPE\_3} \end{cases}$$

The user has to provide working space which is pointed by input parameter work. The input parameter lwork is size of the working space, and it is returned by sygvd_bufferSize().

If output parameter devInfo = -i (less than zero), the i-th parameter is wrong (not counting handle). If devInfo = i (i > 0 and i<=n) and jobz = CUSOLVER_EIG_MODE_NOVECTOR, i off-diagonal elements of an intermediate tridiagonal form did not converge to zero. If devInfo = N + i (i > 0), then the leading minor of order i of B is not positive definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

if jobz = CUSOLVER_EIG_MODE_VECTOR, A contains the orthogonal eigenvectors of the matrix A. The eigenvectors are computed by divide and conquer algorithm.

Please visit cuSOLVER Library Samples - sygvd for a code example.

API of sygvd

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
itype	host	input	Specifies the problem type to be solved: ▶ itype=CUSOLVER_EIG_TYPE_1: A*x = (lambda)*B*x. ▶ itype=CUSOLVER_EIG_TYPE_2: A*B*x = (lambda)*x. ▶ itype=CUSOLVER_EIG_TYPE_3: B*A*x = (lambda)*x.
jobz	host	input	Specifies options to either compute eigenvalue only or compute eigen-pair: jobz = CUSOLVER_EIG_MODE_NOVECTOR : Compute eigenvalues only; jobz = CUSOLVER_EIG_MODE_VECTOR : Compute eigenvalues and eigenvectors.
uplo	host	input	Specifies which part of A and B are stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of A and B are stored. uplo

Parameter	Memory	In/out	Meaning
			= CUBLAS_FILL_MODE_UPPER: Upper triangle of A and B are stored.
n	host	input	Number of rows (or columns) of matrix A and B.
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A. On exit, if jobz = CUSOLVER_EIG_MODE_VECTOR, and devInfo = 0, A contains the orthonormal eigenvectors of the matrix A. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A. lda is not less than max(1,n).
B	device	in/out	<type> array of dimension ldb * n. If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of B contains the upper triangular part of the matrix B. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of B contains the lower triangular part of the matrix B. On exit, if devInfo is less than n, B is overwritten by triangular factor U or L from the Cholesky factorization of B.
ldb	host	input	Leading dimension of two-dimensional array used to store matrix B. ldb is not less than max(1,n).
W	device	output	A real array of dimension n. The eigenvalue values of A, sorted so that W(i) >= W(i+1).
work	device	in/out	Working space, <type> array of size lwork.
Lwork	host	input	Size of work, returned by sygvd_bufferSize.
devInfo	device	output	If devInfo = 0, the operation is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle). If devInfo = i (> 0), devInfo indicates either potrf or syevd is wrong.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, or lda<max(1,n), or ldb<max(1,n), or itype is not 1, 2 or 3, or jobz is not 'N' or 'V', or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.16. cusolverDn<t>sygvdx()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSsygvdx_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    const float *A,
    int lda,
    const float *B,
    int ldb,
    float vl,
    float vu,
    int il,
    int iu,
    int *h_meig,
    const float *W,
    int *lwork);

cusolverStatus_t
cusolverDnDsygvdx_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    const double *A,
    int lda,
    const double *B,
    int ldb,
    double vl,
    double vu,
    int il,
    int iu,
    int *h_meig,
    const double *W,
    int *lwork);
```

```
cusolverStatus_t
cusolverDnChegvdx_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    const cuComplex *A,
    int lda,
    const cuComplex *B,
    int ldb,
    float vl,
    float vu,
    int il,
    int iu,
    int *h_meig,
    const float *W,
    int *lwork);

cusolverStatus_t
cusolverDnZhegvdx_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *B,
    int ldb,
    double vl,
    double vu,
    int il,
    int iu,
    int *h_meig,
    const double *W,
    int *lwork);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSsygvdx(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    float *A,
    int lda,
    float *B,
    int ldb,
    float vl,
    float vu,
    int il,
    int iu,
```

```
    int *h_meig,
    float *W,
    float *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnDsygvdx(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    double *A,
    int lda,
    double *B,
    int ldb,
    double vl,
    double vu,
    int il,
    int iu,
    int *h_meig,
    double *W,
    double *work,
    int lwork,
    int *devInfo);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnChegvdx(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int n,
    cuComplex *A,
    int lda,
    cuComplex *B,
    int ldb,
    float vl,
    float vu,
    int il,
    int iu,
    int *h_meig,
    float *W,
    cuComplex *work,
    int lwork,
    int *devInfo);

cusolverStatus_t
cusolverDnZhegvdx(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
```

```
    int n,
    cuDoubleComplex *A,
    int lda,
    cuDoubleComplex *B,
    int ldb,
    double vl,
    double vu,
    int il,
    int iu,
    int *h_meig,
    double *W,
    cuDoubleComplex *work,
    int lwork,
    int *devInfo);
```

This function computes all or selection of the eigenvalues and optionally eigenvectors of a symmetric (Hermitian) `n×n` matrix-pair (`A`,`B`). The generalized symmetric-definite eigenvalue problem is

$$\text{eig(A,B)} = \begin{cases} A*V = B*V*\Lambda & \text{if itype = CUSOLVER\_EIG\_TYPE\_1} \\ A*B*V = V*\Lambda & \text{if itype = CUSOLVER\_EIG\_TYPE\_2} \\ B*A*V = V*\Lambda & \text{if itype = CUSOLVER\_EIG\_TYPE\_3} \end{cases}$$

where the matrix `B` is positive definite. `Λ` is a real `n×h_meig` diagonal matrix. The diagonal elements of `Λ` are the eigenvalues of (`A`, `B`) in ascending order. `V` is an `n×h_meig` orthogonal matrix. `h_meig` is the number of eigenvalues/eigenvectors computed by the routine, `h_meig` is equal to `n` when the whole spectrum (e.g., `range` = `CUSOLVER_EIG_RANGE_ALL`) is requested. The eigenvectors are normalized as follows:

$$\begin{cases} V^H * B * V = I & \text{if itype = CUSOLVER\_EIG\_TYPE\_1, CUSOLVER\_EIG\_TYPE\_2} \\ V^H * \text{inv(B)} * V = I & \text{if itype = CUSOLVER\_EIG\_TYPE\_3} \end{cases}$$

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is size of the working space, and it is returned by `sygvdx_bufferSize()`.

If output parameter `devInfo = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). If `devInfo = i` (i > 0 and i<=n) and `jobz` = CUSOLVER_EIG_MODE_NOVECTOR, `i` off-diagonal elements of an intermediate tridiagonal form did not converge to zero. If `devInfo = n + i` (i > 0), then the leading minor of order `i` of `B` is not positive definite. The factorization of `B` could not be completed and no eigenvalues or eigenvectors were computed.

If `jobz` = CUSOLVER_EIG_MODE_VECTOR, `A` contains the orthogonal eigenvectors of the matrix `A`. The eigenvectors are computed by divide and conquer algorithm.

Please visit cuSOLVER Library Samples - sygvdx for a code example.

API of sygvdx

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`itype`	`host`	`input`	Specifies the problem type to be solved: ▶ `itype`=CUSOLVER_EIG_TYPE_1: A*x = (lambda)*B*x

Parameter	Memory	In/out	Meaning
			▶ itype=CUSOLVER_EIG_TYPE_2: A*B*x = (lambda)*x ▶ itype=CUSOLVER_EIG_TYPE_3: B*A*x = (lambda)*x
jobz	host	input	Specifies options to either compute eigenvalue only or compute eigen-pair: jobz = CUSOLVER_EIG_MODE_NOVECTOR : Compute eigenvalues only; jobz = CUSOLVER_EIG_MODE_VECTOR : Compute eigenvalues and eigenvectors.
range	host	input	Specifies options to which selection of eigenvalues and optionally eigenvectors that need to be computed: range = CUSOLVER_EIG_RANGE_ALL : all eigenvalues/eigenvectors will be found, will becomes the classical syevd/heevd routine; range = CUSOLVER_EIG_RANGE_V : all eigenvalues/eigenvectors in the half-open interval (vl,vu] will be found; range = CUSOLVER_EIG_RANGE_I : the il-th through iu-th eigenvalues/ eigenvectors will be found;
uplo	host	input	Specifies which part of A and B are stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of A and B are stored. uplo = CUBLAS_FILL_MODE_UPPER: Upper triangle of A and B are stored.
n	host	input	Number of rows (or columns) of matrix A and B.
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A. On exit, if jobz = CUSOLVER_EIG_MODE_VECTOR, and devInfo = 0, A contains the orthonormal eigenvectors of the matrix A. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A. lda is not less than max(1,n).

Parameter	Memory	In/out	Meaning
B	device	in/out	<type> array of dimension ldb * n. If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of B contains the upper triangular part of the matrix B. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of B contains the lower triangular part of the matrix B. On exit, if devInfo is less than n, B is overwritten by triangular factor U or L from the Cholesky factorization of B.
ldb	host	input	Leading dimension of two-dimensional array used to store matrix B. ldb is not less than max(1,n).
vl,vu	host	input	Real values float or double for (C, S) or (Z, D) precision respectively. If range = CUSOLVER_EIG_RANGE_V, the lower and upper bounds of the interval to be searched for eigenvalues. vl > vu. Not referenced if range = CUSOLVER_EIG_RANGE_ALL or range = CUSOLVER_EIG_RANGE_I. Note that, if eigenvalues are very close to each other, it is well known that two different eigenvalues routines might find slightly different number of eigenvalues inside the same interval. This is due to the fact that different eigenvalue algorithms, or even same algorithm but different run might find eigenvalues within some rounding error close to the machine precision. Thus, if the user want to be sure not to miss any eigenvalue within the interval bound, we suggest that, the user substract/add epsilon (machine precision) to the interval bound such as (vl=vl-eps, vu=vu+eps). this suggestion is valid for any selective routine from cuSolver or LAPACK.
il,iu	host	input	Integer. If range = CUSOLVER_EIG_RANGE_I, the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= il <= iu <= n, if n > 0; il = 1 and iu = 0 if n = 0. Not referenced if range = CUSOLVER_EIG_RANGE_ALL or range = CUSOLVER_EIG_RANGE_V.
h_meig	host	output	Integer. The total number of eigenvalues found. 0 <= h_meig <= n. If range = CUSOLVER_EIG_RANGE_ALL, h_meig = n,

Parameter	Memory	In/out	Meaning
			and if range = CUSOLVER_EIG_RANGE_I, h_meig = iu-il+1.
W	device	output	A real array of dimension n. The eigenvalue values of A, sorted so that W(i) >= W(i+1).
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of work, returned by sygvdx_bufferSize.
devInfo	device	output	If devInfo = 0, the operation is successful. if devInfo = -i, the i-th parameter is wrong (not counting handle). If devInfo = i (> 0), devInfo indicates either potrf or syevd is wrong.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, or lda<max(1,n), or ldb<max(1,n), or itype is not CUSOLVER_EIG_TYPE_1 or CUSOLVER_EIG_TYPE_2 or CUSOLVER_EIG_TYPE_3 or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTORL, or range is not CUSOLVER_EIG_RANGE_ALL or CUSOLVER_EIG_RANGE_V or CUSOLVER_EIG_RANGE_I, or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.17. cusolverDn<t>syevj()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSsyevj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const float *A,
    int lda,
    const float *W,
    int *lwork,
    syevjInfo_t params);
```

```
cusolverStatus_t
cusolverDnDsyevj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const double *A,
    int lda,
    const double *W,
    int *lwork,
    syevjInfo_t params);

cusolverStatus_t
cusolverDnCheevj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuComplex *A,
    int lda,
    const float *W,
    int *lwork,
    syevjInfo_t params);

cusolverStatus_t
cusolverDnZheevj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const double *W,
    int *lwork,
    syevjInfo_t params);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSsyevj(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    float *A,
    int lda,
    float *W,
    float *work,
    int lwork,
    int *info,
    syevjInfo_t params);

cusolverStatus_t
cusolverDnDsyevj(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    double *A,
    int lda,
    double *W,
    double *work,
    int lwork,
    int *info,
```

```
    syevjInfo_t params);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCheevj(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuComplex *A,
    int lda,
    float *W,
    cuComplex *work,
    int lwork,
    int *info,
    syevjInfo_t params);

cusolverStatus_t
cusolverDnZheevj(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuDoubleComplex *A,
    int lda,
    double *W,
    cuDoubleComplex *work,
    int lwork,
    int *info,
    syevjInfo_t params);
```

This function computes eigenvalues and eigenvectors of a symmetric (Hermitian) `n×n` matrix `A`. The standard symmetric eigenvalue problem is

$$A*Q = Q*\Lambda$$

where `Λ` is a real `n×n` diagonal matrix. `Q` is an `n×n` unitary matrix. The diagonal elements of `Λ` are the eigenvalues of `A` in ascending order.

`syevj` has the same functionality as `syevd`. The difference is that `syevd` uses QR algorithm and `syevj` uses Jacobi method. The parallelism of Jacobi method gives GPU better performance on small and medium size matrices. Moreover the user can configure `syevj` to perform approximation up to certain accuracy.

How does it work?

`syevj` iteratively generates a sequence of unitary matrices to transform matrix `A` to the following form

$$V^H * A * V = W + E$$

where `W` is diagonal and `E` is symmetric without diagonal.

During the iterations, the Frobenius norm of `E` decreases monotonically. As `E` goes down to zero, `W` is the set of eigenvalues. In practice, Jacobi method stops if

$$||E||_F \leq \text{eps} * ||A||_F$$

where eps is the given tolerance.

syevj has two parameters to control the accuracy. First parameter is tolerance (eps). The default value is machine accuracy but The user can use function cusolverDnXsyevjSetTolerance to set a priori tolerance. The second parameter is maximum number of sweeps which controls number of iterations of Jacobi method. The default value is 100 but the user can use function cusolverDnXsyevjSetMaxSweeps to set a proper bound. The experimentis show 15 sweeps are good enough to converge to machine accuracy. syevj stops either tolerance is met or maximum number of sweeps is met.

The Jacobi method has quadratic convergence, so the accuracy is not proportional to number of sweeps. To guarantee certain accuracy, the user should configure tolerance only.

After syevj, the user can query residual by function cusolverDnXsyevjGetResidual and number of executed sweeps by function cusolverDnXsyevjGetSweeps. However the user needs to be aware that residual is the Frobenius norm of E, not accuracy of individual eigenvalue, i.e.

$$\text{residual} = ||E||_F = ||\Lambda - W||_F$$

The same as syevd, the user has to provide working space pointed by input parameter work. The input parameter lwork is the size of the working space, and it is returned by syevj_bufferSize().

If output parameter info = -i (less than zero), the i-th parameter is wrong (not counting handle). If info = n+1, syevj does not converge under given tolerance and maximum sweeps.

If the user sets an improper tolerance, syevj may not converge. For example, tolerance should not be smaller than machine accuracy.

If jobz = CUSOLVER_EIG_MODE_VECTOR, A contains the orthonormal eigenvectors V.

Please visit cuSOLVER Library Samples - syevj for a code example.

API of syevj

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
jobz	host	input	Specifies options to either compute eigenvalue only or compute eigen-pair: jobz = CUSOLVER_EIG_MODE_NOVECTOR : Compute eigenvalues only; jobz = CUSOLVER_EIG_MODE_VECTOR : Compute eigenvalues and eigenvectors.
uplo	host	input	Specifies which part of A is stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of A is stored. uplo = CUBLAS_FILL_MODE_UPPER: Upper triangle of A is stored.
n	host	input	Number of rows (or columns) of matrix A.

Parameter	Memory	In/out	Meaning
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A. On exit, if jobz = CUSOLVER_EIG_MODE_VECTOR, and info = 0, A contains the orthonormal eigenvectors of the matrix A. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
W	device	output	A real array of dimension n. The eigenvalue values of A, in ascending order ie, sorted so that W(i) <= W(i+1).
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of work, returned by syevj_bufferSize.
info	device	output	If info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle). If info = n +1, syevj does not converge under given tolerance and maximum sweeps.
params	host	in/out	Structure filled with parameters of Jacobi algorithm and results of syevj.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, or lda<max(1,n), or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.18. cusolverDn<t>sygvj()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSsygvj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const float *A,
    int lda,
    const float *B,
    int ldb,
    const float *W,
    int *lwork,
    syevjInfo_t params);

cusolverStatus_t
cusolverDnDsygvj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const double *A,
    int lda,
    const double *B,
    int ldb,
    const double *W,
    int *lwork,
    syevjInfo_t params);

cusolverStatus_t
cusolverDnChegvj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuComplex *A,
    int lda,
    const cuComplex *B,
    int ldb,
    const float *W,
    int *lwork,
    syevjInfo_t params);

cusolverStatus_t
cusolverDnZhegvj_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const cuDoubleComplex *B,
    int ldb,
    const double *W,
    int *lwork,
    syevjInfo_t params);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSsygvj(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    float *A,
    int lda,
    float *B,
    int ldb,
    float *W,
    float *work,
    int lwork,
    int *info,
    syevjInfo_t params);

cusolverStatus_t
cusolverDnDsygvj(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    double *A,
    int lda,
    double *B,
    int ldb,
    double *W,
    double *work,
    int lwork,
    int *info,
    syevjInfo_t params);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnChegvj(
    cusolverDnHandle_t handle,
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuComplex *A,
    int lda,
    cuComplex *B,
    int ldb,
    float *W,
    cuComplex *work,
    int lwork,
    int *info,
    syevjInfo_t params);

cusolverStatus_t
cusolverDnZhegvj(
    cusolverDnHandle_t handle,
```

```
    cusolverEigType_t itype,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuDoubleComplex *A,
    int lda,
    cuDoubleComplex *B,
    int ldb,
    double *W,
    cuDoubleComplex *work,
    int lwork,
    int *info,
    syevjInfo_t params);
```

This function computes eigenvalues and eigenvectors of a symmetric (Hermitian) `n×n` matrix-pair (`A`,`B`). The generalized symmetric-definite eigenvalue problem is

$$\text{eig(A,B)} = \begin{cases} A*V = B*V*\Lambda & \text{if itype = CUSOLVER\_EIG\_TYPE\_1} \\ A*B*V = V*\Lambda & \text{if itype = CUSOLVER\_EIG\_TYPE\_2} \\ B*A*V = V*\Lambda & \text{if itype = CUSOLVER\_EIG\_TYPE\_3} \end{cases}$$

where the matrix `B` is positive definite. `Λ` is a real `n×n` diagonal matrix. The diagonal elements of `Λ` are the eigenvalues of (`A`, `B`) in ascending order. `V` is an `n×n` orthogonal matrix. The eigenvectors are normalized as follows:

$$\begin{cases} V^H * B * V = I & \text{if itype = CUSOLVER\_EIG\_TYPE\_1, CUSOLVER\_EIG\_TYPE\_2} \\ V^H * \text{inv(B)} * V = I & \text{if itype = CUSOLVER\_EIG\_TYPE\_3} \end{cases}$$

This function has the same functionality as `sygvd` except that `syevd` in `sygvd` is replaced by `syevj` in `sygvj`. Therefore, `sygvj` inherits properties of `syevj`, the user can use `cusolverDnXsyevjSetTolerance` and `cusolverDnXsyevjSetMaxSweeps` to configure tolerance and maximum sweeps.

However the meaning of residual is different from `syevj`. `sygvj` first computes Cholesky factorization of matrix `B`,

$$B = L * L^H$$

transform the problem to standard eigenvalue problem, then calls `syevj`.

For example, the standard eigenvalue problem of type I is

$$M * Q = Q * \Lambda$$

where matrix `M` is symmtric

$$M = L^{-1} * A * L^{-\mathrm{H}}$$

The residual is the result of `syevj` on matrix `M`, not `A`.

The user has to provide working space which is pointed by input parameter `work`. The input parameter `lwork` is the size of the working space, and it is returned by `sygvj_bufferSize()`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). If `info = i` (i > 0 and i<=n), `B` is not positive definite, the factorization of `B` could not

be completed and no eigenvalues or eigenvectors were computed. If `info = n+1`, `syevj` does not converge under given tolerance and maximum sweeps. In this case, the eigenvalues and eigenvectors are still computed because non-convergence comes from improper tolerance of maximum sweeps.

if `jobz` = CUSOLVER_EIG_MODE_VECTOR, `A` contains the orthogonal eigenvectors `V`.

Please visit cuSOLVER Library Samples - sygvj for a code example.

API of sygvj

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`itype`	`host`	`input`	Specifies the problem type to be solved: `itype`=CUSOLVER_EIG_TYPE_1: A*x = (lambda)*B*x. `itype`=CUSOLVER_EIG_TYPE_2: A*B*x = (lambda)*x. `itype`=CUSOLVER_EIG_TYPE_3: B*A*x = (lambda)*x.
`jobz`	`host`	`input`	Specifies options to either compute eigenvalue only or compute eigen-pair: `jobz` = `CUSOLVER_EIG_MODE_NOVECTOR` : Compute eigenvalues only; `jobz` = `CUSOLVER_EIG_MODE_VECTOR` : Compute eigenvalues and eigenvectors.
`uplo`	`host`	`input`	Specifies which part of `A` and `B` are stored. `uplo` = `CUBLAS_FILL_MODE_LOWER`: Lower triangle of `A` and `B` are stored. `uplo` = `CUBLAS_FILL_MODE_UPPER`: Upper triangle of `A` and `B` are stored.
`n`	`host`	`input`	Number of rows (or columns) of matrix `A` and `B`.
`A`	`device`	`in/out`	<type> array of dimension `lda * n` with `lda` is not less than `max(1,n)`. If `uplo` = `CUBLAS_FILL_MODE_UPPER`, the leading n-by-n upper triangular part of `A` contains the upper triangular part of the matrix `A`. If `uplo` = `CUBLAS_FILL_MODE_LOWER`, the leading n-by-n lower triangular part of `A` contains the lower triangular part of the matrix `A`. On exit, if `jobz` = `CUSOLVER_EIG_MODE_VECTOR`, and `info` = 0, `A` contains the orthonormal eigenvectors of the matrix `A`. If `jobz` = `CUSOLVER_EIG_MODE_NOVECTOR`, the contents of `A` are destroyed.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `A`. `lda` is not less than `max(1,n)`.

Parameter	Memory	In/out	Meaning
B	device	in/out	<type> array of dimension ldb * n. If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of B contains the upper triangular part of the matrix B. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of B contains the lower triangular part of the matrix B. On exit, if info is less than n, B is overwritten by triangular factor U or L from the Cholesky factorization of B.
ldb	host	input	Leading dimension of two-dimensional array used to store matrix B. ldb is not less than max(1,n).
W	device	output	A real array of dimension n. The eigenvalue values of A, sorted so that W(i) >= W(i+1).
work	device	in/out	Working space, <type> array of size lwork.
lwork	host	input	Size of work, returned by sygvj_bufferSize.
info	device	output	if info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle). If info = i (> 0), info indicates either B is not positive definite or syevj (called by sygvj) does not converge.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, or lda<max(1,n), or ldb<max(1,n), or itype is not 1, 2 or 3, or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.3.19. cusolverDn<t>syevjBatched()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnSsyevjBatched_bufferSize(
    cusolverDnHandle_t handle,
```

```
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const float *A,
    int lda,
    const float *W,
    int *lwork,
    syevjInfo_t params,
    int batchSize
    );

cusolverStatus_t
cusolverDnDsyevjBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const double *A,
    int lda,
    const double *W,
    int *lwork,
    syevjInfo_t params,
    int batchSize
    );

cusolverStatus_t
cusolverDnCheevjBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuComplex *A,
    int lda,
    const float *W,
    int *lwork,
    syevjInfo_t params,
    int batchSize
    );

cusolverStatus_t
cusolverDnZheevjBatched_bufferSize(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    const cuDoubleComplex *A,
    int lda,
    const double *W,
    int *lwork,
    syevjInfo_t params,
    int batchSize
    );
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnSsyevjBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
```

```
    cublasFillMode_t uplo,
    int n,
    float *A,
    int lda,
    float *W,
    float *work,
    int lwork,
    int *info,
    syevjInfo_t params,
    int batchSize
    );

cusolverStatus_t
cusolverDnDsyevjBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    double *A,
    int lda,
    double *W,
    double *work,
    int lwork,
    int *info,
    syevjInfo_t params,
    int batchSize
    );
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverDnCheevjBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuComplex *A,
    int lda,
    float *W,
    cuComplex *work,
    int lwork,
    int *info,
    syevjInfo_t params,
    int batchSize
    );

cusolverStatus_t
cusolverDnZheevjBatched(
    cusolverDnHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int n,
    cuDoubleComplex *A,
    int lda,
    double *W,
    cuDoubleComplex *work,
    int lwork,
    int *info,
    syevjInfo_t params,
```

```
    int batchSize
    );
```

This function computes eigenvalues and eigenvectors of a squence of symmetric (Hermitian) `n×n` matrices

$$A_j * Q_j = Q_j * \Lambda_j$$

where Λ_j is a real `n×n` diagonal matrix. Q_j is an `n×n` unitary matrix. The diagonal elements of Λ_j are the eigenvalues of A_j in either ascending order or non-sorting order.

`syevjBatched` performs `syevj` on each matrix. It requires that all matrices are of the same size `n` and are packed in contiguous way,

$$A = (\mathrm{A0} \quad \mathrm{A1} \quad \cdots)$$

Each matrix is column-major with leading dimension `lda`, so the formula for random access is $A_k(\mathrm{i,j}) = \mathrm{A[\,i + lda*j + lda*n*k]}$.

The parameter `W` also contains eigenvalues of each matrix in contiguous way,

$$W = (\mathrm{W0} \quad \mathrm{W1} \quad \cdots)$$

The formula for random access of `W` is $W_k(\mathrm{j}) = \mathrm{W[\,j + n*k]}$.

Except for tolerance and maximum sweeps, `syevjBatched` can either sort the eigenvalues in ascending order (default) or chose as-is (without sorting) by the function `cusolverDnXsyevjSetSortEig`. If the user packs several tiny matrices into diagonal blocks of one matrix, non-sorting option can separate spectrum of those tiny matrices.

`syevjBatched` cannot report residual and executed sweeps by function `cusolverDnXsyevjGetResidual` and `cusolverDnXsyevjGetSweeps`. Any call of the above two returns `CUSOLVER_STATUS_NOT_SUPPORTED`. The user needs to compute residual explicitly.

The user has to provide working space pointed by input parameter `work`. The input parameter `lwork` is the size of the working space, and it is returned by `syevjBatched_bufferSize()`.

The output parameter `info` is an integer array of size `batchSize`. If the function returns `CUSOLVER_STATUS_INVALID_VALUE`, the first element `info[0] = -i` (less than zero) indicates `i-th` parameter is wrong (not counting handle). Otherwise, if `info[i] = n+1`, `syevjBatched` does not converge on `i-th` matrix under given tolerance and maximum sweeps.

If `jobz` = CUSOLVER_EIG_MODE_VECTOR, A_j contains the orthonormal eigenvectors V_j.

Please visit cuSOLVER Library Samples - syevjBatched for a code example.

API of syevjBatched

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`jobz`	`host`	`input`	Specifies options to either compute eigenvalue only or compute eigen-pair:

Parameter	Memory	In/out	Meaning
			jobz = CUSOLVER_EIG_MODE_NOVECTOR : Compute eigenvalues only; jobz = CUSOLVER_EIG_MODE_VECTOR : Compute eigenvalues and eigenvectors.
uplo	host	input	Specifies which part of Aj is stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of Aj is stored. uplo = CUBLAS_FILL_MODE_UPPER: Upper triangle of Aj is stored.
n	host	input	Number of rows (or columns) of matrix each Aj.
A	device	in/out	<type> array of dimension lda * n * batchSize with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of Aj contains the upper triangular part of the matrix Aj. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of Aj contains the lower triangular part of the matrix Aj. On exit, if jobz = CUSOLVER_EIG_MODE_VECTOR, and info[j] = 0, Aj contains the orthonormal eigenvectors of the matrix Aj. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of Aj are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix Aj.
W	device	output	A real array of dimension n*batchSize. It stores the eigenvalues of Aj in ascending order or non-sorting order.
work	device	in/out	<type> array of size lwork, workspace.
lwork	host	input	Size of work, returned by syevjBatched_bufferSize.
info	device	output	An integer array of dimension batchSize. If CUSOLVER_STATUS_INVALID_VALUE is returned, info[0] = -i (less than zero) indicates i-th parameter is wrong (not counting handle). Otherwise, if info[i] = 0, the operation is successful. If info[i] = n+1, syevjBatched does not converge on i-th matrix under given tolerance and maximum sweeps.
params	host	in/out	Structure filled with parameters of Jacobi algorithm.

Parameter	Memory	In/out	Meaning
batchSize	host	input	Number of matrices.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, or lda<max(1,n), or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER), or batchSize<0.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.4. Dense Linear Solver Reference (64-bit API)

This section describes linear solver 64-bit API of cuSolverDN, including Cholesky factorization, LU with partial pivoting and QR factorization.

2.4.4.1. cusolverDnXpotrf()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnXpotrf_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cublasFillMode_t uplo,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType computeType,
    size_t *workspaceInBytesOnDevice,
    size_t *workspaceInBytesOnHost)
```

The following routine:

```
cusolverStatus_t
cusolverDnXpotrf(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cublasFillMode_t uplo,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    cudaDataType computeType,
    void *bufferOnDevice,
    size_t workspaceInBytesOnDevice,
    void *bufferOnHost,
    size_t workspaceInBytesOnHost,
```

```
    int *info )
```

computes the Cholesky factorization of a Hermitian positive-definite matrix using the generic API interfacte.

`A` is a `n×n` Hermitian matrix, only lower or upper part is meaningful. The input parameter `uplo` indicates which part of the matrix is used. The function would leave other part untouched.

If input parameter `uplo` is `CUBLAS_FILL_MODE_LOWER`, only lower triangular part of `A` is processed, and replaced by lower triangular Cholesky factor `L`.

$$A = L * L^H$$

If input parameter `uplo` is `CUBLAS_FILL_MODE_UPPER`, only upper triangular part of `A` is processed, and replaced by upper triangular Cholesky factor `U`.

$$A = U^H * U$$

The user has to provide device and host working spaces which are pointed by input parameters `bufferOnDevice` and `bufferOnHost`. The input parameters `workspaceInBytesOnDevice` (and `workspaceInBytesOnHost`) is size in bytes of the device (and host) working space, and it is returned by `cusolverDnXpotrf_bufferSize()`.

If Cholesky factorization failed, i.e. some leading minor of `A` is not positive definite, or equivalently some diagonal elements of `L` or `U` is not a real number. The output parameter `info` would indicate smallest leading minor of `A` which is not positive definite.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

Currently, `cusolverDnXpotrf` supports only the default algorithm.

Please visit cuSOLVER Library Samples - Xpotrf for a code example.

Table of algorithms supported by cusolverDnXpotrf

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

List of input arguments for `cusolverDnXpotrf_bufferSize` and `cusolverDnXpotrf`:

API of potrf

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`params`	`host`	`input`	Structure with information collected by `cusolverDnSetAdvOptions`.
`uplo`	`host`	`input`	Indicates if matrix `A` lower or upper part is stored, the other part is not referenced.
`n`	`host`	`input`	Number of rows and columns of matrix `A`.
`dataTypeA`	`host`	`in`	Data type of array `A`.
`A`	`device`	`in/out`	Array of dimension `lda * n` with `lda` is not less than `max(1,n)`.

Parameter	Memory	In/out	Meaning
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
computeType	host	in	Data type of computation.
bufferOnDevice	device	in/out	Device workspace. Array of type void of size workspaceInBytesOnDevice bytes.
workspaceInBytesOnDevice	host	input	Size in bytes of bufferOnDevice, returned by cusolverDnXpotrf_bufferSize.
bufferOnHost	host	in/out	Host workspace. Array of type void of size workspaceInBytesOnHost bytes.
workspaceInBytesOnHost	host	input	Size in bytes of bufferOnHost, returned by cusolverDnXpotrf_bufferSize.
info	device	output	If info = 0, the Cholesky factorization is successful. If info = -i, the i-th parameter is wrong (not counting handle). If info = i, the leading minor of order i is not positive definite.

The generic API has two different types, dataTypeA is data type of the matrix A, computeType is compute type of the operation. cusolverDnXpotrf only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	SPOTRF
CUDA_R_64F	CUDA_R_64F	DPOTRF
CUDA_C_32F	CUDA_C_32F	CPOTRF
CUDA_C_64F	CUDA_C_64F	ZPOTRF

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n)).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.4.2. cusolverDnXpotrs()

```
cusolverStatus_t
cusolverDnXpotrs(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cublasFillMode_t uplo,
    int64_t n,
    int64_t nrhs,
```

```
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType dataTypeB,
    void *B,
    int64_t ldb,
    int *info)
```

This function solves a system of linear equations

$$A*X=B$$

where A is a n×n Hermitian matrix, only lower or upper part is meaningful using the generic API interface. The input parameter uplo indicates which part of the matrix is used. The function would leave other part untouched.

The user has to call cusolverDnXpotrf first to factorize matrix A. If input parameter uplo is CUBLAS_FILL_MODE_LOWER, A is lower triangular Cholesky factor L correspoding to $A=L*L^H$. If input parameter uplo is CUBLAS_FILL_MODE_UPPER, A is upper triangular Cholesky factor U corresponding to $A=U^H*U$.

The operation is in-place, i.e. matrix X overwrites matrix B with the same leading dimension ldb.

If output parameter info = -i (less than zero), the i-th parameter is wrong (not counting handle).

Currently, cusolverDnXpotrs supports only the default algorithm.

Please visit cuSOLVER Library Samples - Xpotrf for a code example.

Table of algorithms supported by cusolverDnXpotrs

CUSOLVER_ALG_0 or NULL	Default algorithm.

List of input arguments for cusolverDnXpotrs:

API of potrs

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolveDN library context.
params	host	input	Structure with information collected by cusolverDnSetAdvOptions.
uplo	host	input	Indicates if matrix A lower or upper part is stored, the other part is not referenced.
n	host	input	Number of rows and columns of matrix A.
nrhs	host	input	Number of columns of matrix X and B.
dataTypeA	host	in	Data type of array A.
A	device	input	Array of dimension lda * n with lda is not less than max(1,n). A is either lower cholesky factor L or upper Cholesky factor U.

Parameter	Memory	In/out	Meaning
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
dataTypeB	host	in	Data type of array B.
B	device	in/out	Array of dimension ldb * nrhs. ldb is not less than max(1,n). As an input, B is right hand side matrix. As an output, B is the solution matrix.
info	device	output	If info = 0, the Cholesky factorization is successful. if info = -i, the i-th parameter is wrong (not counting handle).

The generic API has two different types, dataTypeA is data type of the matrix A, dataTypeB is data type of the matrix B. cusolverDnXpotrs only supports the following four combinations.

Valid combination of data type and compute type

dataTypeA	dataTypeB	Meaning
CUDA_R_32F	CUDA_R_32F	SPOTRS
CUDA_R_64F	CUDA_R_64F	DPOTRS
CUDA_C_32F	CUDA_C_32F	CPOTRS
CUDA_C_64F	CUDA_C_64F	ZPOTRS

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, nrhs<0, lda<max(1,n) or ldb<max(1,n)).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.4.3. cusolverDnXgetrf()

The helper function below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t cusolverDnXgetrf_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType computeType,
    size_t *workspaceInBytesOnDevice,
    size_t *workspaceInBytesOnHost)
```

The function bellow

```
cusolverStatus_t
cusolverDnXgetrf(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    int64_t *ipiv,
    cudaDataType computeType,
    void *bufferOnDevice,
    size_t workspaceInBytesOnDevice,
    void *bufferOnHost,
    size_t workspaceInBytesOnHost,
    int *info )
```

computes the LU factorization of a `m×n` matrix

$$P*A = L*U$$

where `A` is a `m×n` matrix, `P` is a permutation matrix, `L` is a lower triangular matrix with unit diagonal, and `U` is an upper triangular matrix using the generic API interface.

If LU factorization failed, i.e. matrix `A` (`U`) is singular, The output parameter `info=i` indicates `U(i,i) = 0`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

If `ipiv` is null, no pivoting is performed. The factorization is `A=L*U`, which is not numerically stable.

No matter LU factorization failed or not, the output parameter `ipiv` contains pivoting sequence, row `i` is interchanged with row `ipiv(i)`.

The user has to provide device and host working spaces which are pointed by input parameters `bufferOnDevice` and `bufferOnHost`. The input parameters `workspaceInBytesOnDevice` (and `workspaceInBytesOnHost`) is size in bytes of the device (and host) working space, and it is returned by `cusolverDnXgetrf_bufferSize()`.

The user can combine `cusolverDnXgetrf` and `cusolverDnGetrs` to complete a linear solver.

Currently, `cusolverDnXgetrf` supports two algorithms. To select legacy implementation, the user has to call `cusolverDnSetAdvOptions`.

Please visit cuSOLVER Library Samples - Xgetrf for a code example.

Table of algorithms supported by cusolverDnXgetrf

`CUSOLVER_ALG_0` or `NULL`	Default algorithm. The fastest, requires a large workspace of `m*n` elements.
`CUSOLVER_ALG_1`	Legacy implementation

List of input arguments for `cusolverDnXgetrf_bufferSize` and `cusolverDnXgetrf`:

API of cusolverDnXgetrf

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
params	host	input	Structure with information collected by cusolverDnSetAdvOptions.
m	host	input	Number of rows of matrix A.
n	host	input	Number of columns of matrix A.
dataTypeA	host	in	Data type of array A.
A	device	in/out	<type> array of dimension lda * n with lda is not less than max(1,m).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
ipiv	device	output	Array of size at least min(m,n), containing pivot indices.
computeType	host	in	Data type of computation.
bufferOnDevice	device	in/out	Device workspace. Array of type void of size workspaceInBytesOnDevice bytes.
workspaceInBytesOnDevice	host	input	Size in bytes of bufferOnDevice, returned by cusolverDnXpotrf_bufferSize.
bufferOnHost	host	in/out	Host workspace. Array of type void of size workspaceInBytesOnHost bytes.
workspaceInBytesOnHost	host	input	Size in bytes of bufferOnHost, returned by cusolverDnXpotrf_bufferSize.
info	device	output	If info = 0, the LU factorization is successful. if info = -i, the i-th parameter is wrong (not counting handle). If info = i, the U(i,i) = 0.

The generic API has two different types, dataTypeA is data type of the matrix A, computeType is compute type of the operation. cusolverDnXgetrf only supports the following four combinations.

valid combination of data type and compute type

DataTypeA	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	SGETRF
CUDA_R_64F	CUDA_R_64F	DGETRF
CUDA_C_32F	CUDA_C_32F	CGETRF
CUDA_C_64F	CUDA_C_64F	ZGETRF

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or lda<max(1,m)).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.4.4. cusolverDnXgetrs()

```
cusolverStatus_t
cusolverDnXgetrs(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cublasOperation_t trans,
    int64_t n,
    int64_t nrhs,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    const int64_t *ipiv,
    cudaDataType dataTypeB,
    void *B,
    int64_t ldb,
    int *info )
```

This function solves a linear system of multiple right-hand sides

$$\mathrm{op(A)} * X = B$$

where A is a n×n matrix, and was LU-factored by cusolverDnXgetrf, that is, lower trianular part of A is L, and upper triangular part (including diagonal elements) of A is U. B is a n×nrhs right-hand side matrix using the generic API interface.

The input parameter trans is defined by

$$\mathrm{op}(A) = \begin{cases} A & \text{if trans == CUBLAS\_OP\_N} \\ A^T & \text{if trans == CUBLAS\_OP\_T} \\ A^H & \text{if trans == CUBLAS\_OP\_C} \end{cases}$$

The input parameter ipiv is an output of cusolverDnXgetrf. It contains pivot indices, which are used to permutate right-hand sides.

If output parameter info = -i (less than zero), the i-th parameter is wrong (not counting handle).

The user can combine cusolverDnXgetrf and cusolverDnXgetrs to complete a linear solver.

Currently, cusolverDnXgetrs supports only the default algorithm.

Please visit cuSOLVER Library Samples - Xgetrf for a code example.

Table of algorithms supported by cusolverDnXgetrs

CUSOLVER_ALG_0 or NULL	Default algorithm.

List of input arguments for cusolverDnXgetrss:

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
params	host	input	Structure with information collected by cusolverDnSetAdvOptions.
trans	host	input	Operation op(A) that is non- or (conj.) transpose.
n	host	input	Number of rows and columns of matrix A.
nrhs	host	input	Number of right-hand sides.
dataTypeA	host	in	Data type of array A.
A	device	input	Array of dimension lda * n with lda is not less than max(1,n).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
ipiv	device	input	Array of size at least n, containing pivot indices.
dataTypeB	host	in	Data type of array B.
B	device	output	<type> array of dimension ldb * nrhs with ldb is not less than max(1,n).
ldb	host	input	Leading dimension of two-dimensional array used to store matrix B.
info	device	output	If info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle).

The generic API has two different types: dataTypeA is data type of the matrix A and dataTypeB is data type of the matrix B. cusolverDnXgetrs only supports the following four combinations:

Valid combination of data type and compute type

DataTypeA	dataTypeB	Meaning
CUDA_R_32F	CUDA_R_32F	SGETRS
CUDA_R_64F	CUDA_R_64F	DGETRS
CUDA_C_32F	CUDA_C_32F	CGETRS
CUDA_C_64F	CUDA_C_64F	ZGETRS

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n) or ldb<max(1,n)).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.4.5. cusolverDnXgeqrf()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnXgeqrf_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType dataTypeTau,
    const void *tau,
    cudaDataType computeType,
    size_t *workspaceInBytesOnDevice,
    size_t *workspaceInBytesOnHost)
```

The following routine:

```
cusolverStatus_t cusolverDnXgeqrf(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    cudaDataType dataTypeTau,
    void *tau,
    cudaDataType computeType,
    void *bufferOnDevice,
    size_t workspaceInBytesOnDevice,
    void *bufferOnHost,
    size_t workspaceInBytesOnHost,
    int *info )
```

computes the QR factorization of a `m×n` matrix

$$A = Q * R$$

where `A` is an `m×n` matrix, `Q` is a `m×n` matrix, and `R` is an `n×n` upper triangular matrix using the generic API interface.

The user has to provide device and host working spaces which are pointed by input parameters `bufferOnDevice` and `bufferOnHost`. The input parameters `workspaceInBytesOnDevice` (and `workspaceInBytesOnHost`) is size in bytes of the device (and host) working space, and it is returned by `cusolverDnXgeqrf_bufferSize()`.

The matrix `R` is overwritten in upper triangular part of `A`, including diagonal elements.

The matrix `Q` is not formed explicitly, instead, a sequence of householder vectors are stored in lower triangular part of `A`. The leading nonzero element of householder vector is assumed to

be 1 such that output parameter `TAU` contains the scaling factor `τ`. If `v` is original householder vector, `q` is the new householder vector corresponding to `τ`, satisying the following relation

$$I - 2 * v * v^H = I - \tau * q * q^H$$

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

Currently, `cusolverDnXgeqrf` supports only the default algorithm.

Please visit cuSOLVER Library Samples - Xgeqrf for a code example.

Table of algorithms supported by cusolverDnXgeqrf

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

List of input arguments for `cusolverDnXgeqrf_bufferSize` and `cusolverDnXgeqrf`:

API of geqrf

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`params`	`host`	`input`	Structure with information collected by `cusolverDnSetAdvOptions`.
`m`	`host`	`input`	Number of rows of matrix `A`.
`n`	`host`	`input`	Number of columns of matrix `A`.
`dataTypeA`	`host`	`in`	Data type of array `A`.
`A`	`device`	`in/out`	Array of dimension `lda * n` with `lda` is not less than `max(1,m)`.
`lda`	`host`	`input`	Leading dimension of two-dimensional array used to store matrix `A`.
`TAU`	`device`	`output`	Array of dimension at least `min(m,n)`.
`computeType`	`host`	`in`	Data type of computation.
`bufferOnDevice`	`device`	`in/out`	Device workspace. Array of type `void` of size `workspaceInBytesOnDevice` bytes.
`workspaceInBytesOnDevice`	`host`	`input`	Size in bytes of `bufferOnDevice`, returned by `cusolverDnXpotrf_bufferSize`.
`bufferOnHost`	`host`	`in/out`	Host workspace. Array of type `void` of size `workspaceInBytesOnHost` bytes.
`workspaceInBytesOnHost`	`host`	`input`	Size in bytes of `bufferOnHost`, returned by `cusolverDnXpotrf_bufferSize`.
`info`	`device`	`output`	If `info = 0`, the LU factorization is successful. If `info = -i`, the `i-th` parameter is wrong (not counting handle).

The generic API has two different types, `dataTypeA` is data type of the matrix `A` and array `tau` and `computeType` is compute type of the operation. `cusolverDnXgeqrf` only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	SGEQRF
CUDA_R_64F	CUDA_R_64F	DGEQRF
CUDA_C_32F	CUDA_C_32F	CGEQRF
CUDA_C_64F	CUDA_C_64F	ZGEQRF

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or lda<max(1,m)).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.4.6. cusolverDnXsytrs()

The helper functions below can calculate the sizes needed for pre-allocated buffers.

```
cusolverStatus_t
cusolverDnXsytrs_bufferSize(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int64_t n,
    int64_t nrhs,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    const int64_t *ipiv,
    cudaDataType dataTypeB,
    void *B,
    int64_t ldb,
    size_t *workspaceInBytesOnDevice,
    size_t *workspaceInBytesOnHost);
```

The following routine:

```
cusolverStatus_t CUSOLVERAPI cusolverDnXsytrs(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    int64_t n,
    int64_t nrhs,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    const int64_t *ipiv,
    cudaDataType dataTypeB,
    void *B,
    int64_t ldb,
    void *bufferOnDevice,
    size_t workspaceInBytesOnDevice,
    void *bufferOnHost,
```

```
    size_t workspaceInBytesOnHost,
    int *info);
```

solves a system of linear equations using the generic API interfacte.

A contains the factorization from cusolverDnXsytrf(), only lower or upper part is meaningful, the other part is not touched.

If input parameter uplo is CUBLAS_FILL_MODE_LOWER, the details of the factorization are stores as:

$$A = L * D * L^T$$

If input parameter uplo is CUBLAS_FILL_MODE_UPPER, the details of the factorization are stores as:

$$A = U * D * U^T$$

The user has to provide the pivot indices that can be otained by cusolverDnXsytrf() as well as device and host work spaces which are pointed by input parameters bufferOnDevice and bufferOnHost. The input parameters workspaceInBytesOnDevice and workspaceInBytesOnHost are sizes in bytes of the device and host work spaces, and they are returned by cusolverDnXsytrs_bufferSize().

If output parameter info = -i (less than zero), the i-th parameter is wrong (not counting handle).

List of input arguments for cusolverDnXsytrs_bufferSize and cusolverDnXsytrs:

API of sytrs

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
uplo	host	input	Indicates if matrix A lower or upper part is stored, the other part is not referenced.
n	host	input	Number of rows and columns of matrix A.
nrhs	host	input	Number of right-hand sides.
dataTypeA	host	in	Data type of array A.
A	device	input	Array of dimension lda * n with lda is not less than max(1,n).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
ipiv	device	input	Array of size at least n, containing pivot indices.
dataTypeB	host	in	Data type of array B.
B	device	in/out	Array of dimension ldb * nrhs with ldb is not less than max(1,nrhs).
ldb	host	input	Leading dimension of two-dimensional array used to store matrix B.

Parameter	Memory	In/out	Meaning
bufferOnDevice	device	in/out	Device workspace. Array of type void of size workspaceInBytesOnDevice bytes.
workspaceInBytesOnDevice	host	input	Size in bytes of bufferOnDevice, returned by cusolverDnXsytrs_bufferSize.
bufferOnHost	host	in/out	Host workspace. Array of type void of size workspaceInBytesOnHost bytes.
workspaceInBytesOnHost	host	input	Size in bytes of bufferOnHost, returned by cusolverDnXsytrs_bufferSize.
info	device	output	If info = 0, the Cholesky factorization is successful. If info = -i, the i-th parameter is wrong (not counting handle). If info = i, the leading minor of order i is not positive definite.

The generic API has two different types: dataTypeA is data type of the matrix A, dataTypeB is data type of the matrix A. cusolverDnXsytrs only supports the following four combinations:

Valid combination of data type and compute type

DataTypeA	DataTypeB	Meaning
CUDA_R_32F	CUDA_R_32F	SSYTRS
CUDA_R_64F	CUDA_R_64F	DSYTRS
CUDA_C_32F	CUDA_C_32F	CSYTRS
CUDA_C_64F	CUDA_C_64F	ZSYTRS

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n)).
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	Data type is not supported.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.4.7. cusolverDnXtrtri()

The helper functions below can calculate the sizes needed for pre-allocated buffers.

```
cusolverStatus_t
cusolverDnXtrtri_bufferSize(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    cublasDiagType_t diag,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
```

```
    size_t *workspaceInBytesOnDevice,
    size_t *workspaceInBytesOnHost);
```

The following routine:

```
cusolverStatus_t
cusolverDnXtrtri(
    cusolverDnHandle_t handle,
    cublasFillMode_t uplo,
    cublasDiagType_t diag,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    void *bufferOnDevice,
    size_t workspaceInBytesOnDevice,
    void *bufferOnHost,
    size_t workspaceInBytesOnHost,
    int *info);
```

computes the inverse of a triangular matrix using the generic API interfacte.

`A` is an `n×n` triangular matrix, only lower or upper part is meaningful. The input parameter `uplo` indicates which part of the matrix is used. The function would leave other part untouched.

If input parameter `uplo` is `CUBLAS_FILL_MODE_LOWER`, only lower triangular part of `A` is processed, and replaced by lower triangular inverse.

If input parameter `uplo` is `CUBLAS_FILL_MODE_UPPER`, only upper triangular part of `A` is processed, and replaced by upper triangular inverse.

The user has to provide device and host work spaces which are pointed by input parameters `bufferOnDevice` and `bufferOnHost`. The input parameters `workspaceInBytesOnDevice` and `workspaceInBytesOnHost` are sizes in bytes of the device and host work spaces, and they are returned by `cusolverDnXtrtri_bufferSize()`.

If matrix inversion fails, the output parameter `info = i` shows `A(i,i) = 0`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

Please visit cuSOLVER Library Samples - Xtrtri for a code example.

List of input arguments for `cusolverDnXtrtri_bufferSize` and `cusolverDnXtrtri`:

API of trtri

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`uplo`	`host`	`input`	Indicates if matrix `A` lower or upper part is stored, the other part is not referenced.
`diag`	`host`	`input`	The enumerated unit diagonal type.
`n`	`host`	`input`	Number of rows and columns of matrix `A`.

Parameter	Memory	In/out	Meaning
dataTypeA	host	in	Data type of array A.
A	device	in/out	Array of dimension lda * n with lda is not less than max(1,n).
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
bufferOnDevice	device	in/out	Device workspace. Array of type void of size workspaceInBytesOnDevice bytes.
workspaceInBytesOnDevice	host	input	Size in bytes of bufferOnDevice, returned by cusolverDnXtrtri_bufferSize.
bufferOnHost	host	in/out	Host workspace. Array of type void of size workspaceInBytesOnHost bytes.
workspaceInBytesOnHost	host	input	Size in bytes of bufferOnHost, returned by cusolverDnXtrtri_bufferSize.
info	device	output	If info = 0, the matrix inversion succeeded. If info = -i, the i-th parameter is wrong (not counting handle). If info = i, A(i,i) = 0.

Valid data types

DataTypeA	Meaning
CUDA_R_32F	STRTRI
CUDA_R_64F	DTRTRI
CUDA_C_32F	CTRTRI
CUDA_C_64F	ZTRTRI

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_NOT_SUPPORTED	Data type is not supported.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0 or lda<max(1,n)).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.5. Dense Eigenvalue Solver Reference (64-bit API)

This section describes eigenvalue solver API of cuSolverDN, including bidiagonalization and SVD.

2.4.5.1. cusolverDnXgesvd()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnXgesvd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    signed char jobu,
    signed char jobvt,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType dataTypeS,
    const void *S,
    cudaDataType dataTypeU,
    const void *U,
    int64_t ldu,
    cudaDataType dataTypeVT,
    const void *VT,
    int64_t ldvt,
    cudaDataType computeType,
    size_t *workspaceInBytesOnDevice,
    size_t *workspaceInBytesOnHost)
```

The following routine:

```
cusolverStatus_t
cusolverDnXgesvd(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    signed char jobu,
    signed char jobvt,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    cudaDataType dataTypeS,
    void *S,
    cudaDataType dataTypeU,
    void *U,
    int64_t ldu,
    cudaDataType dataTypeVT,
    void *VT,
    int64_t ldvt,
    cudaDataType computeType,
    void *bufferOnDevice,
    size_t workspaceInBytesOnDevice,
    void *bufferOnHost,
    size_t workspaceInBytesOnHost,
    int *info)
```

This function computes the singular value decomposition (SVD) of a `m×n` matrix `A` and corresponding the left and/or right singular vectors. The SVD is written

$$A = U * \Sigma * V^H$$

where Σ is an m×n matrix which is zero except for its min(m,n) diagonal elements, U is an m×m unitary matrix, and V is an n×n unitary matrix. The diagonal elements of Σ are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

The user has to provide device and host working spaces which are pointed by input parameters bufferOnDevice and bufferOnHost. The input parameters workspaceInBytesOnDevice (and workspaceInBytesOnHost) is size in bytes of the device (and host) working space, and it is returned by cusolverDnXgesvd_bufferSize().

If output parameter info = -i (less than zero), the i-th parameter is wrong (not counting handle). if bdsqr did not converge, info specifies how many superdiagonals of an intermediate bidiagonal form did not converge to zero.

Currently, cusolverDnXgesvd supports only the default algorithm.

Table of algorithms supported by cusolverDnXgesvd

CUSOLVER_ALG_0 or NULL	Default algorithm.

Please visit cuSOLVER Library Samples - Xgesvd for a code example.

Remark 1: gesvd only supports m>=n.

Remark 2: the routine returns V^H, not V.

List of input arguments for cusolverDnXgesvd_bufferSize and cusolverDnXgesvd:

API of cusolverDnXgesvd

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverDN library context.
params	host	input	Structure with information collected by cusolverDnSetAdvOptions.
jobu	host	input	Specifies options for computing all or part of the matrix U: = 'A': all m columns of U are returned in array U: = 'S': the first min(m,n) columns of U (the left singular vectors) are returned in the array U; = 'O': the first min(m,n) columns of U (the left singular vectors) are overwritten on the array A; = 'N': no columns of U (no left singular vectors) are computed.
jobvt	host	input	Specifies options for computing all or part of the matrix V**T: = 'A': all N rows of V**T are returned in the array VT; = 'S': the first min(m,n) rows of V**T (the right singular vectors) are returned in the array VT; = 'O': the first min(m,n) rows of V**T (the right singular vectors) are overwritten on the array A; = 'N': no rows of V**T (no right singular vectors) are computed.
m	host	input	Number of rows of matrix A.

Parameter	Memory	In/out	Meaning
n	host	input	Number of columns of matrix A.
dataTypeA	host	input	Data type of array A.
A	device	in/out	Array of dimension lda * n with lda is not less than max(1,m). On exit, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
dataTypeS	host	input	Data type of array S.
S	device	output	Real array of dimension min(m,n). The singular values of A, sorted so that S(i) >= S(i+1).
dataTypeU	host	input	Data type of array U.
U	device	output	Array of dimension ldu * m with ldu is not less than max(1,m). U contains the m×m unitary matrix U.
ldu	host	input	Leading dimension of two-dimensional array used to store matrix U.
dataTypeVT	host	input	Data type of array VT.
VT	device	output	Array of dimension ldvt * n with ldvt is not less than max(1,n). VT contains the n×n unitary matrix V**T.
ldvt	host	input	Leading dimension of two-dimensional array used to store matrix Vt.
computeType	host	input	Data type of computation.
bufferOnDevice	device	in/out	Device workspace. Array of type void of size workspaceInBytesOnDevice bytes.
workspaceInBytesOnDevice	host	input	Dize in bytes of bufferOnDevice, returned by cusolverDnXpotrf_bufferSize.
bufferOnHost	host	in/out	Host workspace. Array of type void of size workspaceInBytesOnHost bytes.
workspaceInBytesOnHost	host	input	Size in bytes of bufferOnHost, returned by cusolverDnXpotrf_bufferSize.
info	device	output	If info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle). if info > 0, info indicates how many superdiagonals of an intermediate bidiagonal form did not converge to zero.

The generic API has three different types, dataTypeA is data type of the matrix A, dataTypeS is data type of the vector S and dataTypeU is data type of the matrix U, dataTypeVT is data type of the matrix VT, computeType is compute type of the operation. cusolverDnXgesvd only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	DataTypeS	DataTypeU	DataTypeVT	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SGESVD
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DGESVD
CUDA_C_32F	CUDA_R_32F	CUDA_C_32F	CUDA_C_32F	CUDA_C_32F	CGESVD
CUDA_C_64F	CUDA_R_64F	CUDA_C_64F	CUDA_C_64F	CUDA_C_64F	ZGESVD

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (`m,n<0` or `lda<max(1,m)` or `ldu<max(1,m)` or `ldvt<max(1,n)`).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.5.2. cusolverDnXgesvdp()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnXgesvdp_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cusolverEigMode_t jobz,
    int econ,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType dataTypeS,
    const void *S,
    cudaDataType dataTypeU,
    const void *U,
    int64_t ldu,
    cudaDataType dataTypeV,
    const void *V,
    int64_t ldv,
    cudaDataType computeType,
    size_t *workspaceInBytesOnDevice,
    size_t *workspaceInBytesOnHost)
```

The routine below:

```
cusolverStatus_t
cusolverDnXgesvdp(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cusolverEigMode_t jobz,
    int econ,
    int64_t m,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    cudaDataType dataTypeS,
    void *S,
```

```
    cudaDataType dataTypeU,
    void *U,
    int64_t ldu,
    cudaDataType dataTypeV,
    void *V,
    int64_t ldv,
    cudaDataType computeType,
    void *bufferOnDevice,
    size_t workspaceInBytesOnDevice,
    void *bufferOnHost,
    size_t workspaceInBytesOnHost,
    int *d_info,
    double *h_err_sigma)
```

This function computes the singular value decomposition (SVD) of a `m×n` matrix `A` and corresponding the left and/or right singular vectors. The SVD is written

$$A = U * \Sigma * V^H$$

where `Σ` is an `m×n` matrix which is zero except for its `min(m,n)` diagonal elements, `U` is an `m×m` unitary matrix, and `V` is an `n×n` unitary matrix. The diagonal elements of `Σ` are the singular values of `A`; they are real and non-negative, and are returned in descending order. The first `min(m,n)` columns of `U` and `V` are the left and right singular vectors of `A`.

`cusolverDnXgesvdp` combines polar decomposition in [14] and `cusolverDnXsyevd` to compute SVD. It is much faster than `cusolverDnXgesvd` which is based on QR algorithm. However polar decomposition in [14] may not deliver a full unitary matrix when the matrix A has a singular value close to zero. To workaround the issue when the singular value is close to zero, we add a small perturbation so polar decomposition can deliver the correct result. The consequence is inaccurate singular values shifted by this perturbation. The output parameter `h_err_sigma` is the magnitude of this perturbation. In other words, `h_err_sigma` shows the accuracy of SVD.

The user has to provide device and host working spaces which are pointed by input parameters `bufferOnDevice` and `bufferOnHost`. The input parameters `workspaceInBytesOnDevice` (and `workspaceInBytesOnHost`) is size in bytes of the device (and host) working space, and it is returned by `cusolverDnXgesvdp_bufferSize()`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

Currently, `cusolverDnXgesvdp` supports only the default algorithm.

Table of algorithms supported by cusolverDnXgesvdp

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

Please visit cuSOLVER Library Samples - Xgesvdp for a code example.

Remark 1: `gesvdp` supports `n>=m` as well.

Remark 2: the routine returns `V`, not V^H

List of input arguments for `cusolverDnXgesvdp_bufferSize` and `cusolverDnXgesvdp`:

API of cusolverDnXgesvdp

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.

Parameter	Memory	In/out	Meaning
params	host	input	Structure with information collected by cusolverDnSetAdvOptions.
jobz	host	input	Specifies options to either compute singular values only or compute singular vectors as well: jobz = CUSOLVER_EIG_MODE_NOVECTOR : Compute singular values only. jobz = CUSOLVER_EIG_MODE_VECTOR : Compute singular values and singular vectors.
econ	host	input	econ = 1 for economy size for U and V.
m	host	input	Number of rows of matrix A.
n	host	input	Number of columns of matrix A.
dataTypeA	host	input	Data type of array A.
A	device	in/out	Array of dimension lda * n with lda is not less than max(1,m). On exit, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
dataTypeS	host	input	Data type of array S.
S	device	output	Real array of dimension min(m,n). The singular values of A, sorted so that S(i) >= S(i+1).
dataTypeU	host	input	Data type of array U.
U	device	output	Array of dimension ldu * m with ldu is not less than max(1,m). U contains the m×m unitary matrix U. If econ=1, only reports first min(m,n) columns of U.
ldu	host	input	Leading dimension of two-dimensional array used to store matrix U.
dataTypeV	host	input	Data type of array V.
V	device	output	Array of dimension ldv * n with ldv is not less than max(1,n). V contains the n×n unitary matrix V. if econ=1, only reports first min(m,n) columns of V.
ldv	host	input	Leading dimension of two-dimensional array used to store matrix V.
computeType	host	input	Data type of computation.
bufferOnDevice	device	in/out	Device workspace. Array of type void of size workspaceInBytesOnDevice bytes.
workspaceInBytesOnDevice	host	input	Size in bytes of bufferOnDevice, returned by cusolverDnXpotrf_bufferSize.

Parameter	Memory	In/out	Meaning
bufferOnHost	host	in/out	Host workspace. Array of type void of size workspaceInBytesOnHost bytes.
workspaceInBytesOnHost	host	input	Size in bytes of bufferOnHost, returned by cusolverDnXpotrf_bufferSize.
info	device	output	If info = 0, the operation is successful. if info = -i, the i-th parameter is wrong (not counting handle).
h_err_sigma	host	output	Magnitude of the perturbation, showing the accuracy of SVD.

The generic API has three different types, dataTypeA is data type of the matrix A, dataTypeS is data type of the vector S and dataTypeU is data type of the matrix U, dataTypeV is data type of the matrix V, computeType is compute type of the operation. cusolverDnXgesvdp only supports the following four combinations:

Valid combination of data type and compute type

DataTypeA	DataTypeS	DataTypeU	DataTypeV	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SGESVDP
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DGESVDP
CUDA_C_32F	CUDA_R_32F	CUDA_C_32F	CUDA_C_32F	CUDA_C_32F	CGESVDP
CUDA_C_64F	CUDA_R_64F	CUDA_C_64F	CUDA_C_64F	CUDA_C_64F	ZGESVDP

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n<0 or lda<max(1,m) or ldu<max(1,m) or ldv<max(1,n)).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.5.3. cusolverDnXgesvdr()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnXgesvdr_bufferSize (
 cusolverDnHandle_t handle,
 cusolverDnParams_t params,
 signed char jobu,
 signed char jobv,
 int64_t m,
 int64_t n,
 int64_t k,
 int64_t p,
 int64_t niters,
 cudaDataType dataTypeA,
 const void *A,
```

```
 int64_t lda,
 cudaDataType dataTypeSrand,
 const void *Srand,
 cudaDataType dataTypeUrand,
 const void *Urand,
 int64_t ldUrand,
 cudaDataType dataTypeVrand,
 const void *Vrand,
 int64_t ldVrand,
 cudaDataType computeType,
 size_t *workspaceInBytesOnDevice,
 size_t *workspaceInBytesOnHost )
```

The routine below

```
cusolverStatus_t
cusolverDnXgesvdr(
 cusolverDnHandle_t handle,
 cusolverDnParams_t params,
 signed char jobu,
 signed char jobv,
 int64_t m,
 int64_t n,
 int64_t k,
 int64_t p,
 int64_t niters,
 cudaDataType dataTypeA,
 void *A,
 int64_t lda,
 cudaDataType dataTypeSrand,
 void *Srand,
 cudaDataType  dataTypeUrand,
 void *Urand,
 int64_t ldUrand,
 cudaDataType dataTypeVrand,
 void *Vrand,
 int64_t ldVrand,
 cudaDataType computeType,
 void *bufferOnDevice,
 size_t workspaceInBytesOnDevice,
 void *bufferOnHost,
 size_t workspaceInBytesOnHost,
 int *d_info)
```

This function computes the approximated rank-k singular value decomposition (k-SVD) of an `m×n` matrix `A` and the corresponding left and/or right singular vectors. The k-SVD is written as

$$A_k \approx U^* \Sigma^* V^H$$

where `Σ` is a `k×k` matrix which is zero except for its diagonal elements, `U` is an `m×k` orthonormal matrix, and `V` is an `k×n` orthonormal matrix. The diagonal elements of `Σ` are the approximated singular values of `A`; they are real and non-negative, and are returned in descending order. The columns of `U` and `V` are the top-`k` left and right singular vectors of `A`.

`cusolverDnXgesvdr` implements randomized methods described in [15] to compute k-SVD that is accurate with high probablity if the conditions described in [15] hold. `cusolverDnXgesvdr` is intended to compute a very small portion of the spectrum (meaning

that `k` is very small compared to `min(m,n)`). of `A` fast and with good quality, specially when the dimenions of the matrix are large.

The accuracy of the method depends on the spectrum of `A`, the number of power iterations `niters`, the oversampling parameter `p` and the ratio between `p` and the dimensions of the matrix `A`. Larger values of oversampling `p` or larger number of iterations `niters` might produce more accurate approximations, but it will also increase the run time of `cusolverDnXgesvdr`.

Our recommendation is to use two iterations and set the oversampling to at least `2k`. Once the solver provides enough accuracy, adjust the values of `k` and `niters` for better performance.

The user has to provide device and host working spaces which are pointed by input parameters `bufferOnDevice` and `bufferOnHost`. The input parameters `workspaceInBytesOnDevice` (and `workspaceInBytesOnHost`) is size in bytes of the device (and host) working space, and it is returned by `cusolverDnXgesvdr_bufferSize()`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

Currently, `cusolverDnXgesvdr` supports only the default algorithm.

Table of algorithms supported by cusolverDnXgesvdr

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

Please visit cuSOLVER Library Samples - Xgesvdr for a code example.

Remark 1: `gesvdr` supports `n>=m` as well.

Remark 2: the routine returns `V`, not V^H

List of input arguments for `cusolverDnXgesvdr_bufferSize` and `cusolverDnXgesvdr`:

API of cusolverDnXgesvdr

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`params`	`host`	`input`	Structure with information collected by `cusolverDnSetAdvOptions`.
`jobu`	`host`	`input`	Specifies options for computing all or part of the matrix `U`: = 'S': the first k columns of U (the left singular vectors) are returned in the array U; = 'N': no columns of U (no left singular vectors) are computed.
`jobv`	`host`	`input`	Specifies options for computing all or part of the matrix V: = 'S': the first k rows of V (the right singular vectors) are returned in the array V; = 'N': no rows of V (no right singular vectors) are computed.
`m`	`host`	`input`	Number of rows of matrix `A`.
`n`	`host`	`input`	Number of columns of matrix `A`.

Parameter	Memory	In/out	Meaning
k	host	input	Rank of the k-SVD decomposition of matrix A. rank is less than min(m,n).
p	host	input	Oversampling. The size of the subspace will be (k + p). (k+p) is less than min(m,n).
niters	host	input	Number of iteration of power method.
dataTypeA	host	input	Data type of array A.
A	device	in/out	Array of dimension lda * n with lda is not less than max(1,m). On exit, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
dataTypeS	host	input	Data type of array S.
S	device	output	Real array of dimension min(m,n). The singular values of A, sorted so that S(i) >= S(i+1).
dataTypeU	host	input	Data type of array U.
U	device	output	Array of dimension ldu * m with ldu is not less than max(1,m). U contains the m×m unitary matrix U. if jobu=S, only reports first min(m,n) columns of U.
ldu	host	input	Leading dimension of two-dimensional array used to store matrix U.
dataTypeV	host	input	Data type of array V.
V	device	output	Array of dimension ldv * n with ldv is not less than max(1,n). V contains the n×n unitary matrix V. If jobv=S, only reports first min(m,n) columns of V.
ldv	host	input	Leading dimension of two-dimensional array used to store matrix V.
computeType	host	input	Data type of computation.
bufferOnDevice	device	in/out	Device workspace. Array of type void of size workspaceInBytesOnDevice bytes.
workspaceInBytesOnDevice	host	input	Size in bytes of bufferOnDevice, returned by cusolverDnXgesvdr_bufferSize.
bufferOnHost	host	in/out	Host workspace. Array of type void of size workspaceInBytesOnHost bytes.
workspaceInBytesOnHost	host	input	Size in bytes of bufferOnHost, returned by cusolverDnXgesvdr_bufferSize.
d_info	device	output	If info = 0, the operation is successful. If info = -i, the i-th parameter is wrong (not counting handle).

The generic API has five different types, `dataTypeA` is data type of the matrix `A`, `dataTypeS` is data type of the vector `S` and `dataTypeU` is data type of the matrix `U`, `dataTypeV` is data type of the matrix `V`, `computeType` is compute type of the operation. `cusolverDnXgesvdr` only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	DataTypeS	DataTypeU	DataTypeV	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SGESVDR
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DGESVDR
CUDA_C_32F	CUDA_R_32F	CUDA_C_32F	CUDA_C_32F	CUDA_C_32F	CGESVDR
CUDA_C_64F	CUDA_R_64F	CUDA_C_64F	CUDA_C_64F	CUDA_C_64F	ZGESVDR

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (`m,n<0` or `lda<max(1,m)` or `ldu<max(1,m)` or `ldv<max(1,n)`).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.5.4. cusolverDnXsyevd()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnXsyevd_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    cudaDataType dataTypeW,
    const void *W,
    cudaDataType computeType,
    size_t *workspaceInBytesOnDevice,
    size_t *workspaceInBytesOnHost)
```

The following routine:

```
cusolverStatus_t
cusolverDnXsyevd(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
```

```
    int64_t lda,
    cudaDataType dataTypeW,
    void *W,
    cudaDataType computeType,
    void *bufferOnDevice,
    size_t workspaceInBytesOnDevice,
    void *bufferOnHost,
    size_t workspaceInBytesOnHost,
    int *info)
```

computes eigenvalues and eigenvectors of a symmetric (Hermitian) `n×n` matrix `A` using the generic API interface. The standard symmetric eigenvalue problem is

$$A*V = V*\Lambda$$

where `Λ` is a real `n×n` diagonal matrix. `V` is an `n×n` unitary matrix. The diagonal elements of `Λ` are the eigenvalues of `A` in ascending order.

The user has to provide device and host working spaces which are pointed by input parameters `bufferOnDevice` and `bufferOnHost`. The input parameters `workspaceInBytesOnDevice` (and `workspaceInBytesOnHost`) is size in bytes of the device (and host) working space, and it is returned by `cusolverDnXsyevd_bufferSize()`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). If `info = i` (greater than zero), `i` off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

If `jobz` = CUSOLVER_EIG_MODE_VECTOR, `A` contains the orthonormal eigenvectors of the matrix `A`. The eigenvectors are computed by a divide and conquer algorithm.

Please visit cuSOLVER Library Samples - Xsyevd for a code example.

Currently, `cusolverDnXsyevd` supports only the default algorithm.

Table of algorithms supported by cusolverDnXsyevd

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

List of input arguments for `cusolverDnXsyevd_bufferSize` and `cusolverDnXsyevd`:

API of cusolverDnXsyevd

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`params`	`host`	`input`	Structure with information collected by `cusolverDnSetAdvOptions`.
`jobz`	`host`	`input`	Specifies options to either compute eigenvalue only or compute eigen-pair: `jobz` = `CUSOLVER_EIG_MODE_NOVECTOR` : Compute eigenvalues only; `jobz` = `CUSOLVER_EIG_MODE_VECTOR` : Compute eigenvalues and eigenvectors.
`uplo`	`host`	`input`	Specifies which part of `A` is stored. `uplo` = `CUBLAS_FILL_MODE_LOWER`: Lower triangle of `A` is stored. `uplo` =

Parameter	Memory	In/out	Meaning
			CUBLAS_FILL_MODE_UPPER: Upper triangle of A is stored.
n	host	input	Number of rows (or columns) of matrix A.
dataTypeA	host	in	Data type of array A.
A	device	in/out	Array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A. On exit, if jobz = CUSOLVER_EIG_MODE_VECTOR, and info = 0, A contains the orthonormal eigenvectors of the matrix A. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.
dataTypeW	host	in	Data type of array W.
W	device	output	A real array of dimension n. The eigenvalue values of A, in ascending order, i.e., sorted so that W(i) <= W(i+1).
computeType	host	in	Data type of computation.
bufferOnDevice	device	in/out	Device workspace. Array of type void of size workspaceInBytesOnDevice bytes.
workspaceInBytesOnDevice	host	input	Size in bytes of bufferOnDevice, returned by cusolverDnXpotrf_bufferSize.
bufferOnHost	host	in/out	Host workspace. Array of type void of size workspaceInBytesOnHost bytes.
workspaceInBytesOnHost	host	input	Size in bytes of bufferOnHost, returned by cusolverDnXpotrf_bufferSize.
info	device	output	If info = 0, the operation is successful. If info = -i, the i-th parameter is wrong (not counting handle). If info = i (> 0), info indicates i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

The generic API has three different types, dataTypeA is data type of the matrix A, dataTypeW is data type of the matrix W and computeType is compute type of the operation. cusolverDnXsyevd only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	DataTypeW	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SSYEVD
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DSYEVD
CUDA_C_32F	CUDA_R_32F	CUDA_C_32F	CHEEVD
CUDA_C_64F	CUDA_R_64F	CUDA_C_64F	ZHEEVD

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n<0, or lda<max(1,n), or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or uplo is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.4.5.5. cusolverDnXsyevdx()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverDnXsyevdx_bufferSize(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
    int64_t n,
    cudaDataType dataTypeA,
    const void *A,
    int64_t lda,
    void *vl,
    void *vu,
    int64_t il,
    int64_t iu,
    int64_t *h_meig,
    cudaDataType dataTypeW,
    const void *W,
    cudaDataType computeType,
    size_t *workspaceInBytesOnDevice,
    size_t *workspaceInBytesOnHost)
```

The following routine:

```
cusolverStatus_t CUSOLVERAPI cusolverDnXsyevdx(
    cusolverDnHandle_t handle,
    cusolverDnParams_t params,
    cusolverEigMode_t jobz,
    cusolverEigRange_t range,
    cublasFillMode_t uplo,
```

```
    int64_t n,
    cudaDataType dataTypeA,
    void *A,
    int64_t lda,
    void * vl,
    void * vu,
    int64_t il,
    int64_t iu,
    int64_t *meig64,
    cudaDataType dataTypeW,
    void *W,
    cudaDataType computeType,
    void *bufferOnDevice,
    size_t workspaceInBytesOnDevice,
    void *bufferOnHost,
    size_t workspaceInBytesOnHost,
    int *info)
```

computes all or selection of the eigenvalues and optionally eigenvectors of a symmetric (Hermitian) `n×n` matrix `A` using the generic API interface. The standard symmetric eigenvalue problem is

$$A*V = V*\Lambda$$

where Λ is a real `n×h_meig` diagonal matrix. `V` is an `n×h_meig` unitary matrix. `h_meig` is the number of eigenvalues/eigenvectors computed by the routine, `h_meig` is equal to `n` when the whole spectrum (e.g., `range` = `CUSOLVER_EIG_RANGE_ALL`) is requested. The diagonal elements of Λ are the eigenvalues of `A` in ascending order.

The user has to provide device and host working spaces which are pointed by input parameters `bufferOnDevice` and `bufferOnHost`. The input parameters `workspaceInBytesOnDevice` (and `workspaceInBytesOnHost`) is size in bytes of the device (and host) working space, and it is returned by `cusolverDnXsyevdx_bufferSize()`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). If `info = i` (greater than zero), `i` off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

if `jobz` = CUSOLVER_EIG_MODE_VECTOR, `A` contains the orthonormal eigenvectors of the matrix `A`. The eigenvectors are computed by a divide and conquer algorithm.

Currently, `cusolverDnXsyevdx` supports only the default algorithm.

Please visit cuSOLVER Library Samples - Xsyevdx for a code example.

Table of algorithms supported by cusolverDnXsyevdx

`CUSOLVER_ALG_0` or `NULL`	Default algorithm.

List of input arguments for `cusolverDnXsyevdx_bufferSize` and `cusolverDnXsyevdx`:

API of cusolverDnXsyevdx

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverDN library context.
`params`	`host`	`input`	Structure with information collected by `cusolverDnSetAdvOptions`.

Parameter	Memory	In/out	Meaning
jobz	host	input	Specifies options to either compute eigenvalue only or compute eigen-pair: jobz = CUSOLVER_EIG_MODE_NOVECTOR : Compute eigenvalues only; jobz = CUSOLVER_EIG_MODE_VECTOR : Compute eigenvalues and eigenvectors.
range	host	input	Specifies options to which selection of eigenvalues and optionally eigenvectors that need to be computed: range = CUSOLVER_EIG_RANGE_ALL : all eigenvalues/eigenvectors will be found, will becomes the classical syevd/heevd routine; range = CUSOLVER_EIG_RANGE_V : all eigenvalues/eigenvectors in the half-open interval (vl,vu] will be found; range = CUSOLVER_EIG_RANGE_I : the il-th through iu-th eigenvalues/eigenvectors will be found;
uplo	host	input	Specifies which part of A is stored. uplo = CUBLAS_FILL_MODE_LOWER: Lower triangle of A is stored. uplo = CUBLAS_FILL_MODE_UPPER: Upper triangle of A is stored.
n	host	input	Number of rows (or columns) of matrix A.
dataTypeA	host	in	Data type of array A.
A	device	in/out	Array of dimension lda * n with lda is not less than max(1,n). If uplo = CUBLAS_FILL_MODE_UPPER, the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A. If uplo = CUBLAS_FILL_MODE_LOWER, the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A. On exit, if jobz = CUSOLVER_EIG_MODE_VECTOR, and info = 0, A contains the orthonormal eigenvectors of the matrix A. If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
lda	host	input	Leading dimension of two-dimensional array used to store matrix A.lda is not less than max(1,n).
vl,vu	host	input	If range = CUSOLVER_EIG_RANGE_V, the lower and upper bounds of the interval to be searched for eigenvalues. vl > vu. Not referenced if range = CUSOLVER_EIG_RANGE_ALL or range

Parameter	Memory	In/out	Meaning
			= CUSOLVER_EIG_RANGE_I. Note that, if eigenvalues are very close to each other, it is well known that two different eigenvalues routines might find slightly different number of eigenvalues inside the same interval. This is due to the fact that different eigenvalue algorithms, or even same algorithm but different run might find eigenvalues within some rounding error close to the machine precision. Thus, if the user want to be sure not to miss any eigenvalue within the interval bound, we suggest that, the user substract/add epsilon (machine precision) to the interval bound such as (vl=vl-eps, vu=vu+eps]. this suggestion is valid for any selective routine from cuSolver or LAPACK.
il,iu	host	input	Integer. If range = CUSOLVER_EIG_RANGE_I, the indices (in ascending order) of the smallest and largest eigenvalues to be returned. 1 <= il <= iu <= n, if n > 0; il = 1 and iu = 0 if n = 0. Not referenced if range = CUSOLVER_EIG_RANGE_ALL or range = CUSOLVER_EIG_RANGE_V.
h_meig	host	output	Integer. The total number of eigenvalues found. 0 <= h_meig <= n. If range = CUSOLVER_EIG_RANGE_ALL, h_meig = n, and if range = CUSOLVER_EIG_RANGE_I, h_meig = iu-il+1.
dataTypeW	host	in	Data type of array W.
W	device	output	A real array of dimension n. The eigenvalue values of A, in ascending order, i.e., sorted so that W(i) <= W(i+1).
computeType	host	in	Data type of computation.
bufferOnDevice	device	in/out	Device workspace. Array of type void of size workspaceInBytesOnDevice bytes.
workspaceInBytesOnDevice	host	input	Size in bytes of bufferOnDevice, returned by cusolverDnXpotrf_bufferSize.
bufferOnHost	host	in/out	Host workspace. Array of type void of size workspaceInBytesOnHost bytes.
workspaceInBytesOnHost	host	input	Size in bytes of bufferOnHost, returned by cusolverDnXpotrf_bufferSize.
info	device	output	If info = 0, the operation is successful. if info = -i, the i-th parameter is

Parameter	Memory	In/out	Meaning
			wrong (not counting handle). If `info = i (> 0)`, `info` indicates `i` off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

The generic API has three different types, `dataTypeA` is data type of the matrix `A`, `dataTypeW` is data type of the matrix `W` and `computeType` is compute type of the operation. `cusolverDnXsyevdx` only supports the following four combinations:

Valid combination of data type and compute type

DataTypeA	DataTypeW	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SSYEVDX
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DSYEVDX
CUDA_C_32F	CUDA_R_32F	CUDA_C_32F	CHEEVDX
CUDA_C_64F	CUDA_R_64F	CUDA_C_64F	ZHEEVDX

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (`n<0`, or `lda<max(1,n)`, or `jobz` is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or `range` is not CUSOLVER_EIG_RANGE_ALL or CUSOLVER_EIG_RANGE_V or CUSOLVER_EIG_RANGE_I, or `uplo` is not CUBLAS_FILL_MODE_LOWER or CUBLAS_FILL_MODE_UPPER).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.5. cuSolverSP: sparse LAPACK Function Reference

This section describes the API of cuSolverSP, which provides a subset of LAPACK funtions for sparse matrices in CSR or CSC format.

2.5.1. Helper Function Reference

2.5.1.1. cusolverSpCreate()

```
cusolverStatus_t
cusolverSpCreate(cusolverSpHandle_t *handle)
```

This function initializes the cuSolverSP library and creates a handle on the cuSolver context. It must be called before any other cuSolverSP API function is invoked. It allocates hardware resources necessary for accessing the GPU.

Output

`handle`	The pointer to the handle to the cuSolverSP context.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The initialization succeeded.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The CUDA Runtime initialization failed.
`CUSOLVER_STATUS_ALLOC_FAILED`	The resources could not be allocated.
`CUSOLVER_STATUS_ARCH_MISMATCH`	The device only supports compute capability 2.0 and above.

2.5.1.2. cusolverSpDestroy()

```
cusolverStatus_t
cusolverSpDestroy(cusolverSpHandle_t handle)
```

This function releases CPU-side resources used by the cuSolverSP library.

Input

`handle`	The handle to the cuSolverSP context.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The shutdown succeeded.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.

2.5.1.3. cusolverSpSetStream()

```
cusolverStatus_t
cusolverSpSetStream(cusolverSpHandle_t handle, cudaStream_t streamId)
```

This function sets the stream to be used by the cuSolverSP library to execute its routines.

Input

`handle`	The handle to the cuSolverSP context.
`streamId`	The stream to be used by the library.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The stream was set successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.

2.5.1.4. cusolverSpXcsrissym()

```
cusolverStatus_t
cusolverSpXcsrissymHost(cusolverSpHandle_t handle,
              int m,
              int nnzA,
              const cusparseMatDescr_t descrA,
              const int *csrRowPtrA,
              const int *csrEndPtrA,
              const int *csrColIndA,
              int *issym);
```

This function checks if `A` has symmetric pattern or not. The output parameter `issym` reports 1 if `A` is symmetric; otherwise, it reports 0.

The matrix `A` is an `m×m` sparse matrix that is defined in CSR storage format by the four arrays `csrValA`, `csrRowPtrA`, `csrEndPtrA` and `csrColIndA`.

The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`.

The `csrlsvlu` and `csrlsvqr` do not accept non-general matrix. the user has to extend the matrix into its missing upper/lower part, otherwise the result is not expected. The user can use `csrissym` to check if the matrix has symmetric pattern or not.

Remark 1: only CPU path is provided.

Remark 2: the user has to check returned status to get valid information. The function converts `A` to CSC format and compare CSR and CSC format. If the CSC failed because of insufficient resources, `issym` is undefined, and this state can only be detected by the return status code.

Input

Parameter	MemorySpace	Description
`handle`	`host`	Handle to the cuSolverSP library context.
`m`	`host`	Number of rows and columns of matrix `A`.
`nnzA`	`host`	Number of nonzeros of matrix `A`. It is the size of `csrValA` and `csrColIndA`.
`descrA`	`host`	The descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrRowPtrA`	`host`	Integer array of `m` elements that contains the start of every row.
`csrEndPtrA`	`host`	Integer array of `m` elements that contains the end of the last row plus one.
`csrColIndA`	`host`	Integer array of `nnzA`column indices of the nonzero elements of matrix `A`.

Output

Parameter	MemorySpace	Description
issym	host	1 if A is symmetric; 0 otherwise.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m, nnzA<=0), base index is not 0 or 1.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.2. High Level Function Reference

This section describes high level API of cuSolverSP, including linear solver, least-square solver and eigenvalue solver. The high-level API is designed for ease-of-use, so it allocates any required memory under the hood automatically. If the host or GPU system memory is not enough, an error is returned.

2.5.2.1. cusolverSp<t>csrlsvlu()

```
cusolverStatus_t
cusolverSpScsrlsvlu[Host](cusolverSpHandle_t handle,
                  int n,
                  int nnzA,
                  const cusparseMatDescr_t descrA,
                  const float *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  const float *b,
                  float tol,
                  int reorder,
                  float *x,
                  int *singularity);

cusolverStatus_t
cusolverSpDcsrlsvlu[Host](cusolverSpHandle_t handle,
                  int n,
                  int nnzA,
                  const cusparseMatDescr_t descrA,
                  const double *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  const double *b,
                  double tol,
                  int reorder,
                  double *x,
                  int *singularity);
```

```
cusolverStatus_t
cusolverSpCcsrlsvlu[Host](cusolverSpHandle_t handle,
                  int n,
                  int nnzA,
                  const cusparseMatDescr_t descrA,
                  const cuComplex *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  const cuComplex *b,
                  float tol,
                  int reorder,
                  cuComplex *x,
                  int *singularity);

cusolverStatus_t
cusolverSpZcsrlsvlu[Host](cusolverSpHandle_t handle,
                  int n,
                  int nnzA,
                  const cusparseMatDescr_t descrA,
                  const cuDoubleComplex *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  const cuDoubleComplex *b,
                  double tol,
                  int reorder,
                  cuDoubleComplex *x,
                  int *singularity);
```

This function solves the linear system

$$A*x=b$$

where `A` is an `n×n` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`. `b` is the right-hand-side vector of size `n`, and `x` is the solution vector of size `n`.

The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. If matrix `A` is symmetric/Hermitian and only lower/upper part is used or meaningful, the user has to extend the matrix into its missing upper/lower part, otherwise the result would be wrong.

The linear system is solved by sparse LU with partial pivoting:

$$P*A=L*U$$

`cusolver` library provides three reordering schemes, `symrcm` `symamd`, and `csrmetisnd` to reduce zero fill-in which dramactically affects the performance of LU factorization. The input parameter `reorder` can enable `symrcm` (`symamd` or `csrmetisnd`) if `reorder` is 1 (2, or 3), otherwise, no reordering is performed.

If `reorder` is nonzero, `csrlsvlu` does

$$P*A*Q^T=L*U$$

where $Q=\text{symrcm}(A+A^T)$.

If A is singular under given tolerance (max(tol,0)), then some diagonal elements of U is zero, i.e.

$$|U(j,j)| < \text{tol for some } j$$

The output parameter singularity is the smallest index of such j. If A is non-singular, singularity is -1. The index is base-0, independent of base index of A. For example, if 2nd column of A is the same as first column, then A is singular and singularity = 1 which means U(1,1)≈0.

Remark 1: csrlsvlu performs traditional LU with partial pivoting, the pivot of k-th column is determined dynamically based on the k-th column of intermediate matrix. csrlsvlu follows Gilbert and Peierls's algorithm [4] which uses depth-first-search and topological ordering to solve triangular system (Davis also describes this algorithm in detail in his book [1]). since cuda 10.1, csrlsvlu will incrementally reallocate the memory to store L and U. This feature can avoid over-estimate size from QR factorization. In some cases, zero fill-in of QR can be order of magnitude higher than LU.

Remark 2: only CPU (Host) path is provided.

Input

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
handle	host	host	Handle to the cuSolverSP library context.
n	host	host	Number of rows and columns of matrix A.
nnzA	host	host	Number of nonzeros of matrix A.
descrA	host	host	The descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	device	host	<type> array of nnzA (= csrRowPtrA(n) – csrRowPtrA(0)) nonzero elements of matrix A.
csrRowPtrA	device	host	Integer array of n + 1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	device	host	Integer array of nnzA (= csrRowPtrA(n) – csrRowPtrA(0)) column indices of the nonzero elements of matrix A.
b	device	host	Right hand side vector of size n.
tol	host	host	Tolerance to decide if singular or not.
reorder	host	host	No ordering if reorder=0. Otherwise, symrcm, symamd, or csrmetisnd is used to reduce zero fill-in.

Output

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
x	device	host	Solution vector of size n, x = inv(A)*b.
singularity	host	host	-1 if A is invertible. Otherwise, first index j such that U(j,j)≈0

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n,nnzA<=0, base index is not 0 or 1, reorder is not 0,1,2,3)
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.2.2. cusolverSp<t>csrlsvqr()

```
cusolverStatus_t
cusolverSpScsrlsvqr[Host](cusolverSpHandle_t handle,
                  int m,
                  int nnz,
                  const cusparseMatDescr_t descrA,
                  const float *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  const float *b,
                  float tol,
                  int reorder,
                  float *x,
                  int *singularity);

cusolverStatus_t
cusolverSpDcsrlsvqr[Host](cusolverSpHandle_t handle,
                  int m,
                  int nnz,
                  const cusparseMatDescr_t descrA,
                  const double *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  const double *b,
                  double tol,
                  int reorder,
                  double *x,
                  int *singularity);

cusolverStatus_t
cusolverSpCcsrlsvqr[Host](cusolverSpHandle_t handle,
                  int m,
                  int nnz,
                  const cusparseMatDescr_t descrA,
```

```
                  const cuComplex *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  const cuComplex *b,
                  float tol,
                  int reorder,
                  cuComplex *x,
                  int *singularity);

cusolverStatus_t
cusolverSpZcsrlsvqr[Host](cusolverSpHandle_t handle,
                  int m,
                  int nnz,
                  const cusparseMatDescr_t descrA,
                  const cuDoubleComplex *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  const cuDoubleComplex *b,
                  double tol,
                  int reorder,
                  cuDoubleComplex *x,
                  int *singularity);
```

This function solves the linear system

$$A*x=b$$

`A` is an `m×m` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`. `b` is the right-hand-side vector of size `m`, and `x` is the solution vector of size `m`.

The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. If matrix `A` is symmetric/Hermitian and only lower/upper part is used or meaningful, the user has to extend the matrix into its missing upper/lower part, otherwise the result would be wrong.

The linear system is solved by sparse QR factorization,

$$A = Q*R$$

If `A` is singular under given tolerance (`max(tol,0)`), then some diagonal elements of `R` is zero, i.e.

$$|R(j,j)| < \text{tol for some } j$$

The output parameter `singularity` is the smallest index of such `j`. If `A` is non-singular, `singularity` is -1. The `singularity` is base-0, independent of base index of `A`. For example, if 2nd column of `A` is the same as first column, then `A` is singular and `singularity = 1` which means `R(1,1)≈0`.

`cusolver` library provides three reordering schemes, `symrcm` `symamd`, and `csrmetisnd` to reduce zero fill-in which dramactically affects the performance of QR factorization. The input parameter `reorder` can enable `symrcm` (`symamd` or `csrmetisnd`) if `reorder` is 1 (2, or 3), otherwise, no reordering is performed.

Input

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
handle	host	host	Handle to the cuSolverSP library context.
m	host	host	Number of rows and columns of matrix A.
nnz	host	host	Number of nonzeros of matrix A.
descrA	host	host	The descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	device	host	<type> array of nnz (= csrRowPtrA(m) – csrRowPtrA(0)) nonzero elements of matrix A.
csrRowPtrA	device	host	Integer array of m + 1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	device	host	Integer array of nnz (= csrRowPtrA(m) – csrRowPtrA(0)) column indices of the nonzero elements of matrix A.
b	device	host	Right hand side vector of size m.
tol	host	host	Tolerance to decide if singular or not.
reorder	host	host	No ordering if reorder=0. Otherwise, symrcm, symamd, or csrmetisnd is used to reduce zero fill-in.

Output

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
x	device	host	solution vector of size m, x = inv(A)*b.
singularity	host	host	-1 if A is invertible. Otherwise, first index j such that R(j,j)≈0

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,nnz<=0, base index is not 0 or 1, reorder is not 0,1,2,3)
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.2.3. cusolverSp<t>csrlsvchol()

```
cusolverStatus_t
cusolverSpScsrlsvchol[Host](cusolverSpHandle_t handle,
                      int m,
                      int nnz,
                      const cusparseMatDescr_t descrA,
                      const float *csrVal,
                      const int *csrRowPtr,
                      const int *csrColInd,
                      const float *b,
                      float tol,
                      int reorder,
                      float *x,
                      int *singularity);

cusolverStatus_t
cusolverSpDcsrlsvchol[Host](cusolverSpHandle_t handle,
                      int m,
                      int nnz,
                      const cusparseMatDescr_t descrA,
                      const double *csrVal,
                      const int *csrRowPtr,
                      const int *csrColInd,
                      const double *b,
                      double tol,
                      int reorder,
                      double *x,
                      int *singularity);

cusolverStatus_t
cusolverSpCcsrlsvchol[Host](cusolverSpHandle_t handle,
                      int m,
                      int nnz,
                      const cusparseMatDescr_t descrA,
                      const cuComplex *csrVal,
                      const int *csrRowPtr,
                      const int *csrColInd,
                      const cuComplex *b,
                      float tol,
                      int reorder,
                      cuComplex *x,
                      int *singularity);

cusolverStatus_t
cusolverSpZcsrlsvchol[Host](cusolverSpHandle_t handle,
                      int m,
                      int nnz,
                      const cusparseMatDescr_t descrA,
                      const cuDoubleComplex *csrVal,
                      const int *csrRowPtr,
                      const int *csrColInd,
                      const cuDoubleComplex *b,
                      double tol,
                      int reorder,
                      cuDoubleComplex *x,
                      int *singularity);
```

This function solves the linear system

$$A*x=b$$

A is an m×m symmetric postive definite sparse matrix that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA. b is the right-hand-side vector of size m, and x is the solution vector of size m.

The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL and upper triangular part of A is ignored (if parameter reorder is zero). In other words, suppose input matrix A is decomposed as $A=L+D+U$, where L is lower triangular, D is diagonal and U is upper triangular. The function would ignore U and regard A as a symmetric matrix with the formula $A=L+D+L^H$. If parameter reorder is nonzero, the user has to extend A to a full matrix, otherwise the solution would be wrong.

The linear system is solved by sparse Cholesky factorization,

$$A=G*G^H$$

where G is the Cholesky factor, a lower triangular matrix.

The output parameter singularity has two meanings:

- If A is not postive definite, there exists some integer k such that A(0:k, 0:k) is not positive definite. singularity is the minimum of such k.
- If A is postive definite but near singular under tolerance (max(tol,0)), i.e. there exists some integer k such that $G(\mathrm{k},\mathrm{k}) <= \mathrm{tol}$. singularity is the minimum of such k.

singularity is base-0. If A is positive definite and not near singular under tolerance, singularity is -1. If the user wants to know if A is postive definite or not, tol=0 is enough.

cusolver library provides three reordering schemes, symrcm symamd, and csrmetisnd to reduce zero fill-in which dramactically affects the performance of Cholesky factorization. The input parameter reorder can enable symrcm (symamd or csrmetisnd) if reorder is 1 (2, or 3), otherwise, no reordering is performed.

Remark 1: the function works for in-place (x and b point to the same memory block) and out-of-place.

Remark 2: the function only works on 32-bit index, if matrix G has large zero fill-in such that number of nonzeros is bigger than 2^{31} , then CUSOLVER_STATUS_ALLOC_FAILED is returned.

Input

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
handle	host	host	Handle to the cuSolverSP library context.
m	host	host	Number of rows and columns of matrix A.
nnz	host	host	Number of nonzeros of matrix A.
descrA	host	host	The descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
			Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrValA`	`device`	`host`	<type> array of `nnz` (= `csrRowPtrA(m)` – `csrRowPtrA(0)`) nonzero elements of matrix `A`.
`csrRowPtrA`	`device`	`host`	Integer array of `m` + 1 elements that contains the start of every row and the end of the last row plus one.
`csrColIndA`	`device`	`host`	Integer array of `nnz` (= `csrRowPtrA(m)` – `csrRowPtrA(0)`) column indices of the nonzero elements of matrix `A`.
`b`	`device`	`host`	Right hand side vector of size `m`.
`tol`	`host`	`host`	Tolerance to decide singularity.
`reorder`	`host`	`host`	No ordering if `reorder`=0. Otherwise, `symrcm`, `symamd`, or `csrmetisnd` is used to reduce zero fill-in.

Output

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
`x`	`device`	`host`	solution vector of size `m`, x = inv(A)*b.
`singularity`	`host`	`host`	-1 if `A` is symmetric postive definite.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.
`CUSOLVER_STATUS_ALLOC_FAILED`	The resources could not be allocated.
`CUSOLVER_STATUS_INVALID_VALUE`	Invalid parameters were passed (`m,nnz<=0`, base index is not 0 or 1, `reorder` is not 0,1,2,3)
`CUSOLVER_STATUS_ARCH_MISMATCH`	The device only supports compute capability 2.0 and above.
`CUSOLVER_STATUS_INTERNAL_ERROR`	An internal operation failed.
`CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED`	The matrix type is not supported.

2.5.2.4. cusolverSp<t>csrlsqvqr()

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverSpScsrlsqvqr[Host](cusolverSpHandle_t handle,
                  int m,
                  int n,
                  int nnz,
```

```
                    const cusparseMatDescr_t descrA,
                    const float *csrValA,
                    const int *csrRowPtrA,
                    const int *csrColIndA,
                    const float *b,
                    float tol,
                    int *rankA,
                    float *x,
                    int *p,
                    float *min_norm);

cusolverStatus_t
cusolverSpDcsrlsqvqr[Host](cusolverSpHandle_t handle,
                    int m,
                    int n,
                    int nnz,
                    const cusparseMatDescr_t descrA,
                    const double *csrValA,
                    const int *csrRowPtrA,
                    const int *csrColIndA,
                    const double *b,
                    double tol,
                    int *rankA,
                    double *x,
                    int *p,
                    double *min_norm);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverSpCcsrlsqvqr[Host](cusolverSpHandle_t handle,
                    int m,
                    int n,
                    int nnz,
                    const cusparseMatDescr_t descrA,
                    const cuComplex *csrValA,
                    const int *csrRowPtrA,
                    const int *csrColIndA,
                    const cuComplex *b,
                    float tol,
                    int *rankA,
                    cuComplex *x,
                    int *p,
                    float *min_norm);

cusolverStatus_t
cusolverSpZcsrlsqvqr[Host](cusolverSpHandle_t handle,
                    int m,
                    int n,
                    int nnz,
                    const cusparseMatDescr_t descrA,
                    const cuDoubleComplex *csrValA,
                    const int *csrRowPtrA,
                    const int *csrColIndA,
                    const cuDoubleComplex *b,
                    double tol,
                    int *rankA,
                    cuDoubleComplex *x,
                    int *p,
                    double *min_norm);
```

This function solves the following least-square problem:

$$x = \arg\min ||A*z - b||$$

A is an m×n sparse matrix that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA. b is the right-hand-side vector of size m, and x is the least-square solution vector of size n.

The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. If A is square, symmetric/Hermitian and only lower/upper part is used or meaningful, the user has to extend the matrix into its missing upper/lower part, otherwise the result is wrong.

This function only works if m is greater or equal to n, in other words, A is a tall matrix.

The least-square problem is solved by sparse QR factorization with column pivoting,

$$A*P^T = Q*R$$

If A is of full rank (i.e. all columns of A are linear independent), then matrix P is an identity. Suppose rank of A is k, less than n, the permutation matrix P reorders columns of A in the following sense:

$$A*P^T = (A_1 \quad A_2) = (Q_1 \quad Q_2)\begin{pmatrix} R_{11} & R_{12} \\ & R_{22} \end{pmatrix}$$

where R_{11} and A have the same rank, but R_{22} is almost zero, i.e. every column of A_2 is linear combination of A_1.

The input parameter tol decides numerical rank. The absolute value of every entry in R_{22} is less than or equal to tolerance=max(tol,0).

The output parameter rankA denotes numerical rank of A.

Suppose $y = P*x$ and $c = Q^H*b$, the least square problem can be reformed by

$$\min||A*x - b|| = \min||R*y - c||$$

or in matrix form

$$\begin{pmatrix} R_{11} & R_{12} \\ & R_{22} \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

The output parameter min_norm is $||c_2||$, which is minimum value of least-square problem.

If A is not of full rank, above equation does not have a unique solution. The least-square problem is equivalent to

$$\min||y||$$
$$\text{subject to} R_{11}*y_1 + R_{12}*y_2 = c_1$$

Or equivalently another least-square problem

$$\min||\begin{pmatrix} R_{11} \backslash R_{12} \\ I \end{pmatrix}*y_2 - \begin{pmatrix} R_{11} \backslash c_1 \\ O \end{pmatrix}||$$

The output parameter x is $P^T * y$, the solution of least-square problem.

The output parameter p is a vector of size n. It corresponds to a permutation matrix P. p(i)=j means (P*x)(i) = x(j). If A is of full rank, p=0:n-1.

Remark 1: p is always base 0, independent of base index of A.

Remark 2: only CPU (Host) path is provided.

Input

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
handle	host	host	Handle to the cuSolver library context.
m	host	host	Number of rows of matrix A.
n	host	host	Number of columns of matrix A.
nnz	host	host	Number of nonzeros of matrix A.
descrA	host	host	The descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	device	host	<type> array of nnz (= csrRowPtrA(m) – csrRowPtrA(0)) nonzero elements of matrix A.
csrRowPtrA	device	host	Integer array of m + 1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	device	host	Integer array of nnz (= csrRowPtrA(m) – csrRowPtrA(0)) column indices of the nonzero elements of matrix A.
b	device	host	Right hand side vector of size m.
tol	host	host	Tolerance to decide rank of A.

Output

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
rankA	host	host	Numerical rank of A.
x	device	host	Solution vector of size n, x=pinv(A)*b.
p	device	host	A vector of size n, which represents the permuation matrix P satisfying A*P^T=Q*R.
min_norm	host	host	||A*x-b||, x=pinv(A)*b.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n,nnz<=0), base index is not 0 or 1.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.2.5. cusolverSp<t>csreigvsi()

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverSpScsreigvsi[Host](cusolverSpHandle_t handle,
                  int m,
                  int nnz,
                  const cusparseMatDescr_t descrA,
                  const float *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  float mu0,
                  const float *x0,
                  int maxite,
                  float tol,
                  float *mu,
                  float *x);

cusolverStatus_t
cusolverSpDcsreigvsi[Host](cusolverSpHandle_t handle,
                  int m,
                  int nnz,
                  const cusparseMatDescr_t descrA,
                  const double *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  double mu0,
                  const double *x0,
                  int maxite,
                  double tol,
                  double *mu,
                  double *x);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverSpCcsreigvsi[Host](cusolverSpHandle_t handle,
                  int m,
                  int nnz,
                  const cusparseMatDescr_t descrA,
                  const cuComplex *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  cuComplex mu0,
                  const cuComplex *x0,
                  int maxite,
```

```
                  float tol,
                  cuComplex *mu,
                  cuComplex *x);

cusolverStatus_t
cusolverSpZcsreigvsi(cusolverSpHandle_t handle,
                  int m,
                  int nnz,
                  const cusparseMatDescr_t descrA,
                  const cuDoubleComplex *csrValA,
                  const int *csrRowPtrA,
                  const int *csrColIndA,
                  cuDoubleComplex mu0,
                  const cuDoubleComplex *x0,
                  int maxite,
                  double tol,
                  cuDoubleComplex *mu,
                  cuDoubleComplex *x);
```

This function solves the simple eigenvalue problem $A*x = \lambda*x$ by shift-inverse method.

`A` is an `m×m` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`. The output paramter `x` is the approximated eigenvector of size `m`,

The following shift-inverse method corrects eigenpair step-by-step until convergence.

It accepts several parameters:

`mu0` is an initial guess of eigenvalue. The shift-inverse method will converge to the eigenvalue `mu` nearest `mu0` if `mu` is a singleton. Otherwise, the shift-inverse method may not converge.

`x0` is an initial eigenvector. If the user has no preference, just chose `x0` randomly. `x0` must be nonzero. It can be non-unit length.

`tol` is the tolerance to decide convergence. If `tol` is less than zero, it would be treated as zero.

`maxite` is maximum number of iterations. It is useful when shift-inverse method does not converge because the tolerance is too small or the desired eigenvalue is not a singleton.

Shift-Inverse Method

Given a initial guess of eigenvalue μ0 and initial vector x0

$x^{(0)}$ = x0 of unit length

for j = 0 : maxite

solve (A − μ0 * I

normalize $x^{(k+1)}$ to unit length

compute approx. eigenvalue μ = x^H * A *

if || A * $x^{(k+1)}$ − μ * x

endfor

The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. If `A` is symmetric/Hermitian and only lower/upper part is used or meaningful, the user has to extend the matrix into its missing upper/lower part, otherwise the result is wrong.

Remark 1: `[cu|h]solver[S|D]csreigvsi` only allows `mu0` as a real number. This works if `A` is symmetric. Otherwise, the non-real eigenvalue has a conjugate counterpart on the complex plan, and shift-inverse method would not converge to such eigevalue even the eigenvalue is a singleton. The user has to extend `A` to complex numbre and call `[cu|h]solver[C|Z]csreigvsi` with `mu0` not on real axis.

Remark 2: the tolerance `tol` should not be smaller than |mu0|*eps, where eps is machine zero. Otherwise, shift-inverse may not converge because of small tolerance.

Input

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
`handle`	`host`	`host`	Handle to the cuSolver library context.
`m`	`host`	`host`	Number of rows and columns of matrix `A`.
`nnz`	`host`	`host`	Number of nonzeros of matrix `A`.
`descrA`	`host`	`host`	The descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrValA`	`device`	`host`	<type> array of `nnz` (= `csrRowPtrA(m)` – `csrRowPtrA(0)`) nonzero elements of matrix `A`.
`csrRowPtrA`	`device`	`host`	Integer array of `m` + 1 elements that contains the start of every row and the end of the last row plus one.
`csrColIndA`	`device`	`host`	Integer array of `nnz` (= `csrRowPtrA(m)` – `csrRowPtrA(0)`) column indices of the nonzero elements of matrix `A`.
`mu0`	`host`	`host`	Initial guess of eigenvalue.
`x0`	`device`	`host`	Initial guess of eigenvector, a vecotr of size `m`.
`maxite`	`host`	`host`	Maximum iterations in shift-inverse method.
`tol`	`host`	`host`	Tolerance for convergence.

Output

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
`mu`	`device`	`host`	Approximated eigenvalue nearest `mu0` under tolerance.
`x`	`device`	`host`	Approximated eigenvector of size `m`.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,nnz<=0), base index is not 0 or 1.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.2.6. cusolverSp<t>csreigs()

```
cusolverStatus_t
solverspScsreigs[Host](cusolverSpHandle_t handle,
                int m,
                int nnz,
                const cusparseMatDescr_t descrA,
                const float *csrValA,
                const int *csrRowPtrA,
                const int *csrColIndA,
                cuComplex left_bottom_corner,
                cuComplex right_upper_corner,
                int *num_eigs);

cusolverStatus_t
cusolverSpDcsreigs[Host](cusolverSpHandle_t handle,
                int m,
                int nnz,
                const cusparseMatDescr_t descrA,
                const double *csrValA,
                const int *csrRowPtrA,
                const int *csrColIndA,
                cuDoubleComplex left_bottom_corner,
                cuDoubleComplex right_upper_corner,
                int *num_eigs);

cusolverStatus_t
cusolverSpCcsreigs[Host](cusolverSpHandle_t handle,
                int m,
                int nnz,
                const cusparseMatDescr_t descrA,
                const cuComplex *csrValA,
                const int *csrRowPtrA,
                const int *csrColIndA,
                cuComplex left_bottom_corner,
                cuComplex right_upper_corner,
                int *num_eigs);

cusolverStatus_t
cusolverSpZcsreigs[Host](cusolverSpHandle_t handle,
                int m,
                int nnz,
                const cusparseMatDescr_t descrA,
                const cuDoubleComplex *csrValA,
                const int *csrRowPtrA,
```

```
                const int *csrColIndA,
                cuDoubleComplex left_bottom_corner,
                cuDoubleComplex right_upper_corner,
                int *num_eigs);
```

This function computes number of algebraic eigenvalues in a given box `B` by contour integral

$$\text{number of algebraic eigenvalues in box B} = \frac{1}{2*\pi*\sqrt{-1}} \oint_C \frac{P'(z)}{P(z)} dz$$

where closed line `C` is boundary of the box `B` which is a rectangle specified by two points, one is left bottom corner (input parameter `left_botoom_corner`) and the other is right upper corner (input parameter `right_upper_corner`). `P(z)=det(A - z*I)` is the characteristic polynomial of `A`.

`A` is an `m×m` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`.

The output parameter `num_eigs` is number of algebraic eigenvalues in the box `B`. This number may not be accurate due to several reasons:

1. The contour `C` is close to some eigenvalues or even passes through some eigenvalues.
2. The numerical integration is not accurate due to coarse grid size. The default resolution is 1200 grids along contour `C` uniformly.

Even though `csreigs` may not be accurate, it still can give the user some idea how many eigenvalues in a region where the resolution of disk theorem is bad. For example, standard 3-point stencil of finite difference of Laplacian operator is a tridiagonal matrix, and disk theorem would show "all eigenvalues are in the interval [0, 4*N^2]" where N is number of grids. In this case, `csreigs` is useful for any interval inside [0, 4*N^2].

Remark 1: if `A` is symmetric in real or hermitian in complex, all eigenvalues are real. The user still needs to specify a box, not an interval. The height of the box can be much smaller than the width.

Remark 2: only CPU (Host) path is provided.

Input

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
`handle`	`host`	`host`	Handle to the cuSolverSP library context.
`m`	`host`	`host`	Number of rows and columns of matrix `A`.
`nnz`	`host`	`host`	Number of nonzeros of matrix `A`.
`descrA`	`host`	`host`	The descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
csrValA	device	host	<type> array of nnz (= csrRowPtrA(m) – csrRowPtrA(0)) nonzero elements of matrix A.
csrRowPtrA	device	host	Integer array of m + 1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	device	host	Integer array of nnz (= csrRowPtrA(m) – csrRowPtrA(0)) column indices of the nonzero elements of matrix A.
left_bottom_corner	host	host	Left bottom corner of the box.
right_upper_corner	host	host	Right upper corner of the box.

Output

Parameter	cusolverSp MemSpace	*Host MemSpace	Description
num_eigs	host	host	Number of algebraic eigenvalues in a box.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m, nnz<=0), base index is not 0 or 1.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.3. Low Level Function Reference

This section describes low level API of cuSolverSP, including symrcm and batched QR.

2.5.3.1. cusolverSpXcsrsymrcm()

```
cusolverStatus_t
cusolverSpXcsrsymrcmHost(cusolverSpHandle_t handle,
             int n,
             int nnzA,
             const cusparseMatDescr_t descrA,
             const int *csrRowPtrA,
             const int *csrColIndA,
             int *p);
```

This function implements Symmetric Reverse Cuthill-McKee permutation. It returns a permutation vector `p` such that `A(p,p)` would concentrate nonzeros to diagonal. This is equivalent to `symrcm` in MATLAB, however the result may not be the same because of different heuristics in the pseudoperipheral finder. The `cuSolverSP` library implements `symrcm` based on the following two papers:

E. Chuthill and J. McKee, reducing the bandwidth of sparse symmetric matrices, ACM '69 Proceedings of the 1969 24th national conference, Pages 157-172

Alan George, Joseph W. H. Liu, An Implementation of a Pseudoperipheral Node Finder, ACM Transactions on Mathematical Software (TOMS) Volume 5 Issue 3, Sept. 1979, Pages 284-295

The output parameter `p` is an integer array of `n` elements. It represents a permutation array and it indexed using the base-0 convention. The permutation array `p` corresponds to a permutation matrix `P`, and satisfies the following relation:

$$\mathrm{A(p,p)} = P * A * P^T$$

`A` is an `n×n` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`.

The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Internally `rcm` works on $A + A^T$, the user does not need to extend the matrix if the matrix is not symmetric.

Remark 1: only CPU (Host) path is provided.

Input

Parameter	*Host MemSpace	Description
`handle`	`host`	Handle to the cuSolverSP library context.
`n`	`host`	Number of rows and columns of matrix `A`.
`nnzA`	`host`	Number of nonzeros of matrix `A`. It is the size of `csrValA` and `csrColIndA`.
`descrA`	`host`	The descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrRowPtrA`	`host`	Integer array of `n+1` elements that contains the start of every row and the end of the last row plus one.
`csrColIndA`	`host`	Integer array of `nnzA`column indices of the nonzero elements of matrix `A`.

Output

Parameter	hsolver	Description
`p`	`host`	Permutation vector of size `n`.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.

CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n,nnzA<=0), base index is not 0 or 1.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.3.2. cusolverSpXcsrsymmdq()

```
cusolverStatus_t
cusolverSpXcsrsymmdqHost(cusolverSpHandle_t handle,
             int n,
             int nnzA,
             const cusparseMatDescr_t descrA,
             const int *csrRowPtrA,
             const int *csrColIndA,
             int *p);
```

This function implements Symmetric Minimum Degree Algorithm based on Quotient Graph. It returns a permutation vector p such that A(p,p) would have less zero fill-in during Cholesky factorization. The cuSolverSP library implements symmdq based on the following two papers:

Patrick R. Amestoy, Timothy A. Davis, Iain S. Duff, An Approximate Minimum Degree Ordering Algorithm, SIAM J. Matrix Analysis Applic. Vol 17, no 4, pp. 886-905, Dec. 1996.

Alan George, Joseph W. Liu, A Fast Implementation of the Minimum Degree Algorithm Using Quotient Graphs, ACM Transactions on Mathematical Software, Vol 6, No. 3, September 1980, page 337-358.

The output parameter p is an integer array of n elements. It represents a permutation array with base-0 index. The permutation array p corresponds to a permutation matrix P, and satisfies the following relation:

$$\mathrm{A(p,p)} = P * A * P^T$$

A is an n×n sparse matrix that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA.

The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Internally mdq works on $A + A^T$, the user does not need to extend the matrix if the matrix is not symmetric.

Remark 1: only CPU (Host) path is provided.

Input

Parameter	*Host MemSpace	Description
handle	host	Handle to the cuSolverSP library context.

Parameter	*Host MemSpace	Description
n	host	Number of rows and columns of matrix A.
nnzA	host	Number of nonzeros of matrix A. It is the size of csrValA and csrColIndA.
descrA	host	The descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrRowPtrA	host	Integer array of n+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	host	Integer array of nnzAcolumn indices of the nonzero elements of matrix A.

Output

Parameter	hsolver	Description
p	host	Permutation vector of size n.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n,nnzA<=0), base index is not 0 or 1.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.3.3. cusolverSpXcsrsymamd()

```
cusolverStatus_t
cusolverSpXcsrsymamdHost(cusolverSpHandle_t handle,
             int n,
             int nnzA,
             const cusparseMatDescr_t descrA,
             const int *csrRowPtrA,
             const int *csrColIndA,
             int *p);
```

This function implements Symmetric Approximate Minimum Degree Algorithm based on Quotient Graph. It returns a permutation vector p such that A(p,p) would have less zero fill-

in during Cholesky factorization. The `cuSolverSP` library implements `symamd` based on the following paper:

Patrick R. Amestoy, Timothy A. Davis, Iain S. Duff, An Approximate Minimum Degree Ordering Algorithm, SIAM J. Matrix Analysis Applic. Vol 17, no 4, pp. 886-905, Dec. 1996.

The output parameter `p` is an integer array of `n` elements. It represents a permutation array with base-0 index. The permutation array `p` corresponds to a permutation matrix `P`, and satisfies the following relation:

$$\mathrm{A(p,p)} = P * A * P^T$$

`A` is an `n×n` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`.

The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Internally `amd` works on $A + A^T$, the user does not need to extend the matrix if the matrix is not symmetric.

Remark 1: only CPU (Host) path is provided.

Input

Parameter	*Host MemSpace	Description
`handle`	`host`	Handle to the cuSolverSP library context.
`n`	`host`	Number of rows and columns of matrix `A`.
`nnzA`	`host`	Number of nonzeros of matrix `A`. It is the size of `csrValA` and `csrColIndA`.
`descrA`	`host`	The descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrRowPtrA`	`host`	Integer array of `n+1` elements that contains the start of every row and the end of the last row plus one.
`csrColIndA`	`host`	Integer array of `nnzA`column indices of the nonzero elements of matrix `A`.

Output

Parameter	hsolver	Description
`p`	`host`	Permutation vector of size `n`.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.
`CUSOLVER_STATUS_ALLOC_FAILED`	The resources could not be allocated.
`CUSOLVER_STATUS_INVALID_VALUE`	Invalid parameters were passed (`n,nnzA<=0`), base index is not 0 or 1.

CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

2.5.3.4. cusolverSpXcsrmetisnd()

```
cusolverStatus_t
cusolverSpXcsrmetisndHost(
    cusolverSpHandle_t handle,
    int n,
    int nnzA,
    const cusparseMatDescr_t descrA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    const int64_t *options,
    int *p);
```

This function is a wrapper of METIS_NodeND. It returns a permutation vector p such that A(p,p) would have less zero fill-in during nested dissection. The cuSolverSP library links libmetis_static.a which is 64-bit metis-5.1.0 .

Note: The libmetis_static.a library is deprecated and will be removed in the next major release. Use the libcusolver_metis_static.a instead.

The parameter options is the configuration of metis. For those who do not have experiences of metis, set options = NULL for default setting.

The output parameter p is an integer array of n elements. It represents a permutation array with base-0 index. The permutation array p corresponds to a permutation matrix P, and satisfies the following relation:

$$A(p,p) = P * A * P^T$$

A is an n×n sparse matrix that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA.

The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Internally csrmetisnd works on $A + A^T$, the user does not need to extend the matrix if the matrix is not symmetric.

Remark 1: only CPU (Host) path is provided.

Input

Parameter	*Host MemSpace	Description
handle	host	Handle to the cuSolverSP library context.
n	host	Number of rows and columns of matrix A.
nnzA	host	Number of nonzeros of matrix A. It is the size of csrValA and csrColIndA.

Parameter	*Host MemSpace	Description
descrA	host	The descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrRowPtrA	host	Integer array of n+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	host	Integer array of nnzAcolumn indices of the nonzero elements of matrix A.
options	host	Integer array to configure metis.

Output

Parameter	*Host MemSpace	Description
p	host	Permutation vector of size n.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n,nnzA<=0), base index is not 0 or 1.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.3.5. cusolverSpXcsrzfd()

```
cusolverStatus_t
cusolverSpScsrzfdHost(
    cusolverSpHandle_t handle,
    int n,
    int nnzA,
    const cusparseMatDescr_t descrA,
    const float *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    int *P,
    int *numnz)

cusolverStatus_t
cusolverSpDcsrzfdHost(
    cusolverSpHandle_t handle,
    int n,
    int nnzA,
    const cusparseMatDescr_t descrA,
```

```
    const double *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    int *P,
    int *numnz)

cusolverStatus_t
cusolverSpCcsrzfdHost(
    cusolverSpHandle_t handle,
    int n,
    int nnzA,
    const cusparseMatDescr_t descrA,
    const cuComplex *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    int *P,
    int *numnz)

cusolverStatus_t
cusolverSpZcsrzfdHost(
    cusolverSpHandle_t handle,
    int n,
    int nnzA,
    const cusparseMatDescr_t descrA,
    const cuDoubleComplex *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    int *P,
    int *numnz)
```

This function implements MC21, zero-free diagonal algorithm. It returns a permutation vector `p` such that `A(p,:)` has no zero diagonal.

`A` is an `n×n` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`.

The output parameter `p` is an integer array of `n` elements. It represents a permutation array with base-0 index. The permutation array `p` corresponds to a permutation matrix `P`, and satisfies the following relation:

$$A(p,:) = P * A$$

The output parameter `numnz` describes number of nonzero diagonal in permutated matrix `A(p,:)`. If `numnz` is less than `n`, matrix `A` has structural singularity.

Remark 1: only CPU (Host) path is provided.

Remark 2: this routine does not maximize diagonal value of permuted matrix. The user cannot expect this routine can make "LU without pivoting" stable.

Input

Parameter	*Host MemSpace	Description
`handle`	`host`	Handle to the cuSolverSP library context.
`n`	`host`	Number of rows and columns of matrix `A`.

Parameter	*Host MemSpace	Description
nnzA	host	Number of nonzeros of matrix A. It is the size of csrValA and csrColIndA.
descrA	host	The descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	host	<type> array of nnzA (= csrRowPtrA(m) – csrRowPtrA(0)) nonzero elements of matrix A.
csrRowPtrA	host	Integer array of n+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	host	Integer array of nnzAcolumn indices of the nonzero elements of matrix A.

Output

Parameter	*Host MemSpace	Description
p	host	Permutation vector of size n.
numnz	host	Number of nonzeros on diagonal of permuted matrix.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (n,nnzA<=0), base index is not 0 or 1.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.3.6. cusolverSpXcsrperm()

```
cusolverStatus_t
cusolverSpXcsrperm_bufferSizeHost(cusolverSpHandle_t handle,
                          int m,
                          int n,
                          int nnzA,
                          const cusparseMatDescr_t descrA,
                          int *csrRowPtrA,
                          int *csrColIndA,
```

```
                            const int *p,
                            const int *q,
                            size_t *bufferSizeInBytes);

cusolverStatus_t
cusolverSpXcsrpermHost(cusolverSpHandle_t handle,
                int m,
                int n,
                int nnzA,
                const cusparseMatDescr_t descrA,
                int *csrRowPtrA,
                int *csrColIndA,
                const int *p,
                const int *q,
                int *map,
                void *pBuffer);
```

Given a left permutation vector `p` which corresponds to permutation matrix `P` and a right permutation vector `q` which corresponds to permutation matrix `Q`, this function computes permutation of matrix `A` by

$$B = P^* A^* Q^T$$

`A` is an `m×n` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA` and `csrColIndA`.

The operation is in-place, i.e. the matrix `A` is overwritten by `B`.

The permutation vector `p` and `q` are base 0. `p` performs row permutation while `q` performs column permutation. One can also use MATLAB command $B = \mathrm{A(p,q)}$ to permutate matrix `A`.

This function only computes sparsity pattern of `B`. The user can use parameter `map` to get `csrValB` as well. The parameter `map` is an input/output. If the user sets `map=0:1:(nnzA-1)` before calling `csrperm`, `csrValB=csrValA(map)`.

The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. If `A` is symmetric and only lower/upper part is provided, the user has to pass $A + A^T$ into this function.

This function requires a buffer size returned by `csrperm_bufferSize()`. The address of `pBuffer` must be a multiple of 128 bytes. If it is not, `CUSOLVER_STATUS_INVALID_VALUE` is returned.

For example, if matrix `A` is

$$A = \begin{pmatrix} 1.0 & 2.0 & 3.0 \\ 4.0 & 5.0 & 6.0 \\ 7.0 & 8.0 & 9.0 \end{pmatrix}$$

and left permutation vector `p=(0,2,1)`, right permutation vector `q=(2,1,0)`, then $P^* A^* Q^T$ is

$$P^* A^* Q^T = \begin{pmatrix} 3.0 & 2.0 & 1.0 \\ 9.0 & 8.0 & 7.0 \\ 6.0 & 5.0 & 4.0 \end{pmatrix}$$

Remark 1: only CPU (Host) path is provided.

Remark 2: the user can combine `csrsymrcm` and `csrperm` to get $P*A*P^T$ which has less zero fill-in during QR factorization.

Input

Parameter	cusolverSp MemSpace	Description
`handle`	`host`	Handle to the cuSolver library context.
`m`	`host`	Number of rows of matrix `A`.
`n`	`host`	Number of columns of matrix `A`.
`nnzA`	`host`	Number of nonzeros of matrix `A`. It is the size of `csrValA` and `csrColIndA`.
`descrA`	`host`	The descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrRowPtrA`	`host`	Integer array of `m+1` elements that contains the start of every row and end of last row plus one of matrix `A`.
`csrColIndA`	`host`	Integer array of `nnzA`column indices of the nonzero elements of matrix `A`.
`p`	`host`	Left permutation vector of size `m`.
`q`	`host`	Right permutation vector of size `n`.
`map`	`host`	Integer array of `nnzA` indices. If the user wants to get relationship between `A` and `B`, `map` must be set `0:1:(nnzA-1)`.
`pBuffer`	`host`	Buffer allocated by the user, the size is returned by `csrperm_bufferSize()`.

Output

Parameter	hsolver	Description
`csrRowPtrA`	`host`	Integer array of `m+1` elements that contains the start of every row and end of last row plus one of matrix `B`.
`csrColIndA`	`host`	Integer array of `nnzA`column indices of the nonzero elements of matrix `B`.
`map`	`host`	Integer array of `nnzA` indices that maps matrix `A` to matrix `B`.
`pBufferSizeInBytes`	`host`	Number of bytes of the buffer.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.
`CUSOLVER_STATUS_ALLOC_FAILED`	The resources could not be allocated.

CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n,nnzA<=0), base index is not 0 or 1.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.5.3.7. cusolverSpXcsrqrBatched()

The create and destroy methods start and end the lifetime of a csrqrInfo object.

```
cusolverStatus_t
cusolverSpCreateCsrqrInfo(csrqrInfo_t *info);

cusolverStatus_t
cusolverSpDestroyCsrqrInfo(csrqrInfo_t info);
```

Analysis is the same for all data types, but each data type has a unique buffer size.

```
cusolverStatus_t
cusolverSpXcsrqrAnalysisBatched(cusolverSpHandle_t handle,
                            int m,
                            int n,
                            int nnzA,
                            const cusparseMatDescr_t descrA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csrqrInfo_t info);

cusolverStatus_t
cusolverSpScsrqrBufferInfoBatched(cusolverSpHandle_t handle,
                            int m,
                            int n,
                            int nnzA,
                            const cusparseMatDescr_t descrA,
                            const float *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            int batchSize,
                            csrqrInfo_t info,
                            size_t *internalDataInBytes,
                            size_t *workspaceInBytes);

cusolverStatus_t
cusolverSpDcsrqrBufferInfoBatched(cusolverSpHandle_t handle,
                            int m,
                            int n,
                            int nnzA,
                            const cusparseMatDescr_t descrA,
                            const double *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            int batchSize,
                            csrqrInfo_t info,
                            size_t *internalDataInBytes,
```

```
                                size_t *workspaceInBytes);
```

Calculate buffer sizes for complex valued data types.

```
cusolverStatus_t
cusolverSpCcsrqrBufferInfoBatched(cusolverSpHandle_t handle,
                                int m,
                                int n,
                                int nnzA,
                                const cusparseMatDescr_t descrA,
                                const cuComplex *csrValA,
                                const int *csrRowPtrA,
                                const int *csrColIndA,
                                int batchSize,
                                csrqrInfo_t info,
                                size_t *internalDataInBytes,
                                size_t *workspaceInBytes);

cusolverStatus_t
cusolverSpZcsrqrBufferInfoBatched(cusolverSpHandle_t handle,
                                int m,
                                int n,
                                int nnzA,
                                const cusparseMatDescr_t descrA,
                                const cuDoubleComplex *csrValA,
                                const int *csrRowPtrA,
                                const int *csrColIndA,
                                int batchSize,
                                csrqrInfo_t info,
                                size_t *internalDataInBytes,
                                size_t *workspaceInBytes);
```

The S and D data types are real valued single and double precision, respectively.

```
cusolverStatus_t
cusolverSpScsrqrsvBatched(cusolverSpHandle_t handle,
                          int m,
                          int n,
                          int nnzA,
                          const cusparseMatDescr_t descrA,
                          const float *csrValA,
                          const int *csrRowPtrA,
                          const int *csrColIndA,
                          const float *b,
                          float *x,
                          int batchSize,
                          csrqrInfo_t info,
                          void *pBuffer);

cusolverStatus_t
cusolverSpDcsrqrsvBatched(cusolverSpHandle_t handle,
                          int m,
                          int n,
                          int nnz,
                          const cusparseMatDescr_t descrA,
                          const double *csrValA,
                          const int *csrRowPtrA,
                          const int *csrColIndA,
                          const double *b,
```

```
                           double *x,
                           int batchSize,
                           csrqrInfo_t info,
                           void *pBuffer);
```

The C and Z data types are complex valued single and double precision, respectively.

```
cusolverStatus_t
cusolverSpCcsrqrsvBatched(cusolverSpHandle_t handle,
                           int m,
                           int n,
                           int nnzA,
                           const cusparseMatDescr_t descrA,
                           const cuComplex *csrValA,
                           const int *csrRowPtrA,
                           const int *csrColIndA,
                           const cuComplex *b,
                           cuComplex *x,
                           int batchSize,
                           csrqrInfo_t info,
                           void *pBuffer);

cusolverStatus_t
cusolverSpZcsrqrsvBatched(cusolverSpHandle_t handle,
                           int m,
                           int n,
                           int nnzA,
                           const cusparseMatDescr_t descrA,
                           const cuDoubleComplex *csrValA,
                           const int *csrRowPtrA,
                           const int *csrColIndA,
                           const cuDoubleComplex *b,
                           cuDoubleComplex *x,
                           int batchSize,
                           csrqrInfo_t info,
                           void *pBuffer);
```

The batched sparse QR factorization is used to solve either a set of least-squares problems

$$x_j = \text{argmin} || A_j * z - b_j ||, \text{ j } = 1,2,..., \text{batchSize}$$

or a set of linear systems

$$A_j x_j = b_j, \text{ j } = 1,2,..., \text{batchSize}$$

where each A_j is a `m×n` sparse matrix that is defined in CSR storage format by the four arrays `csrValA`, `csrRowPtrA` and `csrColIndA`.

The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. If `A` is symmetric and only lower/upper part is prvided, the user has to pass $A + A^H$ into this function.

The prerequisite to use batched sparse QR has two-folds. First all matrices A_j must have the same sparsity pattern. Second, no column pivoting is used in least-square problem, so the solution is valid only if A_j is of full rank for all `j = 1,2,..., batchSize` . All matrices have the same sparity pattern, so only one copy of `csrRowPtrA` and `csrColIndA` is used. But the array `csrValA` stores coefficients of A_j one after another. In other words, `csrValA[k*nnzA : (k+1)*nnzA]` is the value of A_k .

The batched QR uses opaque data structure `csrqrInfo` to keep intermediate data, for example, matrix `Q` and matrix `R` of QR factorization. The user needs to create `csrqrInfo` first by `cusolverSpCreateCsrqrInfo` before any function in batched QR operation. The `csrqrInfo` would not release internal data until `cusolverSpDestroyCsrqrInfo` is called.

There are three routines in batched sparse QR, `cusolverSpXcsrqrAnalysisBatched`, `cusolverSp[S|D|C|Z]csrqrBufferInfoBatched` and `cusolverSp[S|D|C|Z]csrqrsvBatched`.

First, `cusolverSpXcsrqrAnalysisBatched` is the analysis phase, used to analyze sparsity pattern of matrix `Q` and matrix `R` of QR factorization. Also parallelism is extracted during analysis phase. Once analysis phase is done, the size of working space to perform QR is known. However `cusolverSpXcsrqrAnalysisBatched` uses CPU to analyze the structure of matrix `A`, and this may consume a lot of memory. If host memory is not sufficient to finish the analysis, `CUSOLVER_STATUS_ALLOC_FAILED` is returned. The required memory for analysis is proportional to zero fill-in in QR factorization. The user may need to perform some kind of reordering to minimize zero fill-in, for example, `colamd` or `symrcm` in MATLAB. `cuSolverSP` library provides `symrcm` (`cusolverSpXcsrsymrcm`).

Second, the user needs to choose proper `batchSize` and to prepare working space for sparse QR. There are two memory blocks used in batched sparse QR. One is internal memory block used to store matrix `Q` and matrix `R`. The other is working space used to perform numerical factorization. The size of the former is proportional to `batchSize`, and the size is specified by returned parameter `internalDataInBytes` of `cusolverSp[S|D|C|Z]csrqrBufferInfoBatched`. while the size of the latter is almost independent of `batchSize`, and the size is specified by returned parameter `workspaceInBytes` of `cusolverSp[S|D|C|Z]csrqrBufferInfoBatched`. The internal memory block is allocated implicitly during first call of `cusolverSp[S|D|C|Z]csrqrsvBatched`. The user only needs to allocate working space for `cusolverSp[S|D|C|Z]csrqrsvBatched`.

Instead of trying all batched matrices, the user can find maximum `batchSize` by querying `cusolverSp[S|D|C|Z]csrqrBufferInfoBatched`. For example, the user can increase `batchSize` till summation of `internalDataInBytes` and `workspaceInBytes` is greater than size of available device memory.

Suppose that the user needs to perform 253 linear solvers and available device memory is 2GB. if `cusolverSp[S|D|C|Z]csrqrsvBatched` can only afford `batchSize` 100, the user has to call `cusolverSp[S|D|C|Z]csrqrsvBatched` three times to finish all. The user calls `cusolverSp[S|D|C|Z]csrqrBufferInfoBatched` with `batchSize` 100. The opaque `info` would remember this `batchSize` and any subsequent call of `cusolverSp[S|D|C|Z]csrqrsvBatched` cannot exceed this value. In this example, the first two calls of `cusolverSp[S|D|C|Z]csrqrsvBatched` will use `batchSize` 100, and last call of `cusolverSp[S|D|C|Z]csrqrsvBatched` will use `batchSize` 53.

Example: suppose that A0, A1, .., A9 have the same sparsity pattern, the following code solves 10 linear systems $A_j x_j = b_j$, $\mathrm{j} = 0,2,...,9$ by batched sparse QR.

```
// Suppose that A0, A1, .., A9 are m x m sparse matrix represented by CSR format,
// Each matrix Aj has nonzero nnzA, and shares the same csrRowPtrA and csrColIndA.
// csrValA is aggregation of A0, A1, ..., A9.
int m ; // number of rows and columns of each Aj
int nnzA ; // number of nonzeros of each Aj
int *csrRowPtrA ; // each Aj has the same csrRowPtrA
```

```
int *csrColIndA ; // each Aj has the same csrColIndA
double *csrValA ; // aggregation of A0,A1,...,A9
cont int batchSize = 10; // 10 linear systems

cusolverSpHandle_t handle; // handle to cusolver library
csrqrInfo_t info = NULL;
cusparseMatDescr_t descrA = NULL;
void *pBuffer = NULL; // working space for numerical factorization

// step 1: create a descriptor
cusparseCreateMatDescr(&descrA);
cusparseSetMatIndexBase(descrA, CUSPARSE_INDEX_BASE_ONE); // A is base-1
cusparseSetMatType(descrA, CUSPARSE_MATRIX_TYPE_GENERAL); // A is a general matrix

// step 2: create empty info structure
cusolverSpCreateCsrqrInfo(&info);

// step 3: symbolic analysis
cusolverSpXcsrqrAnalysisBatched(
    handle, m, m, nnzA,
    descrA, csrRowPtrA, csrColIndA, info);
```

```
// step 4: allocate working space for Aj*xj=bj
cusolverSpDcsrqrBufferInfoBatched(
    handle, m, m, nnzA,
    descrA,
    csrValA, csrRowPtrA, csrColIndA,
    batchSize,
    info,
    &internalDataInBytes,
    &workspaceInBytes);

cudaMalloc(&pBuffer, workspaceInBytes);

// step 5: solve Aj*xj = bj
cusolverSpDcsrqrsvBatched(
    handle, m, m, nnzA,
    descrA, csrValA, csrRowPtrA, csrColIndA,
    b,
    x,
    batchSize,
    info,
    pBuffer);

// step 7: destroy info
cusolverSpDestroyCsrqrInfo(info);
```

Please refer to cuSOLVER Library Samples - csrqr for a code example.

Remark 1: only GPU (device) path is provided.

Input

Parameter	cusolverSp MemSpace	Description
handle	host	Handle to the cuSolverSP library context.
m	host	Number of rows of each matrix Aj.
n	host	Number of columns of each matrix Aj.
nnzA	host	Number of nonzeros of each matrix Aj. It is the size csrColIndA.

Parameter	cusolverSp MemSpace	Description
descrA	host	The descriptor of each matrix Aj. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	device	<type> array of nnzA*batchSize nonzero elements of matrices A0, A1, ...All matrices are aggregated one after another.
csrRowPtrA	device	Integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	device	Integer array of nnzAcolumn indices of the nonzero elements of each matrix Aj.
b	device	<type> array of m*batchSize of right-hand-side vectors b0, b1, ...All vectors are aggregated one after another.
batchSize	host	Number of systems to be solved.
info	host	Opaque structure for QR factorization.
pBuffer	device	Buffer allocated by the user, the size is returned by cusolverSpXcsrqrBufferInfoBatched().

Output

Parameter	cusolverSp MemSpace	Description
x	device	<type> array of m*batchSize of solution vectors x0, x1, ...All vectors are aggregated one after another.
internalDataInBytes	host	Number of bytes of the internal data.
workspaceInBytes	host	Number of bytes of the buffer in numerical factorization.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (m,n,nnzA<=0), base index is not 0 or 1.
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.
CUSOLVER_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported.

2.6. cuSolverRF: Refactorization Reference

This section describes API of cuSolverRF, a library for fast refactorization.

2.6.1. cusolverRfAccessBundledFactors()

```
cusolverStatus_t
cusolverRfAccessBundledFactors(/* Input */
                               cusolverRfHandle_t handle,
                               /* Output (in the host memory) */
                               int* nnzM,
                               /* Output (in the device memory) */
                               int** Mp,
                               int** Mi,
                               double** Mx);
```

This routine allows direct access to the lower `L` and upper `U` triangular factors stored in the cuSolverRF library handle. The factors are compressed into a single matrix `M=(L-I)+U`, where the unitary diagonal of `L` is not stored. It is assumed that a prior call to the `cusolverRfRefactor()` was done in order to generate these triangular factors.

Parameter	MemSpace	In/out	Meaning
`handle`	`host`	`input`	The handle to the cuSolverRF library.
`nnzM`	`host`	`output`	The number of non-zero elements of matrix `M`.
`Mp`	`device`	`output`	The array of offsets corresponding to the start of each row in the arrays `Mi` and `Mx`. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix M. The array size is `n+1`.
`Mi`	`device`	`output`	The array of column indices corresponding to the non-zero elements in the matrix `M`. It is assumed that this array is sorted by row and by column within each row. The array size is `nnzM`.
`Mx`	`device`	`output`	The array of values corresponding to the non-zero elements in the matrix `M`. It is assumed that this array is sorted by row and by column within each row. The array size is `nnzM`.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.

CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.

2.6.2. cusolverRfAnalyze()

```
cusolverStatus_t
cusolverRfAnalyze(cusolverRfHandle_t handle);
```

This routine performs the appropriate analysis of parallelism available in the LU re-factorization depending upon the algorithm chosen by the user.

$$A = L * U$$

It is assumed that a prior call to the `cusolverRfSetup[Host|Device]()` was done in order to create internal data structures needed for the analysis.

This routine needs to be called only once for a single linear system

$$A_i x_i = f_i$$

Parameter	MemSpace	In/out	Meaning
handle	host	in/out	The handle to the cuSolverRF library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.
CUSOLVER_STATUS_ALLOC_FAILED	An allocation of memory failed.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.6.3. cusolverRfSetupDevice()

```
cusolverStatus_t
cusolverRfSetupDevice(/* Input (in the device memory) */
                      int n,
                      int nnzA,
                      int* csrRowPtrA,
                      int* csrColIndA,
                      double* csrValA,
                      int nnzL,
                      int* csrRowPtrL,
                      int* csrColIndL,
                      double* csrValL,
                      int nnzU,
                      int* csrRowPtrU,
                      int* csrColIndU,
                      double* csrValU,
                      int* P,
                      int* Q,
```

```
                    /* Output */
                    cusolverRfHandle_t handle);
```

This routine assembles the internal data structures of the cuSolverRF library. It is often the first routine to be called after the call to the `cusolverRfCreate()` routine.

This routine accepts as input (on the device) the original matrix `A`, the lower `(L)` and upper `(U)` triangular factors, as well as the left `(P)` and the right `(Q)` permutations resulting from the full LU factorization of the first `(i=1)` linear system

$$A_i x_i = f_i$$

The permutations `P` and `Q` represent the final composition of all the left and right reorderings applied to the original matrix `A`, respectively. However, these permutations are often associated with partial pivoting and reordering to minimize fill-in, respectively.

This routine needs to be called only once for a single linear system

$$A_i x_i = f_i$$

Parameter	MemSpace	In/out	Meaning
`n`	`host`	`input`	The number of rows (and columns) of matrix `A`.
`nnzA`	`host`	`input`	The number of non-zero elements of matrix `A`.
`csrRowPtrA`	`device`	`input`	The array of offsets corresponding to the start of each row in the arrays `csrColIndA` and `csrValA`. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix. The array size is `n+1`.
`csrColIndA`	`device`	`input`	The array of column indices corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is `nnzA`.
`csrValA`	`device`	`input`	The array of values corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is `nnzA`.
`nnzL`	`host`	`input`	The number of non-zero elements of matrix `L`.
`csrRowPtrL`	`device`	`input`	The array of offsets corresponding to the start of each row in the arrays `csrColIndL` and `csrValL`. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix `L`. The array size is `n+1`.

Parameter	MemSpace	In/out	Meaning
csrColIndL	device	input	The array of column indices corresponding to the non-zero elements in the matrix L. It is assumed that this array is sorted by row and by column within each row. The array size is nnzL.
csrValL	device	input	The array of values corresponding to the non-zero elements in the matrix L. It is assumed that this array is sorted by row and by column within each row. The array size is nnzL.
nnzU	host	input	the number of non-zero elements of matrix U.
csrRowPtrU	device	input	The array of offsets corresponding to the start of each row in the arrays csrColIndU and csrValU. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix U. The array size is n+1.
csrColIndU	device	input	The array of column indices corresponding to the non-zero elements in the matrix U. It is assumed that this array is sorted by row and by column within each row. The array size is nnzU.
csrValU	device	input	The array of values corresponding to the non-zero elements in the matrix U. It is assumed that this array is sorted by row and by column within each row. The array size is nnzU.
P	device	input	The left permutation (often associated with pivoting). The array size in n.
Q	device	input	The right permutation (often associated with reordering). The array size in n.
handle	host	output	The handle to the GLU library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	An unsupported value or parameter was passed.
CUSOLVER_STATUS_ALLOC_FAILED	An allocation of memory failed.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.6.4. cusolverRfSetupHost()

```
cusolverStatus_t
```

```
cusolverRfSetupHost(/* Input (in the host memory) */
                    int n,
                    int nnzA,
                    int* h_csrRowPtrA,
                    int* h_csrColIndA,
                    double* h_csrValA,
                    int nnzL,
                    int* h_csrRowPtrL,
                    int* h_csrColIndL,
                    double* h_csrValL,
                    int nnzU,
                    int* h_csrRowPtrU,
                    int* h_csrColIndU,
                    double* h_csrValU,
                    int* h_P,
                    int* h_Q,
                    /* Output */
                    cusolverRfHandle_t handle);
```

This routine assembles the internal data structures of the cuSolverRF library. It is often the first routine to be called after the call to the `cusolverRfCreate()` routine.

This routine accepts as input (on the host) the original matrix `A`, the lower `(L)` and upper `(U)` triangular factors, as well as the left `(P)` and the right `(Q)` permutations resulting from the full LU factorization of the first `(i=1)` linear system

$$A_i x_i = f_i$$

The permutations `P` and `Q` represent the final composition of all the left and right reorderings applied to the original matrix `A`, respectively. However, these permutations are often associated with partial pivoting and reordering to minimize fill-in, respectively.

This routine needs to be called only once for a single linear system

$$A_i x_i = f_i$$

Parameter	MemSpace	In/out	Meaning
`n`	`host`	`input`	The number of rows (and columns) of matrix `A`.
`nnzA`	`host`	`input`	The number of non-zero elements of matrix `A`.
`h_csrRowPtrA`	`host`	`input`	The array of offsets corresponding to the start of each row in the arrays `h_csrColIndA` and `h_csrValA`. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix. The array size is `n+1`.
`h_csrColIndA`	`host`	`input`	The array of column indices corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is `nnzA`.

Parameter	MemSpace	In/out	Meaning
h_csrValA	host	input	The array of values corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is nnzA.
nnzL	host	input	The number of non-zero elements of matrix L.
h_csrRowPtrL	host	input	The array of offsets corresponding to the start of each row in the arrays h_csrColIndL and h_csrValL. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix L. The array size is n+1.
h_csrColIndL	host	input	The array of column indices corresponding to the non-zero elements in the matrix L. It is assumed that this array is sorted by row and by column within each row. The array size is nnzL.
h_csrValL	host	input	The array of values corresponding to the non-zero elements in the matrix L. It is assumed that this array is sorted by row and by column within each row. The array size is nnzL.
nnzU	host	input	The number of non-zero elements of matrix U.
h_csrRowPtrU	host	input	The array of offsets corresponding to the start of each row in the arrays h_csrColIndU and h_csrValU. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix U. The array size is n+1.
h_csrColIndU	host	input	The array of column indices corresponding to the non-zero elements in the matrix U. It is assumed that this array is sorted by row and by column within each row. The array size is nnzU.
h_csrValU	host	input	The array of values corresponding to the non-zero elements in the matrix U. It is assumed that this array is sorted by row and by column within each row. The array size is nnzU.
h_P	host	input	The left permutation (often associated with pivoting). The array size in n.
h_Q	host	input	The right permutation (often associated with reordering). The array size in n.
handle	host	output	The handle to the cuSolverRF library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	An unsupported value or parameter was passed.
CUSOLVER_STATUS_ALLOC_FAILED	An allocation of memory failed.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.6.5. cusolverRfCreate()

```
cusolverStatus_t cusolverRfCreate(cusolverRfHandle_t *handle);
```

This routine initializes the cuSolverRF library. It allocates required resources and must be called prior to any other cuSolverRF library routine.

Parameter	MemSpace	In/out	Meaning
handle	host	output	The pointer to the cuSolverRF library handle.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	An allocation of memory failed.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.6.6. cusolverRfExtractBundledFactorsHost()

```
cusolverStatus_t
cusolverRfExtractBundledFactorsHost(/* Input */
                                    cusolverRfHandle_t handle,
                                    /* Output (in the host memory) */
                                    int* h_nnzM,
                                    int** h_Mp,
                                    int** h_Mi,
                                    double** h_Mx);
```

This routine extracts lower (L) and upper (U) triangular factors from the cuSolverRF library handle into the host memory. The factors are compressed into a single matrix M=(L-I)+U, where the unitary diagonal of (L) is not stored. It is assumed that a prior call to the cusolverRfRefactor() was done in order to generate these triangular factors.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The handle to the cuSolverRF library.

Parameter	MemSpace	In/out	Meaning
h_nnzM	host	output	The number of non-zero elements of matrix M.
h_Mp	host	output	The array of offsets corresponding to the start of each row in the arrays h_Mi and h_Mx. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix M. The array size is n+1.
h_Mi	host	output	The array of column indices corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is h_nnzM.
h_Mx	host	output	The array of values corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is h_nnzM.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	An allocation of memory failed.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.

2.6.7. cusolverRfExtractSplitFactorsHost()

```
cusolverStatus_t
cusolverRfExtractSplitFactorsHost(/* Input */
                                  cusolverRfHandle_t handle,
                                  /* Output (in the host memory) */
                                  int* h_nnzL,
                                  int** h_Lp,
                                  int** h_Li,
                                  double** h_Lx,
                                  int* h_nnzU,
                                  int** h_Up,
                                  int** h_Ui,
                                  double** h_Ux);
```

This routine extracts lower (L) and upper (U) triangular factors from the cuSolverRF library handle into the host memory. It is assumed that a prior call to the cusolverRfRefactor() was done in order to generate these triangular factors.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The handle to the cuSolverRF library.

Parameter	MemSpace	In/out	Meaning
h_nnzL	host	output	The number of non-zero elements of matrix L.
h_Lp	host	output	The array of offsets corresponding to the start of each row in the arrays h_Li and h_Lx. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix L. The array size is n+1.
h_Li	host	output	The array of column indices corresponding to the non-zero elements in the matrix L. It is assumed that this array is sorted by row and by column within each row. The array size is h_nnzL.
h_Lx	host	output	The array of values corresponding to the non-zero elements in the matrix L. It is assumed that this array is sorted by row and by column within each row. The array size is h_nnzL.
h_nnzU	host	output	The number of non-zero elements of matrix U.
h_Up	host	output	The array of offsets corresponding to the start of each row in the arrays h_Ui and h_Ux. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix U. The array size is n+1.
h_Ui	host	output	The array of column indices corresponding to the non-zero elements in the matrix U. It is assumed that this array is sorted by row and by column within each row. The array size is h_nnzU.
h_Ux	host	output	The array of values corresponding to the non-zero elements in the matrix U. It is assumed that this array is sorted by row and by column within each row. The array size is h_nnzU.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ALLOC_FAILED	An allocation of memory failed.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.

2.6.8. cusolverRfDestroy()

```
cusolverStatus_t cusolverRfDestroy(cusolverRfHandle_t handle);
```

This routine shuts down the cuSolverRF library. It releases acquired resources and must be called after all the cuSolverRF library routines.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The cuSolverRF library handle.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

2.6.9. cusolverRfGetMatrixFormat()

```
cusolverStatus_t
cusolverRfGetMatrixFormat(cusolverRfHandle_t handle,
                          cusolverRfMatrixFormat_t *format,
                          cusolverRfUnitDiagonal_t *diag);
```

This routine gets the matrix format used in the `cusolverRfSetupDevice()`, `cusolverRfSetupHost()`, `cusolverRfResetValues()`, `cusolverRfExtractBundledFactorsHost()` and `cusolverRfExtractSplitFactorsHost()` routines.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The handle to the cuSolverRF library.
format	host	output	The enumerated matrix format type.
diag	host	output	The enumerated unit diagonal type.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

2.6.10. cusolverRfGetNumericProperties()

```
cusolverStatus_t
cusolverRfGetNumericProperties(cusolverRfHandle_t handle,
                               double *zero,
                               double *boost);
```

This routine gets the numeric values used for checking for "zero" pivot and for boosting it in the `cusolverRfRefactor()` and `cusolverRfSolve()` routines. The numeric boosting will be used only if `boost > 0.0`.

Parameter	MemSpace	In/out	Meaning
`handle`	`host`	`input`	The handle to the cuSolverRF library.
`zero`	`host`	`output`	The value below which zero pivot is flagged.
`boost`	`host`	`output`	The value which is substituted for zero pivot (if the later is flagged).

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.

2.6.11. cusolverRfGetNumericBoostReport()

```
cusolverStatus_t
cusolverRfGetNumericBoostReport(cusolverRfHandle_t handle,
                                cusolverRfNumericBoostReport_t *report);
```

This routine gets the report whether numeric boosting was used in the `cusolverRfRefactor()` and `cusolverRfSolve()` routines.

Parameter	MemSpace	In/out	Meaning
`handle`	`host`	`input`	The handle to the cuSolverRF library.
`report`	`host`	`output`	The enumerated boosting report type.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.

2.6.12. cusolverRfGetResetValuesFastMode()

```
cusolverStatus_t
cusolverRfGetResetValuesFastMode(cusolverRfHandle_t handle,
                                 rfResetValuesFastMode_t *fastMode);
```

This routine gets the mode used in the `cusolverRfResetValues` routine.

Parameter	MemSpace	In/out	Meaning
`handle`	`host`	`input`	The handle to the cuSolverRF library.
`fastMode`	`host`	`output`	The enumerated mode type.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

2.6.13. cusolverRfGet_Algs()

```
cusolverStatus_t
cusolverRfGet_Algs(cusolverRfHandle_t handle,
              cusolverRfFactorization_t* fact_alg,
              cusolverRfTriangularSolve_t* solve_alg);
```

This routine gets the algorithm used for the refactorization in `cusolverRfRefactor()` and the triangular solve in `cusolverRfSolve()`.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The handle to the cuSolverRF library.
alg	host	output	The enumerated algorithm type.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

2.6.14. cusolverRfRefactor()

```
cusolverStatus_t cusolverRfRefactor(cusolverRfHandle_t handle);
```

This routine performs the LU re-factorization:

$$A = L^* U$$

exploring the available parallelism on the GPU. It is assumed that a prior call to the `glu_analyze()` was done in order to find the available paralellism.

This routine may be called multiple times, once for each of the linear systems:

$$A_i x_i = f_i$$

There are some constraints to the combination of algorithms used for refactorization and solving routines, `cusolverRfRefactor()` and `cusolverRfSolve()`. The wrong combination generates the error code `CUSOLVER_STATUS_INVALID_VALUE`. The table bellow summarizes the supported combinations of algorithms:

Compatible algorithms for solving and refactorization routines.

Factorization	Solving
CUSOLVERRF_FACTORIZATION_ALG0	TRIANGULAR_SOLVE_ALG1

Factorization	Solving
CUSOLVERRF_FACTORIZATION_ALG1	TRIANGULAR_SOLVE_ALG2, TRIANGULAR_SOLVE_ALG3
CUSOLVERRF_FACTORIZATION_ALG2	TRIANGULAR_SOLVE_ALG2, TRIANGULAR_SOLVE_ALG3

Parameter	MemSpace	In/out	Meaning
handle	host	in/out	The handle to the cuSolverRF library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.
CUSOLVER_STATUS_ZERO_PIVOT	A zero pivot was encountered during the computation.

2.6.15. cusolverRfResetValues()

```
cusolverStatus_t
cusolverRfResetValues(/* Input (in the device memory) */
                  int n,
                  int nnzA,
                  int* csrRowPtrA,
                  int* csrColIndA,
                  double* csrValA,
                  int* P,
                  int* Q,
                  /* Output */
                  cusolverRfHandle_t handle);
```

This routine updates internal data structures with the values of the new coefficient matrix. It is assumed that the arrays `csrRowPtrA`, `csrColIndA`, `P` and `Q` have not changed since the last call to the `cusolverRfSetup[Host|Device]` routine. This assumption reflects the fact that the sparsity pattern of coefficient matrices as well as reordering to minimize fill-in and pivoting remain the same in the set of linear systems:

$$A_i x_i = f_i$$

This routine may be called multiple times, once for each of the linear systems:

$$A_i x_i = f_i$$

Parameter	MemSpace	In/out	Meaning
n	host	input	The number of rows (and columns) of matrix A.
nnzA	host	input	The number of non-zero elements of matrix A.

Parameter	MemSpace	In/out	Meaning
csrRowPtrA	device	input	The array of offsets corresponding to the start of each row in the arrays csrColIndA and csrValA. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix. The array size is n+1.
csrColIndA	device	input	The array of column indices corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is nnzA.
csrValA	device	input	The array of values corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is nnzA.
P	device	input	The left permutation (often associated with pivoting). The array size in n.
Q	device	input	The right permutation (often associated with reordering). The array size in n.
handle	host	output	The handle to the cuSolverRF library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	An unsupported value or parameter was passed.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.

2.6.16. cusolverRfSetMatrixFormat()

```
cusolverStatus_t
cusolverRfSetMatrixFormat(cusolverRfHandle_t handle,
                          gluMatrixFormat_t format,
                          gluUnitDiagonal_t diag);
```

This routine sets the matrix format used in the cusolverRfSetupDevice(), cusolverRfSetupHost(), cusolverRfResetValues(), cusolverRfExtractBundledFactorsHost() and cusolverRfExtractSplitFactorsHost() routines. It may be called once prior to cusolverRfSetupDevice() and cusolverRfSetupHost() routines.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The handle to the cuSolverRF library.
format	host	input	The enumerated matrix format type.

Parameter	MemSpace	In/out	Meaning
diag	host	input	The enumerated unit diagonal type.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	An enumerated mode parameter is wrong.

2.6.17. cusolverRfSetNumericProperties()

```
cusolverStatus_t
cusolverRfSetNumericProperties(cusolverRfHandle_t handle,
                               double zero,
                               double boost);
```

This routine sets the numeric values used for checking for "zero" pivot and for boosting it in the `cusolverRfRefactor()` and `cusolverRfSolve()` routines. It may be called multiple times prior to `cusolverRfRefactor()` and `cusolverRfSolve()` routines. The numeric boosting will be used only if `boost > 0.0`.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The handle to the cuSolverRF library.
zero	host	input	The value below which zero pivot is flagged.
boost	host	input	The value which is substituted for zero pivot (if the later is flagged).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

2.6.18. cusolverRfSetResetValuesFastMode()

```
cusolverStatus_t
cusolverRfSetResetValuesFastMode(cusolverRfHandle_t handle,
                                 gluResetValuesFastMode_t fastMode);
```

This routine sets the mode used in the `cusolverRfResetValues` routine. The fast mode requires extra memory and is recommended only if very fast calls to `cusolverRfResetValues()` are needed. It may be called once prior to `cusolverRfAnalyze()` routine.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The handle to the cuSolverRF library.

Parameter	MemSpace	In/out	Meaning
fastMode	host	input	The enumerated mode type.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	An enumerated mode parameter is wrong.

2.6.19. cusolverRfSetAlgs()

```
cusolverStatus_t
cusolverRfSetAlgs(cusolverRfHandle_t handle,
             gluFactorization_t fact_alg,
             gluTriangularSolve_t alg);
```

This routine sets the algorithm used for the refactorization in `cusolverRfRefactor()` and the triangular solve in `cusolverRfSolve()`. It may be called once prior to `cusolverRfAnalyze()` routine.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The handle to the cuSolverRF library.
alg	host	input	The enumerated algorithm type.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

2.6.20. cusolverRfSolve()

```
cusolverStatus_t
cusolverRfSolve(/* Input (in the device memory) */
          cusolverRfHandle_t handle,
          int *P,
          int *Q,
          int nrhs,
          double *Temp,
          int ldt,
          /* Input/Output (in the device memory) */
          double *XF,
          /* Input */
          int ldxf);
```

This routine performs the forward and backward solve with the lower $L \in R^{nxn}$ and upper $U \in R^{nxn}$ triangular factors resulting from the LU re-factorization:

$$A = L * U$$

which is assumed to have been computed by a prior call to the `cusolverRfRefactor()` routine.

The routine can solve linear systems with multiple right-hand-sides (RHS):

$$AX = (\mathrm{LU})X = L(\mathrm{UX}) = LY = F \text{ where } UX = Y$$

even though currently only a single RHS is supported.

This routine may be called multiple times, once for each of the linear systems:

$$A_i x_i = f_i$$

Parameter	MemSpace	In/out	Meaning
`handle`	`host`	`output`	The handle to the cuSolverRF library.
`P`	`device`	`input`	The left permutation (often associated with pivoting). The array size in `n`.
`Q`	`device`	`input`	The right permutation (often associated with reordering). The array size in `n`.
`nrhs`	`host`	`input`	The number right-hand-sides to be solved.
`Temp`	`host`	`input`	The dense matrix that contains temporary workspace (of size `ldt*nrhs`).
`ldt`	`host`	`input`	The leading dimension of dense matrix Temp (`ldt >= n`).
`XF`	`host`	`in/out`	The dense matrix that contains the right-hand-sides `F` and solutions `X` (of size `ldxf*nrhs`).
`ldxf`	`host`	`input`	The leading dimension of dense matrix `XF` (`ldxf >= n`).

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.
`CUSOLVER_STATUS_INVALID_VALUE`	An unsupported value or parameter was passed.
`CUSOLVER_STATUS_EXECUTION_FAILED`	A kernel failed to launch on the GPU.
`CUSOLVER_STATUS_INTERNAL_ERROR`	An internal operation failed.

2.6.21. cusolverRfBatchSetupHost()

```
cusolverStatus_t
cusolverRfBatchSetupHost(/* Input (in the host memory) */
                         int batchSize,
                         int n,
                         int nnzA,
                         int* h_csrRowPtrA,
                         int* h_csrColIndA,
                         double *h_csrValA_array[],
```

```
                        int nnzL,
                        int* h_csrRowPtrL,
                        int* h_csrColIndL,
                        double *h_csrValL,
                        int nnzU,
                        int* h_csrRowPtrU,
                        int* h_csrColIndU,
                        double *h_csrValU,
                        int* h_P,
                        int* h_Q,
                        /* Output */
                        cusolverRfHandle_t handle);
```

This routine assembles the internal data structures of the cuSolverRF library for batched operation. It is called after the call to the `cusolverRfCreate()` routine, and before any other batched routines.

The batched operation assumes that the user has the following linear systems:

$$A_j x_j = b_j,\ \mathrm{j} = 1,2,...,\ \mathrm{batchSize}$$

where each matrix in the set: $\{A_j\}$ has the same sparsity pattern, and quite similar such that factorization can be done by the same permutation `P` and `Q`. In other words, $A_j,\ \mathrm{j} > 1$ is a small perturbation of A_1.

This routine accepts as input (on the host) the original matrix `A` (sparsity pattern and batched values), the lower `(L)` and upper `(U)` triangular factors, as well as the left `(P)` and the right `(Q)` permutations resulting from the full LU factorization of the first `(i=1)` linear system:

$$A_i x_i = f_i$$

The permutations `P` and `Q` represent the final composition of all the left and right reorderings applied to the original matrix `A`, respectively. However, these permutations are often associated with partial pivoting and reordering to minimize fill-in, respectively.

Remark 1: the matrices `A`, `L` and `U` must be CSR format and base-0.

Remark 2: to get best performance, `batchSize` should be multiple of 32 and greater or equal to 32. The algorithm is memory-bound, once bandwidth limit is reached, there is no room to improve performance by large `batchSize`. In practice, `batchSize` of 32 - 128 is often enough to obtain good performance, but in some cases larger `batchSize` might be beneficial.

The following routine needs to be called only once for a single linear system:

$$A_i x_i = f_i$$

Parameter	MemSpace	In/out	Meaning
`batchSize`	`host`	`input`	The number of matrices in the batched mode.
`n`	`host`	`input`	The number of rows (and columns) of matrix `A`.
`nnzA`	`host`	`input`	The number of non-zero elements of matrix `A`.

Parameter	MemSpace	In/out	Meaning
h_csrRowPtrA	host	input	The array of offsets corresponding to the start of each row in the arrays h_csrColIndA and h_csrValA. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix. The array size is n+1.
h_csrColIndA	host	input	The array of column indices corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is nnzA.
h_csrValA_array	host	input	Array of pointers of size batchSize, each pointer points to the array of values corresponding to the non-zero elements in the matrix.
nnzL	host	input	The number of non-zero elements of matrix L.
h_csrRowPtrL	host	input	The array of offsets corresponding to the start of each row in the arrays h_csrColIndL and h_csrValL. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix L. The array size is n+1.
h_csrColIndL	host	input	The array of column indices corresponding to the non-zero elements in the matrix L. It is assumed that this array is sorted by row and by column within each row. The array size is nnzL.
h_csrValL	host	input	The array of values corresponding to the non-zero elements in the matrix L. It is assumed that this array is sorted by row and by column within each row. The array size is nnzL.
nnzU	host	input	The number of non-zero elements of matrix U.
h_csrRowPtrU	host	input	The array of offsets corresponding to the start of each row in the arrays h_csrColIndU and h_csrValU. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix U. The array size is n+1.
h_csrColIndU	host	input	The array of column indices corresponding to the non-zero elements in the matrix U. It is assumed that this

Parameter	MemSpace	In/out	Meaning
			array is sorted by row and by column within each row. The array size is nnzU.
h_csrValU	host	input	The array of values corresponding to the non-zero elements in the matrix U. It is assumed that this array is sorted by row and by column within each row. The array size is nnzU.
h_P	host	input	The left permutation (often associated with pivoting). The array size in n.
h_Q	host	input	The right permutation (often associated with reordering). The array size in n.
handle	host	output	The handle to the cuSolverRF library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	An unsupported value or parameter was passed.
CUSOLVER_STATUS_ALLOC_FAILED	An allocation of memory failed.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.6.22. cusolverRfBatchAnalyze()

```
cusolverStatus_t cusolverRfBatchAnalyze(cusolverRfHandle_t handle);
```

This routine performs the appropriate analysis of parallelism available in the batched LU re-factorization.

It is assumed that a prior call to the cusolverRfBatchSetup[Host]() was done in order to create internal data structures needed for the analysis.

The following routine needs to be called only once for a single linear system:

$$A_j x_j = b_j,\ \mathrm{j} = 1,2,\ldots,\ \mathrm{batchSize}$$

Parameter	Memory	In/out	Meaning
handle	host	in/out	The handle to the cuSolverRF library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.
CUSOLVER_STATUS_ALLOC_FAILED	An allocation of memory failed.

CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.6.23. cusolverRfBatchResetValues()

```
cusolverStatus_t
cusolverRfBatchResetValues(/* Input (in the device memory) */
                           int batchSize,
                           int n,
                           int nnzA,
                           int* csrRowPtrA,
                           int* csrColIndA,
                           double* csrValA_array[],
                           int *P,
                           int *Q,
                           /* Output */
                           cusolverRfHandle_t handle);
```

This routine updates internal data structures with the values of the new coefficient matrix. It is assumed that the arrays `csrRowPtrA`, `csrColIndA`, `P` and `Q` have not changed since the last call to the `cusolverRfbatch_setup_host` routine.

This assumption reflects the fact that the sparsity pattern of coefficient matrices as well as reordering to minimize fill-in and pivoting remain the same in the set of linear systems:

$$A_j x_j = b_j,\ \mathrm{j} = 1,2,\ldots,\ \mathrm{batchSize}$$

The input parameter `csrValA_array` is an array of pointers on device memory. `csrValA_array(j)` points to matrix: A_j which is also on device memory.

Parameter	MemSpace	In/out	Meaning
batchSize	host	input	The number of matrices in batched mode.
n	host	input	The number of rows (and columns) of matrix A.
nnzA	host	input	The number of non-zero elements of matrix A.
csrRowPtrA	device	input	The array of offsets corresponding to the start of each row in the arrays csrColIndA and csrValA. This array has also an extra entry at the end that stores the number of non-zero elements in the matrix. The array size is n+1.
csrColIndA	device	input	The array of column indices corresponding to the non-zero elements in the matrix. It is assumed that this array is sorted by row and by column within each row. The array size is nnzA.
csrValA_array	device	input	Array of pointers of size batchSize, each pointer points to the array of values

Parameter	MemSpace	In/out	Meaning
			corresponding to the non-zero elements in the matrix.
P	device	input	The left permutation (often associated with pivoting). The array size in n.
Q	device	input	The right permutation (often associated with reordering). The array size in n.
handle	host	output	The handle to the cuSolverRF library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	An unsupported value or parameter was passed.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.

2.6.24. cusolverRfBatchRefactor()

```
cusolverStatus_t cusolverRfBatchRefactor(cusolverRfHandle_t handle);
```

This routine performs the LU re-factorization:

$$M_j = P * A_j * Q^T = L_j * U_j$$

exploring the available parallelism on the GPU. It is assumed that a prior call to the cusolverRfBatchAnalyze() was done in order to find the available paralellism.

Remark: cusolverRfBatchRefactor() would not report any failure of LU refactorization. The user has to call cusolverRfBatchZeroPivot() to know which matrix failed the LU refactorization.

Parameter	Memory	In/out	Meaning
handle	host	in/out	The handle to the cuSolverRF library.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.

2.6.25. cusolverRfBatchSolve()

```
cusolverStatus_t
cusolverRfBatchSolve(/* Input (in the device memory) */
                cusolverRfHandle_t handle,
                int *P,
                int *Q,
```

```
                int nrhs,
                double *Temp,
                int ldt,
                /* Input/Output (in the device memory) */
                double *XF_array[],
                /* Input */
                int ldxf);
```

To solve $A_j * x_j = b_j$, first we reform the equation by $M_j * Q * x_j = P * b_j$ where $M_j = P * A_j * Q^T$. Then do refactorization $M_j = L_j * U_j$ by `cusolverRfBatch_Refactor()`. Further `cusolverRfBatch_Solve()` takes over the remaining steps, including:

$z_j = P * b_j$

$M_j * y_j = z_j$

$x_j = Q^T * y_j$

The input parameter `XF_array` is an array of pointers on device memory. `XF_array(j)` points to matrix x_j which is also on device memory.

Remark 1: only a single rhs is supported.

Remark 2: no singularity is reported during backward solve. If some matrix A_j failed the refactorization and U_j has some zero diagonal, backward solve would compute NAN. The user has to call `cusolverRfBatch_Zero_Pivot` to check if refactorization is successful or not.

Parameter	MemSpace	In/out	Meaning
`handle`	`host`	`output`	The handle to the cuSolverRF library.
`P`	`device`	`input`	The left permutation (often associated with pivoting). The array size in `n`.
`Q`	`device`	`input`	The right permutation (often associated with reordering). The array size in `n`.
`nrhs`	`host`	`input`	The number right-hand-sides to be solved.
`Temp`	`device`	`input`	The dense matrix that contains temporary workspace (of size `ldt*nrhs`).
`ldt`	`host`	`input`	The leading dimension of dense matrix Temp (`ldt >= n`).
`XF_array`	`device`	`in/out`	Array of pointers of size `batchSize`, each pointer points to the dense matrix that contains the right-hand-sides `F` and solutions `X` (of size `ldxf*nrhs`).
`ldxf`	`host`	`input`	The leading dimension of dense matrix `XF` (`ldxf >= n`).

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_NOT_INITIALIZED`	The library was not initialized.

CUSOLVER_STATUS_INVALID_VALUE	An unsupported value or parameter was passed.
CUSOLVER_STATUS_EXECUTION_FAILED	A kernel failed to launch on the GPU.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

2.6.26. cusolverRfBatchZeroPivot()

```
cusolverStatus_t
cusolverRfBatchZeroPivot(/* Input */
                     cusolverRfHandle_t handle
                     /* Output (in the host memory) */
                     int *position);
```

Although A_j is close to each other, it does not mean $M_j = P * A_j * Q^T = L_j * U_j$ exists for every j. The user can query which matrix failed LU refactorization by checking corresponding value in `position` array. The input parameter `position` is an integer array of size `batchSize`.

The `j-th` component denotes the refactorization result of matrix A_j. If `position(j)` is -1, the LU refactorization of matrix A_j is successful. If `position(j)` is `k >= 0`, matrix A_j is not LU factorizable and its matrix $U_j(j,j)$ is zero.

The return value of `cusolverRfBatch_Zero_Pivot` is `CUSOLVER_STATUS_ZERO_PIVOT` if there exists one A_j which failed LU refactorization. The user can redo LU factorization to get new permutation `P` and `Q` if error code `CUSOLVER_STATUS_ZERO_PIVOT` is returned.

Parameter	MemSpace	In/out	Meaning
handle	host	input	The handle to the cuSolverRF library.
position	host	output	Integer array of size `batchSize`. The value of `position(j)` reports singularity of matrix `Aj`, -1 if no structural/ numerical zero, `k >= 0` if `Aj(k,k)` is either structural zero or numerical zero.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_ZERO_PIVOT	A zero pivot was encountered during the computation.

Chapter 3. Using the CUSOLVERMG API

3.1. General Description

This section describes how to use the cuSolverMG library API. It is not a reference for the cuSolverMG API data types and functions; that is provided in subsequent chapters.

3.1.1. Thread Safety

The library is thread-safe only if there is one cuSolverMG context per thread.

3.1.2. Determinism

Currently all cuSolverMG API routines from a given toolkit version generate the same bit-wise results when the following conditions are respected :

- all GPUs particating to the computation have the same compute-capabilities and the same number of SMs.
- the tiles size is kept the same between run.
- number of logical GPUs is kept the same. The order of GPUs are not important because all have the same compute-capabilities.

3.1.3. Tile Strategy

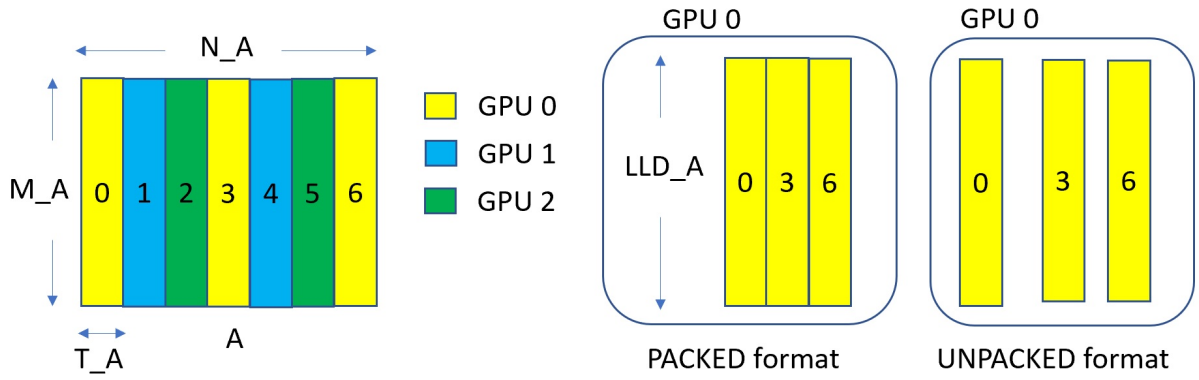
The tiling strategy of cuSolverMG is compatible with ScaLAPACK. The current release only supports 1-D column block cyclic, column-major PACKED format.

Figure 1.a shows a partition of the matrix A of dimension `M_A` by `N_A`. Each column tile has `T_A` columns. There are seven columns of tiles, labeled as 0,1,2,3,4,5,6, distributed into three GPUs in a `cyclic` way, i.e. each GPU takes one column tile in turn. For example, GPU 0 has column tile 0, 3, 6 (yellow tiles) and GPU 1 takes column tiles next to GPU 0 (blue tiles). Not all GPUs have the same number of tiles; in this example, GPU 0 has three tiles, others have only two tiles.

Figure 1.b shows two possilbe formats to store those column tiles locally in each GPU. Left side is called PACKED format and right side is UNPACKED format. PACKED format aggregates three column tiles in a contiguous memory block while UNPACKED format distributes these three column tiles into different memory blocks. The only difference between them is that PACKED format can have a big GEMM call instead of three GEMM calls in UNPACKED format. So theoretically speaking, PACKED format can deliver better performance than UNPACKED format. `cusolveMG` only supports PACKED format in the API. In order to achieve maximal performance, the user just needs to choose the proper tile size `T_A` to partition the matrix, not too small, for example 256 or above is enough.

There is another parameter, called `LLD_A`, to control the leading dimension of the local matrix in each GPU. `LLD_A` must be greater or equal to `M_A`. The purpose of `LLD_A` is for better performance of GEMM. For small problems, GEMM is faster if `LLD_A` is power of 2. However for big problems, `LLD_A` does not show significant improvement. `cuSolverMG` only supports `LLD_A=M_A`.

Figure 1. Example of `cusolveMG` tiling for 3 GPUs



(a) 1D column block cyclic (b) packed or unpacked format

The processing grid in `cuSolverMG` is a list of GPU IDs, similar to the process ID in `ScaLAPACK`. `cuSolverMG` only supports 1D column block cyclic, so only 1D grid is supported as well. Suppose `deviceId` is a list of GPU IDs, both `deviceId=1,1,1` and `deviceId=2,1,0` are valid. The former describes three logical devices that are selected to run `cuSolverMG` routines, and all have the same physical ID, 0. The latter still uses three logical devices, but each has a different physical ID. The current design only accepts 32 logical devices, that is, the length of `deviceId` is less or equal to 32. Figure 1 uses `deviceId=0,1,2`.

In practice, the matrix A is distributed into GPUs listed in `deviceId`. If the user chooses `deviceId=1,1,1`, all columns tile are located in GPU 1, this will limit the size of the problem because of memory capacity of one GPU. Besides, multiGPU routine adds extra overhead on data communication through the off-chip bus, which has a big performance impact if NVLINK is not supported or used. It would be faster to run on a single GPU instead of runing multiGPU version with devices of the same GPU ID.

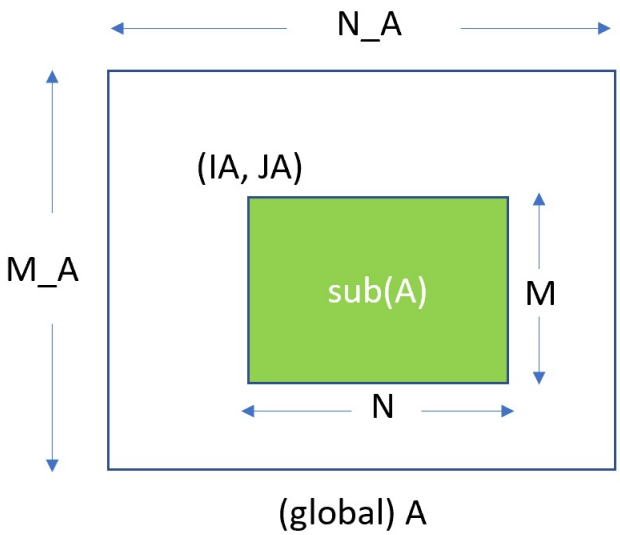
3.1.4. Global Matrix Versus Local Matrix

Operating a submatrix of the matrix A is simple in dense linear algebra, just shift the pointer to the starting point of the submatrix relative to A. For example, `gesvd(10,10, A)` is SVD of `A(0:9,0:9)`. `gesvd(10,10, A + 5 + 2*lda )` is SVD of 10-by-10 submatrix starting at `A(5,2)`.

However it is not simple to operate on a submatrix of a distributed matrix because different starting point of the submatrix changes the distribution of the layout of that submatrix. `ScaLAPACK` introduces two parameters, `IA` and `JA`, to locate the submatrix. Figure 2 shows (global) matrix A of dimension `M_A` by `N_A`. The `sub(A)` is a `M` by `N` submatrix of A, starting at `IA` and `JA`. Please be aware that `IA` and `JA` are base-1.

Given a distributed matrix `A`, the user can compute eigenvalues of the submatrix `sub(A)` by either calling `syevd(A, IA, JA)` or gathering `sub(A)` to another distributed matrix `B` and calling `syevd(B, IB=1, JB=1)`.

Figure 2. global matrix and local matrix



3.1.5. Usage of _bufferSize

There is no cudaMalloc inside `cuSolverMG` library, so the user must allocate the device workspace explicitly. The routine `xyz_bufferSize` is to query the size of workspace of the routine `xyz`, for example `xyz = syevd`. To make the API simple, `xyz_bufferSize` follows almost the same signature of `xyz` even it only depends on some parameters, for example, the device pointer is not used to decide the size of workspace. In most cases, `xyz_bufferSize` is called in the beginning before actual device data (pointing by a device pointer) is prepared or before the device pointer is allocated. In such cases, the user can pass a null pointer to `xyz_bufferSize` without breaking the functionality.

`xyz_bufferSize` returns bufferSize for each device. The size is number of elements, not number of bytes.

3.1.6. Synchronization

All routines are in synchronous (blocking call) manner. The data is ready after the routine. However the user has to prepare the distributed data before calling the routine. For example, if the user has multiple streams to set up the matrix, stream synchronization or device synchronization is necessary to guarantee the distributed matrix is ready.

3.1.7. Context Switch

The user does not need to restore the device by `cudaSetDevice()` after each cuSolverMG call. All routines set the device back to what the caller has.

3.1.8. NVLINK

The peer-to-peer communication via NVLINK can dramatically reduce the overhead of data exchange among GPUs. `cuSolverMG` does not enable NVLINK implicitly, instead, it gives this option back to the user, not to interfere with other libraries. The example code H.1 shows how to enable peer-to-peer communication.

3.2. cuSolverMG Types Reference

3.2.1. cuSolverMG Types

The `float`, `double`, `cuComplex`, and `cuDoubleComplex` data types are supported. The first two are standard C data types, while the last two are exported from `cuComplex.h`. In addition, cuSolverMG uses some familiar types from cuBLAS.

3.2.2. cusolverMgHandle_t

This is a pointer type to an opaque cuSolverMG context, which the user must initialize by calling `cusolverMgCreate()` prior to calling any other library function. An un-initialized handle object will lead to unexpected behavior, including crashes of cuSolverMG. The handle created and returned by `cusolverMgCreate()` must be passed to every cuSolverMG function.

3.2.3. cusolverMgGridMapping_t

The type indicates layout of grids.

Value	Meaning
`CUDALIBMG_GRID_MAPPING_ROW_MAJOR`	Row-major ordering.
`CUDALIBMG_GRID_MAPPING_COL_MAJOR`	Column-major ordering.

3.2.4. cudaLibMgGrid_t

Opaque structure of the distributed grid.

3.2.5. cudaLibMgMatrixDesc_t

Opaque structure of the distributed matrix descriptor.

3.3. Helper Function Reference

3.3.1. cusolverMgCreate()

```
cusolverStatus_t
cusolverMgCreate(cusolverMgHandle_t *handle)
```

This function initializes the cuSolverMG library and creates a handle on the cuSolverMG context. It must be called before any other cuSolverMG API function is invoked. It allocates hardware resources necessary for accessing the GPU.

Output

handle	The pointer to the handle to the cuSolverMG context.

Status Returned

CUSOLVER_STATUS_SUCCESS	The initialization succeeded.
CUSOLVER_STATUS_ALLOC_FAILED	The resources could not be allocated.

3.3.2. cusolverMgDestroy()

```
cusolverStatus_t
cusolverMgDestroy( cusolverMgHandle_t handle)
```

This function releases CPU-side resources used by the cuSolverMG library.

Input

handle	The handle to the cuSolverMG context.

Status Returned

CUSOLVER_STATUS_SUCCESS	The shutdown succeeded.

3.3.3. cusolverMgDeviceSelect()

```
cusolverStatus_t
cusolverMgDeviceSelect(
    cusolverMgHandle_t handle,
    int nbDevices,
    int deviceId[] )
```

This function registers a subset of devices (GPUs) to `cuSolverMG` handle. Such subset of devices is used in subsequent API calls. The array `deviceId` contains a list of logical device ID. The term `logical` means repeated device ID are permitted. For example, suppose the user has only one GPU in the system, say device 0. If the user sets `deviceId=0,0,0`, then `cuSolverMG` treats them as three independent GPUs, one stream each, so concurrent kernel launches still hold. The current design only supports up to 32 logical devices.

Input

`handle`	The pointer to the handle to the cuSolverMG context.
`nbDevices`	The number of logical devices.
`deviceId`	An integer array of size `nbDevices`.

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The initialization succeeded.
`CUSOLVER_STATUS_INVALID_VALUE`	`nbDevices` must be greater than zero, and less or equal to 32.
`CUSOLVER_STATUS_ALLOC_FAILED`	The resources could not be allocated.
`CUSOLVER_STATUS_INTERNAL_ERROR`	Internal error occurred when setting internal streams and events.

3.3.4. cusolverMgCreateDeviceGrid()

```
cusolverStatus_t
cusolverMgCreateDeviceGrid(
    cusolverMgGrid_t* grid,
    int32_t numRowDevices,
    int32_t numColDevices,
    const int32_t deviceId[],
    cusolverMgGridMapping_t mapping)
```

This function sets up a grid of devices.

Only 1-D column block cyclic is supported, so `numRowDevices` must be equal to 1.

WARNING: `cusolverMgCreateDeviceGrid()` must be consistent with `cusolverMgDeviceSelect()`, i.e. `numColDevices` must be equal to `nbDevices` in `cusolverMgDeviceSelect()`.

Parameter	MemSpace	In/out	Meaning
`grid`	`host`	`output`	The pointer to the opaque structure.
`numRowDevices`	`host`	`input`	Number of devices in the row.
`numColDevices`	`host`	`input`	Number of devices in the column.

Parameter	MemSpace	In/out	Meaning
deviceId	host	input	Integer array of size numColDevices, containing device IDs.
mapping	host	input	Row-major or column-major ordering.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_INVALID_VALUE	numColDevices is not greater than 0. numRowDevices is not 1.

3.3.5. cusolverMgDestroyGrid()

```
cusolverStatus_t
cusolverMgDestroyGrid(
    cusolverMgGrid_t grid)
```

This function releases resources of a grid.

Parameter	MemSpace	In/out	Meaning
grid	host	input/output	The pointer to the opaque structure.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.

3.3.6. cusolverMgCreateMatDescr()

```
cusolverStatus_t
cusolverMgCreateMatrixDesc(
    cusolverMgMatrixDesc_t * desc,
    int64_t numRows,
    int64_t numCols,
    int64_t rowBlockSize,
    int64_t colBlockSize,
    cudaDataType_t dataType,
    const cusolverMgGrid_t grid)
```

This function sets up the matrix descriptor desc.

Only 1-D column block cyclic is supported, so numRows must be equal to rowBlockSize.

Parameter	Memory	In/out	Meaning
desc	host	output	The matrix descriptor.
numRows	host	input	The number of rows of global A.
numCols	host	input	The number of columns of global A.
rowBlockSize	host	input	The number of rows per tile.
colBlockSize	host	input	The number of columns per tile.

Parameter	Memory	In/out	Meaning
dataType	host	input	Data type of the matrix.
grid	host	input	The pointer to structure of grid.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_INVALID_VALUE	numRows, numCols, or rowBlockSize or colBlockSize is less than 0. numRows is not equal to rowBlockSize.

3.3.7. cusolverMgDestroyMatrixDesc()

```
cusolverStatus_t
cusolverMgDestroyMatrixDesc(
    cusolverMgMatrixDesc_t desc)
```

This function releases the matrix descriptor desc.

Parameter	Memory	In/out	Meaning
desc	host	input/output	The matrix descriptor.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.

3.4. Dense Linear Solver Reference

This section describes the linear solver API of cuSolverMG.

3.4.1. cusolverMgPotrf()

The following helper function can calculate the sizes needed for pre-allocated buffer for cusolverMgPotrf:

```
cusolverStatus_t
cusolverMgPotrf_bufferSize(
    cusolverMgHandle_t handle,
 cublasFillMode_t uplo,
    int N,
    void *array_d_A[],
    int IA,
    int JA,
    cudaLibMgMatrixDesc_t descrA,
    cudaDataType computeType,
 int64_t *lwork)
```

The following routine:

```
cusolverStatus_t
cusolverMgPotrf(
    cusolverMgHandle_t handle,
 cublasFillMode_t uplo,
    int N,
    void *array_d_A[],
    int IA,
    int JA,
    cudaLibMgMatrixDesc_t descrA,
    cudaDataType computeType,
    void *array_d_work[],
    int64_t lwork,
    int *info)
```

computes the Cholesky factorization of a Hermitian positive-definite matrix using the generic API interface.

`A` is an `n×n` Hermitian matrix; only lower or upper part is meaningful. The input parameter `uplo` indicates which part of the matrix is used. The function would leave other parts untouched.

If input parameter `uplo` is `CUBLAS_FILL_MODE_LOWER`, only lower triangular part of `A` is processed, and replaced by lower triangular Cholesky factor `L`:

$$A = L * L^H$$

If input parameter `uplo` is `CUBLAS_FILL_MODE_UPPER`, only upper triangular part of `A` is processed, and replaced by upper triangular Cholesky factor `U`:

$$A = U^H * U$$

The user has to provide device working space in `array_d_work`. `array_d_work` is a host pointer array of dimension `G`, where `G` is number of devices. `array_d_work[j]` is a device pointer pointing to a device memory in j-th device. The data type of `array_d_work[j]` is `computeType`. The size of `array_d_work[j]` is `lwork` which is the number of elements per device, returned by `cusolverMgPotrf_bufferSize()`.

If Cholesky factorization failed, i.e. some leading minor of `A` is not positive definite, or equivalently some diagonal elements of `L` or `U` is not a real number. The output parameter `info` would indicate smallest leading minor of `A` which is not positive definite.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

The generic API has two different types, `dataTypeA` is data type of the matrix `A`, and`computeType` is compute type of the operation and data type of the workspace (`array_d_work`) `descrA` contains `dataTypeA`, so there is no explicit parameter of `dataTypeA`. `cusolverMgPotrf` only supports the following four combinations.

Please visit cuSOLVER Library Samples - MgPotrf for a code example.

valid combination of data type and compute type

DataTypeA	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	SPOTRF
CUDA_R_64F	CUDA_R_64F	DPOTRF
CUDA_C_32F	CUDA_C_32F	CPOTRF
CUDA_C_64F	CUDA_C_64F	ZPOTRF

API of potrf

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverMg library context.
uplo	host	input	Indicates if matrix A lower or upper part is stored, the other part is not referenced. Only CUBLAS_FILL_MODE_LOWER is supported.
N	host	input	Number of rows and columns of matrix sub(A).
array_d_A	host	in/out	A host pointer array of dimension G. It contains a distributed <type> array containing sub(A) of dimension N * N. On exit, sub(A) contains the factors L or U.
IA	host	input	The row index in the global array A indicating the first row of sub(A).
JA	host	input	The column index in the global array A indicating the first column of sub(A).
descrA	host	input	Matrix descriptor for the distributed matrix A.
computeType	host	input	Data type used for computation.
array_d_work	host	in/out	A host pointer array of dimension G. array_d_work[j] points to a device working space in j-th device, <type> array of size lwork.
lwork	host	input	Sze of array_d_work[j], returned by cusolverMgPotrf_bufferSize. lwork denotes number of elements, not number of bytes.
info	host	output	If info = 0, the Cholesky factorization is successful. If info = -i, the i-th parameter is wrong (not counting handle). If info = i, the leading minor of order i is not positive definite.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.

CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (M,N<0).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

3.4.2. cusolverMgPotrs()

The helper function below can calculate the sizes needed for pre-allocated buffer for cusolverMgPotrs.

```
cusolverStatus_t
cusolverMgPotrs_bufferSize(
 cusolverMgHandle_t handle,
 cublasFillMode_t uplo,
 int n,
 int nrhs,
 void *array_d_A[],
 int IA,
 int JA,
 cudaLibMgMatrixDesc_t descrA,
 void *array_d_B[],
 int IB,
 int JB,
 cudaLibMgMatrixDesc_t descrB,
 cudaDataType computeType,
 int64_t *lwork )
```

The following routine:

```
cusolverStatus_t
cusolverMgPotrs(
 cusolverMgHandle_t handle,
 cublasFillMode_t uplo,
 int n,
 int nrhs,
 void *array_d_A[],
 int IA,
 int JA,
 cudaLibMgMatrixDesc_t descrA,
 void *array_d_B[],
 int IB,
 int JB,
 cudaLibMgMatrixDesc_t descrB,
 cudaDataType computeType,
 void *array_d_work[],
 int64_t lwork,
 int *info)
```

This function solves a system of linear equations:

$$A * X = B$$

where A is a n×n Hermitian matrix, only lower or upper part is meaningful using the generic API interface. The input parameter uplo indicates which part of the matrix is used. The function would leave other parts untouched.

If input parameter uplo is CUBLAS_FILL_MODE_LOWER, the matrix should A contain the lower triangular factor for Cholesky decomposition previously computed by cusolverMgPotrf routine.

$$A = L^* L^H$$

If input parameter uplo is CUBLAS_FILL_MODE_UPPER, the matrix should A contain the upper triangular factor for Cholesky decomposition previously computed by the cusolverMgPotrf routine.

$$A = U^H * U$$

The operation is in-place, i.e. matrix B contains the solution of the linear system on exit.

If output parameter info = -i (less than zero), the i-th parameter is wrong (not counting handle).

The user has to provide device working space in array_d_work. array_d_work is a host pointer array of dimension G, where G is the number of devices. array_d_work[j] is a device pointer pointing to a device memory in the j-th device. The data type of array_d_work[j] is computeType. The size of array_d_work[j] is lwork which is number of elements per device, returned by cusolverMgPotrs_bufferSize().

If output parameter info = -i (less than zero), the i-th parameter is wrong (not counting handle).

The generic API has four different types: dataTypeA is data type of the matrix A, dataTypeB is data type of the matrix B, computeType is compute type of the operation and data type of the workspace (array_d_work) descrA contains dataTypeA and descrB contains dataTypeB and so there is no explicit parameter of dataTypeA and dataTypeB. cusolverMgPotrs only supports the following four combinations.

Please visit cuSOLVER Library Samples - MgPotrf for a code example.

valid combination of data type and compute type

DataTypeA	DataTypeB	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SPOTRS
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DPOTRS
CUDA_C_32F	CUDA_C_32F	CUDA_C_32F	CPOTRS
CUDA_C_64F	CUDA_C_64F	CUDA_C_64F	ZPOTRS

API of potrs

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverMg library context.
uplo	host	input	Indicates if matrix A lower or upper part is stored, the other

Parameter	Memory	In/out	Meaning
			part is not referenced. Only CUBLAS_FILL_MODE_LOWER is supported.
N	host	input	Number of rows and columns of matrix sub(A).
NRHS	host	input	Number of columns of matrix sub(A) and sub(B).
array_d_A	host	in/out	A host pointer array of dimension G. It contains a distributed <type> array containing sub(A) of dimension M * N. On exit, sub(A) contains the factors L and U.
IA	host	input	The row index in the global array A indicating the first row of sub(A).
JA	host	input	The column index in the global array A indicating the first column of sub(A).
descrA	host	input	Matrix descriptor for the distributed matrix A.
array_d_B	host	in/out	A host pointer array of dimension G. It contains a distributed <type> array containing sub(B) of dimension N * NRHS. On exit, sub(A) contains the solution to the linear system.
IB	host	input	The row index in the global array B indicating the first row of sub(B).
JB	host	input	The column index in the global array B indicating the first column of sub(B).
descrB	host	input	Matrix descriptor for the distributed matrix B.
computeType	host	input	Data type used for computation.
array_d_work	host	in/out	A host pointer array of dimension G. array_d_work[j] points to a device working space in j-th device, <type> array of size lwork.
lwork	host	input	Size of array_d_work[j], returned by cusolverMgPotrs_bufferSize. lwork denotes number of elements, not number of bytes.
info	host	output	If info = 0, the routine successful. If info = -i, the i-th parameter is wrong (not counting handle).

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.

CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (M,N<0).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

3.4.3. cusolverMgPotri()

The helper function below can calculate the sizes needed for pre-allocated buffer for cusolverMgPotri.

```
cusolverStatus_t
cusolverMgPotri_bufferSize(
 cusolverMgHandle_t handle,
 cublasFillMode_t uplo,
 int N,
 void *array_d_A[],
 int IA,
 int JA,
 cudaLibMgMatrixDesc_t descrA,
 cudaDataType computeType,
 int64_t *lwork)
```

The following routine:

```
cusolverStatus_t
cusolverMgPotri(
 cusolverMgHandle_t handle,
 cublasFillMode_t uplo,
 int N,
 void *array_d_A[],
 int IA,
 int JA,
 cudaLibMgMatrixDesc_t descrA,
 cudaDataType computeType,
 void *array_d_work[],
 int64_t lwork,
 int *info)
```

This function computes the inverse of a Hermitian positive-definite matrix A using the Cholesky factorization

$$A = L^{*}L^{H} = U^{H}{*}U$$

computed by cusolverMgPotrf().

If the input parameter uplo is CUBLAS_FILL_MODE_LOWER, on input, matrix A contains the lower triangular factor of A computed by cusolverMgPotrf. Only lower triangular part of A is processed, and replaced the by lower triangular part of the inverse of A.

If the input parameter uplo is CUBLAS_FILL_MODE_UPPER, on input, matrix A contains the upper triangular factor of A computed by cusolverMgPotrf. Only upper triangular part of A is processed, and replaced the by upper triangular part of the inverse of A.

The user has to provide device working space in `array_d_work`. `array_d_work` is a host pointer array of dimension `G`, where `G` is number of devices. `array_d_work[j]` is a device pointer pointing to a device memory in the `j`-th device. The data type of `array_d_work[j]` is `computeType`. The size of `array_d_work[j]` is `lwork` which is number of elements per device, returned by `cusolverMgPotri_bufferSize()`.

If the computation of the inverse fails, i.e. some leading minor of `L` or `U`, is null, the output parameter `info` would indicate the smallest leading minor of `L` or `U` which is not positive definite.

If the output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting the handle).

The generic API has two different types, `dataTypeA` is data type of the matrix `A`, `computeType` is compute type of the operation and data type of the workspace (`array_d_work`) `descrA` contains `dataTypeA`, so there is no explicit parameter of `dataTypeA`. `cusolverMgPotri` only supports the following four combinations.

Please visit cuSOLVER Library Samples - MgPotrf for a code example.

valid combination of data type and compute type

DataTypeA	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	SPOTRI
CUDA_R_64F	CUDA_R_64F	DPOTRI
CUDA_C_32F	CUDA_C_32F	CPOTRI
CUDA_C_64F	CUDA_C_64F	ZPOTRI

API of potrf

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverMg library context.
uplo	host	input	Indicates if matrix `A` lower or upper part is stored, the other part is not referenced. Only `CUBLAS_FILL_MODE_LOWER` is supported.
N	host	input	Number of rows and columns of matrix `sub(A)`.
array_d_A	host	in/out	A host pointer array of dimension `G`. It contains a distributed <type> array containing `sub(A)` of dimension `N * N`. On exit, `sub(A)` contains the upper or lower triangular part of the inverse of A depeding on the value of `uplo` argument.
IA	host	input	The row index in the global array A indicating the first row of `sub(A)`.
JA	host	input	The column index in the global array A indicating the first column of `sub(A)`.
descrA	host	input	Matrix descriptor for the distributed matrix A.

Parameter	Memory	In/out	Meaning
computeType	host	input	Data type used for computation.
array_d_work	host	in/out	A host pointer array of dimension G. array_d_work[j] points to a device working space in j-th device, <type> array of size lwork.
lwork	host	input	Size of array_d_work[j], returned by cusolverMgPotri_bufferSize. lwork denotes number of elements, not number of bytes.
info	host	output	If info = 0, the Cholesky factorization is successful. If info = -i, the i-th parameter is wrong (not counting handle). If info = i, the leading minor of order i is zero.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_NOT_INITIALIZED	The library was not initialized.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (M,N<0).
CUSOLVER_STATUS_ARCH_MISMATCH	The device only supports compute capability 2.0 and above.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

3.4.4. cusolverMgGetrf()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverMgGetrf_bufferSize(
    cusolverMgHandle_t handle,
    int M,
    int N,
    void *array_d_A[],
    int IA,
    int JA,
    cudaLibMgMatrixDesc_t descrA,
    int *array_d_IPIV[],
    cudaDataType_t computeType,
    int64_t *lwork);
```

```
cusolverStatus_t
cusolverMgGetrf(
    cusolverMgHandle_t handle,
    int M,
    int N,
    void *array_d_A[],
```

```
    int IA,
    int JA,
    cudaLibMgMatrixDesc_t descrA,
    int *array_d_IPIV[],
    cudaDataType_t computeType,
    void *array_d_work[],
    int64_t lwork,
    int *info );
```

This function computes the LU factorization of a `M×N` matrix

$$P*A=L*U$$

where `A` is a `M×N` matrix, `P` is a permutation matrix, `L` is a lower triangular matrix with unit diagonal, and `U` is an upper triangular matrix.

The user has to provide device working space in `array_d_work`. `array_d_work` is a host pointer array of dimension `G`, where `G` is number of devices. `array_d_work[j]` is a device pointer pointing to a device memory in j-th device. The data type of `array_d_work[j]` is `computeType`. The size of `array_d_work[j]` is `lwork` which is number of elements per device, returned by `cusolverMgGetrf_bufferSize()`.

If LU factorization failed, i.e. matrix `A` (`U`) is singular, The output parameter `info=i` indicates `U(i,i) = 0`.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

If `array_d_IPIV` is null, no pivoting is performed. The factorization is `A=L*U`, which is not numerically stable.

`array_d_IPIV` must be consistent with `array_d_A`, i.e. `JA` is the first column of `sub(A)`, also the first column of `sub(IPIV)`.

No matter LU factorization failed or not, the output parameter `array_d_IPIV` contains pivoting sequence, row `i` is interchanged with row `array_d_IPIV(i)`.

The generic API has three different types, `dataTypeA` is data type of the matrix `A`, `computeType` is compute type of the operation and data type of the workspace (`array_d_work`) `descrA` conatins `dataTypeA`, so there is no explicit parameter of `dataTypeA`. `cusolverMgGetrf` only supports the following four combinations.

Please visit cuSOLVER Library Samples - MgGetrf for a code example.

valid combination of data type and compute type

DataTypeA	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	SGETRF
CUDA_R_64F	CUDA_R_64F	DGETRF
CUDA_C_32F	CUDA_C_32F	CGETRF
CUDA_C_64F	CUDA_C_64F	ZGETRF

Remark 1: tile size `TA` must be less or equal to 512.

API of getrf

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverMg library context.
M	host	input	Number of rows of matrix sub(A).
N	host	input	Number of columns of matrix sub(A).
array_d_A	host	in/out	A host pointer array of dimension G. It contains a distributed <type> array containing sub(A) of dimension M * N. On exit, sub(A) contains the factors L and U.
IA	host	input	The row index in the global array A indicating the first row of sub(A).
JA	host	input	The column index in the global array A indicating the first column of sub(A).
descrA	host	input	Matrix descriptor for the distributed matrix A.
array_d_IPIV	host	output	A host pointer array of dimension G. it contains a distributed integer array containing sub(IPIV) of size min(M,N). sub(IPIV) contains pivot indices.
computeType	host	input	Data type used for computation.
array_d_work	host	in/out	A host pointer array of dimension G. array_d_work[j] points to a device working space in j-th device, <type> array of size lwork.
lwork	host	input	Size of array_d_work[j], returned by cusolverMgGetrf_bufferSize. lwork denotes number of elements, not number of bytes.
info	host	output	If info = 0, the LU factorization is successful. If info = -i, the i-th parameter is wrong (not counting handle). If info = i, the U(i,i) = 0.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (M,N<0).
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

3.4.5. cusolverMgGetrs()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverMgGetrs_bufferSize(
```

```
    cusolverMgHandle_t handle,
    cublasOperation_t TRANS,
    int N,
    int NRHS,
    void *array_d_A[],
    int IA,
    int JA,
    cudaLibMgMatrixDesc_t descrA,
    int *array_d_IPIV[],
    void *array_d_B[],
    int IB,
    int JB,
    cudaLibMgMatrixDesc_t descrB,
    cudaDataType_t computeType,
    int64_t *lwork);
```

```
cusolverStatus_t
cusolverMgGetrs(
    cusolverMgHandle_t handle,
    cublasOperation_t TRANS,
    int N,
    int NRHS,
    void *array_d_A[],
    int IA,
    int JA,
    cudaLibMgMatrixDesc_t descrA,
    int *array_d_IPIV[],
    void *array_d_B[],
    int IB,
    int JB,
    cudaLibMgMatrixDesc_t descrB,
    cudaDataType_t computeType,
    void *array_d_work[],
    int64_t lwork,
    int *info );
```

This function solves a linear system of multiple right-hand sides

$$\mathrm{op(A)} * X = B$$

where `A` is a `N×N` matrix, and was LU-factored by `getrf`, that is, lower trianular part of A is `L`, and upper triangular part (including diagonal elements) of `A` is `U`. `B` is a `N×NRHS` right-hand side matrix. The solution matirx `X` overwrites the right-hand-side matrix `B`.

The input parameter `TRANS` is defined by

$$\mathrm{op}(A) = \begin{cases} A & \text{if TRANS == CUBLAS\_OP\_N} \\ A^T & \text{if TRANS == CUBLAS\_OP\_T} \\ A^H & \text{if TRANS == CUBLAS\_OP\_C} \end{cases}$$

The user has to provide device working space in `array_d_work`. `array_d_work` is a host pointer array of dimension `G`, where `G` is number of devices. `array_d_work[j]` is a device pointer pointing to a device memory in j-th device. The data type of `array_d_work[j]` is `computeType`. The size of `array_d_work[j]` is `lwork` which is number of elements per device, returned by `cusolverMgGetrs_bufferSize()`.

If `array_d_IPIV` is null, no pivoting is performed. Otherwise, `array_d_IPIV` is an output of `getrf`. It contains pivot indices, which are used to permutate right-hand sides.

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle).

The generic API has three different types, `dataTypeA` is data type of the matrix `A`, `dataTypeB` is data type of the matrix `B`, and `computeType` is compute type of the operation and data type of the workspace (`array_d_work`) `descrA` conatins `dataTypeA`, so there is no explicit parameter of `dataTypeA`. `descrB` conatins `dataTypeB`, so there is no explicit parameter of `dataTypeB`. `cusolverMgGetrs` only supports the following four combinations.

Valid combinations of data type and compute type

DataTypeA	DataTypeB	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SGETRS
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DGETRS
CUDA_C_32F	CUDA_C_32F	CUDA_C_32F	CGETRS
CUDA_C_64F	CUDA_C_64F	CUDA_C_64F	ZGETRS

Remark 1: tile size `TA` must be less or equal to 512.

Remark 2: only support `TRANS=CUBLAS_OP_N`.

Please visit cuSOLVER Library Samples - MgGetrf for a code example.

API of getrs

Parameter	Memory	In/out	Meaning
handle	host	input	Handle to the cuSolverMG library context.
TRANS	host	input	Operation `op(A)` that is non- or (conj.) transpose.
N	host	input	Number of rows and columns of matrix `sub(A)`.
NRHS	host	input	Number of columns of matrix `sub(B)`.
array_d_A	host	input	A host pointer array of dimension `G`. It contains a distributed <type> array containing `sub(A)` of dimension `M * N`. `sub(A)` contains the factors `L` and `U`.
IA	host	input	The row index in the global array A indicating the first row of `sub(A)`.
JA	host	input	The column index in the global array A indicating the first column of `sub(A)`.
descrA	host	input	Matrix descriptor for the distributed matrix A.
array_d_IPIV	host	input	A host pointer array of dimension `G`. it contains a distributed integer array containing `sub(IPIV)` of dimension `min(M,N)`. `sub(IPIV)` contains pivot indices.
array_d_B	host	in/out	A host pointer array of dimension `G`. It contains a distributed <type> array

Parameter	Memory	In/out	Meaning
			containing `sub(B)` of dimension `N * NRHS`.
`IB`	`host`	`input`	The row index in the global array B indicating the first row of `sub(B)`.
`JB`	`host`	`input`	The column index in the global array B indicating the first column of `sub(B)`.
`descrB`	`host`	`input`	Matrix descriptor for the distributed matrix B.
`computeType`	`host`	`input`	Data type used for computation.
`array_d_work`	`host`	`in/out`	A host pointer array of dimension `G`. `array_d_work[j]` points to a device working space in `j-th` device, <type> array of size `lwork`.
`lwork`	`host`	`input`	Size of `array_d_work[j]`, returned by `cusolverMgGetrs_bufferSize`. `lwork` denotes number of elements, not number of bytes.
`info`	`host`	`output`	If `info = 0`, the operation is successful. If `info = -i`, the `i-th` parameter is wrong (not counting handle).

Status Returned

`CUSOLVER_STATUS_SUCCESS`	The operation completed successfully.
`CUSOLVER_STATUS_INVALID_VALUE`	Invalid parameters were passed (`N<0` or `NRHS<0`).
`CUSOLVER_STATUS_INTERNAL_ERROR`	An internal operation failed.

3.5. Dense Eigenvalue Solver Reference

This section describes the eigenvalue solver API of cuSolverMG.

3.5.1. cusolverMgSyevd()

The helper functions below can calculate the sizes needed for pre-allocated buffer.

```
cusolverStatus_t
cusolverMgSyevd_bufferSize(
    cusolverMgHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int N,
    void *array_d_A[],
    int IA,
    int JA,
    cudaLibMgMatrixDesc_t descrA,
    void *W,
    cudaDataType_t dataTypeW,
    cudaDataType_t computeType,
```

```
    int64_t *lwork
    );
```

```
cusolverStatus_t
cusolverMgSyevd(
    cusolverMgHandle_t handle,
    cusolverEigMode_t jobz,
    cublasFillMode_t uplo,
    int N,
    void *array_d_A[],
    int IA,
    int JA,
    cudaLibMgMatrixDesc_t descrA,
    void *W,
    cudaDataType_t dataTypeW,
    cudaDataType_t computeType,
    void *array_d_work[],
    int64_t lwork,
    int *info );
```

This function computes eigenvalues and eigenvectors of a symmetric (Hermitian) `N×N` matrix `A`. The standard symmetric eigenvalue problem is:

$$A*V = V*\Lambda$$

where `Λ` is a real `N×N` diagonal matrix. `V` is an `N×N` unitary matrix. The diagonal elements of `Λ` are the eigenvalues of `A` in ascending order.

`cusolverMgSyevd` returns the eigenvalues in `W` and overwrites the eigenvectors in `A`. `W` is a host `1×N` vector.

The generic API has three different types, `dataTypeA` is data type of the matrix `A`, `dataTypeW` is data type of the vector `W`, and `computeType` is compute type of the operation and data type of the workspace (`array_d_work`) `descrA` conatins `dataTypeA`, so there is no explicit parameter of `dataTypeA`. `cusolverMgSyevd` only supports the following four combinations.

Valid combination of data type and compute type

DataTypeA	DataTypeW	ComputeType	Meaning
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	SSYEVD
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	DSYEVD
CUDA_C_32F	CUDA_R_32F	CUDA_C_32F	CHEEVD
CUDA_C_64F	CUDA_R_64F	CUDA_C_64F	ZHEEVD

The user has to provide device working space in `array_d_work`. `array_d_work` is a host pointer array of dimension `G`, where `G` is number of devices. `array_d_work[j]` is a device pointer pointing to a device memory in j-th device. The data type of `array_d_work[j]` is `computeType`. The size of `array_d_work[j]` is `lwork` which is number of elements per device, returned by `cusolverMgSyevd_bufferSize()`.

`array_d_A` is also a host pointer array of dimension `G`. `array_d_A[j]` is a device pointer pointing to a device memory in j-th device. The data type of `array_d_A[j]` is `dataTypeA`. The size of `array_d_A[j]` is about `N*TA*(blocks per device)`. The user has to prepare `array_d_A` manually (see cuSOLVER Library Samples - MgSyevd for a code example.).

If output parameter `info = -i` (less than zero), the `i-th` parameter is wrong (not counting handle). If `info = i` (greater than zero), `i` off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

If `jobz` = CUSOLVER_EIG_MODE_VECTOR, `A` contains the orthonormal eigenvectors of the matrix `A`. The eigenvectors are computed by a divide and conquer algorithm.

Remark 1: only `CUBLAS_FILL_MODE_LOWER` is supported, so the user has to prepare lower triangle of `A`.

Remark 2: only `IA=1` and `JA=1` are supported.

Remark 3: tile size `TA` must be less or equal to 1024. To achieve best performance, `TA` should be 256 or 512.

Please visit cuSOLVER Library Samples - MgSyevd for a code example.

API of syevd

Parameter	Memory	In/out	Meaning
`handle`	`host`	`input`	Handle to the cuSolverMG library context.
`jobz`	`host`	`input`	Specifies options to either compute eigenvalue only or compute eigen-pair: `jobz` = `CUSOLVER_EIG_MODE_NOVECTOR` : Compute eigenvalues only `jobz` = `CUSOLVER_EIG_MODE_VECTOR` : Compute eigenvalues and eigenvectors
`uplo`	`host`	`input`	Specifies which part of `A` is stored. `uplo` = `CUBLAS_FILL_MODE_LOWER`: Lower triangle of `A` is stored. `uplo` = `CUBLAS_FILL_MODE_UPPER`: Upper triangle of `A` is stored. Only `CUBLAS_FILL_MODE_LOWER` is supported.
`N`	`host`	`input`	Number of rows (or columns) of matrix `sub(A)`.
`array_d_A`	`host`	`in/out`	A host pointer array of dimension `G`. It contains a distributed <type> array containing `sub(A)` of dimension `N * N`. If `uplo` = `CUBLAS_FILL_MODE_UPPER`, the leading N-by-N upper triangular part of `sub(A)` contains the upper triangular part of the matrix `sub(A)`. If `uplo` = `CUBLAS_FILL_MODE_LOWER`, the leading N-by-N lower triangular part of `sub(A)` contains the lower triangular part of the matrix `sub(A)`. On exit, if `jobz` = `CUSOLVER_EIG_MODE_VECTOR`, and `info` = 0, `sub(A)` contains the

Parameter	Memory	In/out	Meaning
			orthonormal eigenvectors of the matrix sub(A). If jobz = CUSOLVER_EIG_MODE_NOVECTOR, the contents of A are destroyed.
IA	host	input	The row index in the global array A indicating the first row of sub(A).
JA	host	input	The column index in the global array A indicating the first column of sub(A).
descrA	host	input	Matrix descriptor for the distributed matrix A.
W	host	output	A real array of dimension N. The eigenvalue values of sub(A), in ascending order ie, sorted so that W(i) <= W(i+1).
dataTypeW	host	input	Data type of the vector W.
computeType	host	input	Data type used for computation.
array_d_work	host	in/out	A host pointer array of dimension G. array_d_work[j] points to a device working space in j-th device, <type> array of size lwork.
lwork	host	input	Size of array_d_work[j], returned by cusolverMgSyevd_bufferSize. lwork denotes number of elements, not number of bytes.
info	host	output	If info = 0, the operation is successful. If info = -i, the i-th parameter is wrong (not counting handle). If info = i (> 0), info indicates i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

Status Returned

CUSOLVER_STATUS_SUCCESS	The operation completed successfully.
CUSOLVER_STATUS_INVALID_VALUE	Invalid parameters were passed (N<0, or lda<max(1,N), or jobz is not CUSOLVER_EIG_MODE_NOVECTOR or CUSOLVER_EIG_MODE_VECTOR, or uplo is not CUBLAS_FILL_MODE_LOWER, or IA and JA are not 1, or N is bigger than dimension of global A, or the combination of dataType and computeType is not valid.
CUSOLVER_STATUS_INTERNAL_ERROR	An internal operation failed.

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