cuSPARSE Library

DU-06709-001_v12.0 | December 2022

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Chapter 1. Introduction

The cuSPARSE library contains a set of basic linear algebra subroutines used for handling sparse matrices. The library targets matrices with a number of (structural) zero elements which represent > 95% of the total entries.

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cuSPARSE Release Notes: cuda-toolkit-release-notes

cuSPARSE GitHub Examples: github.com/NVIDIA/CUDALibrarySamples

It is implemented on top of the NVIDIA® CUDA™ runtime (which is part of the CUDA Toolkit) and is designed to be called from C and C++.

The library routines can be classified into four categories:

- Level 1: operations between a vector in sparse format and a vector in dense format
- Level 2: operations between a matrix in sparse format and a vector in dense format
- Level 3: operations between a matrix in sparse format and a set of vectors in dense format (which can also usually be viewed as a dense tall matrix)
- Conversion: operations that allow conversion between different matrix formats, and compression of csr matrices.

The cuSPARSE library allows developers to access the computational resources of the NVIDIA graphics processing unit (GPU), although it does not auto-parallelize across multiple GPUs. The cuSPARSE API assumes that input and output data reside in GPU (device) memory, unless it is explicitly indicated otherwise by the string `DevHostPtr` in a function parameter's name.

It is the responsibility of the developer to allocate memory and to copy data between GPU memory and CPU memory using standard CUDA runtime API routines, such as `cudaMalloc()`, `cudaFree()`, `cudaMemcpy()`, and `cudaMemcpyAsync()`.

1.1. Naming Conventions

The cuSPARSE library functions are available for data types `float`, `double`, `cuComplex`, and `cuDoubleComplex`. The sparse Level 1, Level 2, and Level 3 functions follow this naming convention:

`cusparse<t>[<matrix data format>]<operation>[<output matrix data format>]`

where `<t>` can be `S`, `D`, `C`, `Z`, or `X`, corresponding to the data types `float`, `double`, `cuComplex`, `cuDoubleComplex`, and the generic type, respectively.

The `<matrix data format>` can be `dense`, `coo`, `csr`, or `csc`, corresponding to the dense, coordinate, compressed sparse row, and compressed sparse column formats, respectively.

Finally, the `<operation>` can be `axpyi`, `gthr`, `gthrz`, `roti`, or `sctr`, corresponding to the Level 1 functions; it also can be `mv` or `sv`, corresponding to the Level 2 functions, as well as `mm` or `sm`, corresponding to the Level 3 functions.

All of the functions have the return type `cusparseStatus_t` and are explained in more detail in the chapters that follow.

1.2. Asynchronous Execution

The cuSPARSE library functions are executed asynchronously with respect to the host and may return control to the application on the host before the result is ready. Developers can use the `cudaDeviceSynchronize()` function to ensure that the execution of a particular cuSPARSE library routine has completed.

A developer can also use the `cudaMemcpy()` routine to copy data from the device to the host and vice versa, using the `cudaMemcpyDeviceToHost` and `cudaMemcpyHostToDevice` parameters, respectively. In this case there is no need to add a call to `cudaDeviceSynchronize()` because the call to `cudaMemcpy()` with the above parameters is blocking and completes only when the results are ready on the host.

1.3. Static Library Support

Starting with release 6.5, the cuSPARSE Library is also delivered in a static form as `libcusparse_static.a` on Linux.

For example, to compile a small application using cuSPARSE against the dynamic library, the following command can be used:

```
nvcc myCusparseApp.c  -lcusparse  -o myCusparseApp>
```

Whereas to compile against the static cuSPARSE library, the following command has to be used:

```
nvcc myCusparseApp.c  -lcusparse_static   -o myCusparseApp>
```

It is also possible to use the native Host C++ compiler. Depending on the Host Operating system, some additional libraries like `pthread` or `dl` might be needed on the linking line. The following command on Linux is suggested :

```
g++ myCusparseApp.c  -lcusparse_static   -lcudart_static -lpthread -ldl -I <cuda-
toolkit-path>/include -L <cuda-toolkit-path>/lib64 -o myCusparseApp
```

Note that in the latter case, the library `cuda` is not needed. The CUDA Runtime will try to open explicitly the `cuda` library if needed. In the case of a system which does not have the CUDA driver installed, this allows the application to gracefully manage this issue and potentially run if a CPU-only path is available.

1.4. Library Dependencies

Starting from CUDA 12.0, cuSPARSE will depend on `nvJitLink` library for JIT (just-in-time) LTO (link-time-optimization) capabilities, see `cusparseSpMMOp` APIs for more information.

If the user links to the dynamic library, the environment variables for loading the libraries (e.g., *LD_LIBRARY_PATH* on linux and *PATH* on Windows) must include the path where *libnvjitlink.so* is located. If it is the same directory of cuSPARSE, the user doesn't need any action. While if linking to the static library, the user needs to link with *-lnvjitlink* and set *LIBRARY_PATH/PATH* accordingly.

Chapter 2. Using the cuSPARSE API

This chapter describes how to use the cuSPARSE library API. It is not a reference for the cuSPARSE API data types and functions; that is provided in subsequent chapters.

2.1. Thread Safety

The library is thread safe and its functions can be called from multiple host threads. However, simultaneous read/writes of the same objects (or of the same handle) are not safe. Hence the handle must be private per thread, i.e., only one handle per thread is safe.

2.2. Scalar Parameters

In the cuSPARSE API, the scalar parameters α and β can be passed by reference on the host or the device.

The few functions that return a scalar result, such as `nnz()`, return the resulting value by reference on the host or the device. Even though these functions return immediately, similarly to those that return matrix and vector results, the scalar result is not ready until execution of the routine on the GPU completes. This requires proper synchronization be used when reading the result from the host.

This feature allows the cuSPARSE library functions to execute completely asynchronously using streams, even when α and β are generated by a previous kernel. This situation arises, for example, when the library is used to implement iterative methods for the solution of linear systems and eigenvalue problems [3].

2.3. Parallelism with Streams

If the application performs several small independent computations, or if it makes data transfers in parallel with the computation, CUDA streams can be used to overlap these tasks.

The application can conceptually associate a stream with each task. To achieve the overlap of computation between the tasks, the developer should create CUDA streams using the function `cudaStreamCreate()` and set the stream to be used by each individual cuSPARSE library routine by calling `cusparseSetStream()` just before calling the actual cuSPARSE routine. Then, computations performed in separate streams would be overlapped automatically on the

GPU, when possible. This approach is especially useful when the computation performed by a single task is relatively small and is not enough to fill the GPU with work, or when there is a data transfer that can be performed in parallel with the computation.

When streams are used, we recommend using the new cuSPARSE API with scalar parameters and results passed by reference in the device memory to achieve maximum computational overlap.

Although a developer can create many streams, in practice it is not possible to have more than 16 concurrent kernels executing at the same time.

2.4. Compatibility and Versioning

The cuSPARSE APIs are intended to be backward compatible at the source level with future releases (unless stated otherwise in the release notes of a specific future release). In other words, if a program uses cuSPARSE, it should continue to compile and work correctly with newer versions of cuSPARSE without source code changes. cuSPARSE is not guaranteed to be backward compatible at the binary level. Using different versions of the `cusparse.h` header file and the shared library is not supported. Using different versions of cuSPARSE and the CUDA runtime is not supported. The APIs should be backward compatible at the source level for public functions in most cases

2.5. Optimization Notes

Most of the cuSPARSE routines can be optimized by exploiting *CUDA Graphs capture* and *Hardware Memory Compression* features.

More in details, a single cuSPARSE call or a sequence of calls can be captured by a CUDA Graph and executed in a second moment. This minimizes kernels launch overhead and allows the CUDA runtime to optimize the whole workflow. A full example of CUDA graphs capture applied to a cuSPARSE routine can be found in cuSPARSE Library Samples - CUDA Graph.

Secondly, the data types and functionalities involved in cuSPARSE are suitable for *Hardware Memory Compression* available in Ampere GPU devices (compute capability 8.0) or above. The feature allows memory compression for data with enough zero bytes without no loss of information. The device memory must be allocation with the CUDA driver APIs. A full example of Hardware Memory Compression applied to a cuSPARSE routine can be found in cuSPARSE Library Samples - Memory Compression.

Chapter 3. cuSPARSE Indexing and Data Formats

The cuSPARSE library supports dense and sparse vector, and dense and sparse matrix formats.

3.1. Index Base Format

The library supports zero- and one-based indexing. The index base is selected through the `cusparseIndexBase_t` type, which is passed as a standalone parameter or as a field in the matrix descriptor `cusparseMatDescr_t` type.

3.1.1. Vector Formats

This section describes dense and sparse vector formats.

3.1.1.1. Dense Format

Dense vectors are represented with a single data array that is stored linearly in memory, such as the following 7×1 dense vector.

$$[1.0 \quad 0.0 \quad 0.0 \quad 2.0 \quad 3.0 \quad 0.0 \quad 4.0]$$

(This vector is referenced again in the next section.)

3.1.1.2. Sparse Format

Sparse vectors are represented with two arrays.

- The *data array* has the nonzero values from the equivalent array in dense format.
- The *integer index array* has the positions of the corresponding nonzero values in the equivalent array in dense format.

For example, the dense vector in section 3.2.1 can be stored as a sparse vector with one-based indexing.

$$\begin{matrix}[1.0 & 2.0 & 3.0 & 4.0] \\ [1 & 4 & 5 & 7\ ]\end{matrix}$$

It can also be stored as a sparse vector with zero-based indexing.

$$\begin{matrix}[1.0 & 2.0 & 3.0 & 4.0] \\ [0 & 3 & 4 & 6\ ]\end{matrix}$$

In each example, the top row is the data array and the bottom row is the index array, and it is assumed that the indices are provided in increasing order and that each index appears only once.

3.2. Matrix Formats

Dense and several sparse formats for matrices are discussed in this section.

3.2.1. Dense Format

The dense matrix `X` is assumed to be stored in column-major format in memory and is represented by the following parameters.

`m`	(integer)	The number of rows in the matrix.
`n`	(integer)	The number of columns in the matrix.
`ldX`	(integer)	The leading dimension of `X`, which must be greater than or equal to `m`. If `ldX` is greater than `m`, then `X` represents a sub-matrix of a larger matrix stored in memory
`X`	(pointer)	Points to the data array containing the matrix elements. It is assumed that enough storage is allocated for `X` to hold all of the matrix elements and that cuSPARSE library functions may access values outside of the sub-matrix, but will never overwrite them.

For example, `m×n` dense matrix `X` with leading dimension `ldX` can be stored with one-based indexing as shown.

$$\begin{bmatrix} X_{1,1} & X_{1,2} & \cdots & X_{1,n} \\ X_{2,1} & X_{2,2} & \cdots & X_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{m,1} & X_{m,2} & \cdots & X_{m,n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{ldX,1} & X_{ldX,2} & \cdots & X_{ldX,n} \end{bmatrix}$$

Its elements are arranged linearly in memory in the order below.

$$[X_{1,1} \quad X_{2,1} \quad \cdots \quad X_{m,1} \quad \cdots \quad X_{ldX,1} \quad \cdots \quad X_{1,n} \quad X_{2,n} \quad \cdots \quad X_{m,n} \quad \cdots \quad X_{ldX,n}]$$

Note: This format and notation are similar to those used in the NVIDIA CUDA cuBLAS library.

3.2.2. Coordinate Format (COO)

The `m×n` sparse matrix `A` is represented in COO format by the following parameters.

`nnz`	(integer)	The number of nonzero elements in the matrix.
`cooValA`	(pointer)	Points to the data array of length `nnz` that holds all nonzero values of `A` in row-major format.
`cooRowIndA`	(pointer)	Points to the integer array of length `nnz` that contains the row indices of the corresponding elements in array `cooValA`.
`cooColIndA`	(pointer)	Points to the integer array of length `nnz` that contains the column indices of the corresponding elements in array `cooValA`.

A sparse matrix in COO format is assumed to be stored in row-major format. Each COO entry consists of a row, column pair. The COO format is assumed to be sorted by row. Both sorted and unsorted column indices are supported.

For example, consider the following 4×5 matrix `A`.

$$\begin{bmatrix} 1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 9.0 & 0.0 & 6.0 \end{bmatrix}$$

It is stored in COO format with zero-based indexing this way.

$$\begin{aligned} \text{cooValA} &= [1.0 \quad 4.0 \quad 2.0 \quad 3.0 \quad 5.0 \quad 7.0 \quad 8.0 \quad 9.0 \quad 6.0] \\ \text{cooRowIndA} &= [0 \quad 0 \quad 1 \quad 1 \quad 2 \quad 2 \quad 2 \quad 3 \quad 3] \\ \text{cooColIndA} &= [0 \quad 1 \quad 1 \quad 2 \quad 0 \quad 3 \quad 4 \quad 2 \quad 4] \end{aligned}$$

In the COO format with one-based indexing, it is stored as shown.

$$\begin{aligned} \text{cooValA} &= [1.0 \quad 4.0 \quad 2.0 \quad 3.0 \quad 5.0 \quad 7.0 \quad 8.0 \quad 9.0 \quad 6.0] \\ \text{cooRowIndA} &= [1 \quad 1 \quad 2 \quad 2 \quad 3 \quad 3 \quad 3 \quad 4 \quad 4] \\ \text{cooColIndA} &= [1 \quad 2 \quad 2 \quad 3 \quad 1 \quad 4 \quad 5 \quad 3 \quad 5] \end{aligned}$$

3.2.3. Compressed Sparse Row Format (CSR)

The only way the CSR differs from the COO format is that the array containing the row indices is compressed in CSR format. The `m×n` sparse matrix `A` is represented in CSR format by the following parameters.

`nnz`	(integer)	The number of nonzero elements in the matrix.
`csrValA`	(pointer)	Points to the data array of length `nnz` that holds all nonzero values of A in row-major format.
`csrRowPtrA`	(pointer)	Points to the integer array of length `m+1` that holds indices into the arrays `csrColIndA` and `csrValA`. The first `m` entries of this array contain the indices of the first nonzero element in the `i`th row for `i=i,...,m`, while the last entry contains `nnz+csrRowPtrA(0)`. In general, `csrRowPtrA(0)` is `0` or `1` for zero- and one-based indexing, respectively.

`csrColIndA`	(pointer)	Points to the integer array of length `nnz` that contains the column indices of the corresponding elements in array `csrValA`.

Sparse matrices in CSR format are assumed to be stored in row-major CSR format. Both sorted and unsorted column indices are supported.

Consider again the 4×5matrix`A`.

$$\begin{bmatrix} 1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 9.0 & 0.0 & 6.0 \end{bmatrix}$$

It is stored in CSR format with zero-based indexing as shown.

$$\begin{aligned} \text{csrValA} &= [1.0 \quad 4.0 \quad 2.0 \quad 3.0 \quad 5.0 \quad 7.0 \quad 8.0 \quad 9.0 \quad 6.0] \\ \text{csrRowPtrA} &= [0 \quad 2 \quad 4 \quad 7 \quad 9 \ ] \\ \text{csrColIndA} &= [0 \quad 1 \quad 1 \quad 2 \quad 0 \quad 3 \quad 4 \quad 2 \quad 4 \ ] \end{aligned}$$

This is how it is stored in CSR format with one-based indexing.

$$\begin{aligned} \text{csrValA} &= [1.0 \quad 4.0 \quad 2.0 \quad 3.0 \quad 5.0 \quad 7.0 \quad 8.0 \quad 9.0 \quad 6.0] \\ \text{csrRowPtrA} &= [1 \quad 3 \quad 5 \quad 8 \quad 10 \ ] \\ \text{csrColIndA} &= [1 \quad 2 \quad 2 \quad 3 \quad 1 \quad 4 \quad 5 \quad 3 \quad 5 \ ] \end{aligned}$$

3.2.4. Compressed Sparse Column Format (CSC)

The CSC format is different from the COO format in two ways: the matrix is stored in column-major format, and the array containing the column indices is compressed in CSC format. The `m×n` matrix `A` is represented in CSC format by the following parameters.

`nnz`	(integer)	The number of nonzero elements in the matrix.
`cscValA`	(pointer)	Points to the data array of length `nnz` that holds all nonzero values of `A` in column-major format.
`cscRowIndA`	(pointer)	Points to the integer array of length `nnz` that contains the row indices of the corresponding elements in array `cscValA`.
`cscColPtrA`	(pointer)	Points to the integer array of length `n+1` that holds indices into the arrays `cscRowIndA` and `cscValA`. The first `n` entries of this array contain the indices of the first nonzero element in the `i`th row for `i=i,...,n`, while the last entry contains `nnz+cscColPtrA(0)`. In general, `cscColPtrA(0)` is `0` or `1` for zero- and one-based indexing, respectively.

Note: The matrix `A` in CSR format has exactly the same memory layout as its transpose in CSC format (and vice versa).

For example, consider once again the 4×5 matrix `A`.

$$\begin{bmatrix} 1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 9.0 & 0.0 & 6.0 \end{bmatrix}$$

It is stored in CSC format with zero-based indexing this way.

$$\begin{aligned} \text{cscValA} &= [1.0 \quad 5.0 \quad 4.0 \quad 2.0 \quad 3.0 \quad 9.0 \quad 7.0 \quad 8.0 \quad 6.0] \\ \text{cscRowIndA} &= [0 \quad 2 \quad 0 \quad 1 \quad 1 \quad 3 \quad 2 \quad 2 \quad 3\ ] \\ \text{cscColPtrA} &= [0 \quad 2 \quad 4 \quad 6 \quad 7 \quad 9\ ] \end{aligned}$$

In CSC format with one-based indexing, this is how it is stored.

$$\begin{aligned} \text{cscValA} &= [1.0 \quad 5.0 \quad 4.0 \quad 2.0 \quad 3.0 \quad 9.0 \quad 7.0 \quad 8.0 \quad 6.0] \\ \text{cscRowIndA} &= [1 \quad 3 \quad 1 \quad 2 \quad 2 \quad 4 \quad 3 \quad 3 \quad 4\ ] \\ \text{cscColPtrA} &= [1 \quad 3 \quad 5 \quad 7 \quad 8 \quad 10\ ] \end{aligned}$$

Each pair of row and column indices appears only once.

3.2.5. Block Compressed Sparse Row Format (BSR)

The only difference between the CSR and BSR formats is the format of the storage element. The former stores primitive data types (`single`, `double`, `cuComplex`, and `cuDoubleComplex`) whereas the latter stores a two-dimensional square block of primitive data types. The dimension of the square block is $blockDim$. The `m×n` sparse matrix `A` is equivalent to a block sparse matrix A_b with $mb = \frac{m + blockDim - 1}{blockDim}$ block rows and $nb = \frac{n + blockDim - 1}{blockDim}$ block columns. If m or n is not multiple of $blockDim$, then zeros are filled into A_b.

`A` is represented in BSR format by the following parameters.

`blockDim`	(integer)	Block dimension of matrix `A`.
`mb`	(integer)	The number of block rows of `A`.
`nb`	(integer)	The number of block columns of `A`.
`nnzb`	(integer)	The number of nonzero blocks in the matrix.
`bsrValA`	(pointer)	Points to the data array of length $nnzb * blockDim^2$ that holds all elements of nonzero blocks of `A`. The block elements are stored in either column-major order or row-major order.
`bsrRowPtrA`	(pointer)	Points to the integer array of length `mb+1` that holds indices into the arrays `bsrColIndA` and `bsrValA`. The first `mb` entries of this array contain the indices of the first nonzero block in the `i`th block row for `i=1,...,mb`, while the last entry contains `nnzb+bsrRowPtrA(0)`. In general, `bsrRowPtrA(0)` is `0` or `1` for zero- and one-based indexing, respectively.
`bsrColIndA`	(pointer)	Points to the integer array of length `nnzb` that contains the column indices of the corresponding blocks in array `bsrValA`.

As with CSR format, (row, column) indices of BSR are stored in row-major order. The index arrays are first sorted by row indices and then within the same row by column indices.

For example, consider again the `4×5` matrix `A`.

$$\begin{bmatrix} 1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 9.0 & 0.0 & 6.0 \end{bmatrix}$$

If $blockDim$ is equal to 2, then mb is 2, nb is 3, and matrix `A` is split into `2×3` block matrix A_b. The dimension of A_b is `4×6`, slightly bigger than matrix A, so zeros are filled in the last column of A_b. The element-wise view of A_b is this.

$$\begin{bmatrix} 1.0 & 4.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 3.0 & 0.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 0.0 & 7.0 & 8.0 & 0.0 \\ 0.0 & 0.0 & 9.0 & 0.0 & 6.0 & 0.0 \end{bmatrix}$$

Based on zero-based indexing, the block-wise view of A_b can be represented as follows.

$$A_b = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \end{bmatrix}$$

The basic element of BSR is a nonzero A_{ij} block, one that contains at least one nonzero element of `A`. Five of six blocks are nonzero in A_b.

$$A_{00} = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}, A_{01} = \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}, A_{10} = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}, A_{11} = \begin{bmatrix} 0 & 7 \\ 9 & 0 \end{bmatrix}, A_{12} = \begin{bmatrix} 8 & 0 \\ 6 & 0 \end{bmatrix}$$

BSR format only stores the information of nonzero blocks, including block indices (i, j) and values A_{ij}. Also row indices are compressed in CSR format.

$$\begin{aligned} \text{bsrValA} &= [A_{00} \;\; A_{01} \;\; A_{10} \;\; A_{11} \;\; A_{12}] \\ \text{bsrRowPtrA} &= [0 \quad 2 \quad 5] \\ \text{bsrColIndA} &= [0 \quad 1 \quad 0 \quad 1 \quad 2] \end{aligned}$$

There are two ways to arrange the data element of block A_{ij}: row-major order and column-major order. Under column-major order, the physical storage of `bsrValA` is this.

$$bsrValA = [1 \;\; 0 \;\; 4 \;\; 2 \;\; | 0 \;\; 3 \;\; 0 \;\; 0 \;\; | 5 \;\; 0 \;\; 0 \;\; 0 \;\; | 0 \;\; 9 \;\; 7 \;\; 0 \;\; | 8 \;\; 6 \;\; 0 \;\; 0 \;\;]$$

Under row-major order, the physical storage of `bsrValA` is this.

$$bsrValA = [1 \;\; 4 \;\; 0 \;\; 2 \;\; | 0 \;\; 0 \;\; 3 \;\; 0 \;\; | 5 \;\; 0 \;\; 0 \;\; 0 \;\; | 0 \;\; 7 \;\; 9 \;\; 0 \;\; | 8 \;\; 0 \;\; 6 \;\; 0 \;\;]$$

Similarly, in BSR format with one-based indexing and column-major order, `A` can be represented by the following.

$$A_b = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

$$bsrValA = [1 \;\; 0 \;\; 4 \;\; 2 \;\; | 0 \;\; 3 \;\; 0 \;\; 0 \;\; | 5 \;\; 0 \;\; 0 \;\; 0 \;\; | 0 \;\; 9 \;\; 7 \;\; 0 \;\; | 8 \;\; 6 \;\; 0 \;\; 0 \;\;]$$

$$\text{bsrRowPtrA} = \begin{bmatrix} 1 & 3 & 6 \end{bmatrix}$$
$$\text{bsrColIndA} = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 \end{bmatrix}$$

Note: The general BSR format has two parameters, `rowBlockDim` and `colBlockDim`. `rowBlockDim` is number of rows within a block and `colBlockDim` is number of columns within a block. If `rowBlockDim=colBlockDim`, general BSR format is the same as BSR format. If `rowBlockDim=colBlockDim=1`, general BSR format is the same as CSR format. The conversion routine gebsr2gebsr is used to do conversion among CSR, BSR and general BSR.

Note: In the cuSPARSE Library, the storage format of blocks in BSR format can be column-major or row-major, independently of the base index. However, if the developer uses BSR format from the Math Kernel Library (MKL) and wants to directly interface with the cuSPARSE Library, then `cusparseDirection_t CUSPARSE_DIRECTION_COLUMN` should be used if the base index is one; otherwise, `cusparseDirection_t CUSPARSE_DIRECTION_ROW` should be used.

3.2.6. Extended BSR Format (BSRX)

BSRX is the same as the BSR format, but the array `bsrRowPtrA` is separated into two parts. The first nonzero block of each row is still specified by the array `bsrRowPtrA`, which is the same as in BSR, but the position next to the last nonzero block of each row is specified by the array `bsrEndPtrA`. Briefly, BSRX format is simply like a 4-vector variant of BSR format.

Matrix `A` is represented in BSRX format by the following parameters.

`blockDim`	(integer)	Block dimension of matrix `A`.
`mb`	(integer)	The number of block rows of `A`.
`nb`	(integer)	The number of block columns of `A`.
`nnzb`	(integer)	number of nonzero blocks in the matrix `A`.
`bsrValA`	(pointer)	Points to the data array of length $nnzb * blockDim^2$ that holds all the elements of the nonzero blocks of `A`. The block elements are stored in either column-major order or row-major order.
`bsrRowPtrA`	(pointer)	Points to the integer array of length `mb` that holds indices into the arrays `bsrColIndA` and `bsrValA`; `bsrRowPtrA(i)` is the position of the first nonzero block of the `i`th block row in `bsrColIndA` and `bsrValA`.
`bsrEndPtrA`	(pointer)	Points to the integer array of length `mb` that holds indices into the arrays `bsrColIndA` and `bsrValA`; `bsrRowPtrA(i)` is the position next to the last nonzero block of the `i`th block row in `bsrColIndA` and `bsrValA`.
`bsrColIndA`	(pointer)	Points to the integer array of length `nnzb` that contains the column indices of the corresponding blocks in array `bsrValA`.

A simple conversion between BSR and BSRX can be done as follows. Suppose the developer has a `2×3` block sparse matrix A_b represented as shown.

$$A_b = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \end{bmatrix}$$

Assume it has this BSR format.

$$
\begin{aligned}
\text{bsrValA of BSR} &= [A_{00}\ \ A_{01}\ \ A_{10}\ \ A_{11}\ \ A_{12}] \\
\text{bsrRowPtrA of BSR} &= [0\quad 2\quad 5] \\
\text{bsrColIndA of BSR} &= [0\quad 1\quad 0\quad 1\quad 2]
\end{aligned}
$$

The `bsrRowPtrA` of the BSRX format is simply the first two elements of the `bsrRowPtrA` BSR format. The `bsrEndPtrA` of BSRX format is the last two elements of the `bsrRowPtrA` of BSR format.

$$
\begin{aligned}
\text{bsrRowPtrA of BSRX} &= [0\quad 2] \\
\text{bsrEndPtrA of BSRX} &= [2\quad 5]
\end{aligned}
$$

The advantage of the BSRX format is that the developer can specify a submatrix in the original BSR format by modifying `bsrRowPtrA` and `bsrEndPtrA` while keeping `bsrColIndA` and `bsrValA` unchanged.

For example, to create another block matrix $\tilde{A} = \begin{bmatrix} O & O & O \\ O & A_{11} & O \end{bmatrix}$ that is slightly different from A, the developer can keep `bsrColIndA` and `bsrValA`, but reconstruct $\tilde{A}$ by properly setting of `bsrRowPtrA` and `bsrEndPtrA`. The following 4-vector characterizes $\tilde{A}$.

$$
\begin{aligned}
\text{bsrValA of } \tilde{A} &= [A_{00}\ \ A_{01}\ \ A_{10}\ \ A_{11}\ \ A_{12}] \\
\text{bsrColIndA of } \tilde{A} &= [0\quad 1\quad 0\quad 1\quad 2] \\
\text{bsrRowPtrA of } \tilde{A} &= [0\quad 3] \\
\text{bsrEndPtrA of } \tilde{A} &= [0\quad 4]
\end{aligned}
$$

Chapter 4. cuSPARSE Types Reference

4.1. Data types

The `float`, `double`, `cuComplex`, and `cuDoubleComplex` data types are supported. The first two are standard C data types, while the last two are exported from `cuComplex.h`.

4.2. cusparseStatus_t

This data type represents the status returned by the library functions and it can have the following values

Value	Description
`CUSPARSE_STATUS_SUCCESS`	The operation completed successfully
`CUSPARSE_STATUS_NOT_INITIALIZED`	The cuSPARSE library was not initialized. This is usually caused by the lack of a prior call, an error in the CUDA Runtime API called by the cuSPARSE routine, or an error in the hardware setup **To correct:** call `cusparseCreate()` prior to the function call; and check that the hardware, an appropriate version of the driver, and the cuSPARSE library are correctly installed The error also applies to generic APIs (Generic APIs reference) for indicating a matrix/vector descriptor not initialized
`CUSPARSE_STATUS_ALLOC_FAILED`	Resource allocation failed inside the cuSPARSE library. This is usually caused by a device memory allocation (`cudaMalloc()`) or by a host memory allocation failure **To correct:** prior to the function call, deallocate previously allocated memory as much as possible
`CUSPARSE_STATUS_INVALID_VALUE`	An unsupported value or parameter was passed to the function (a negative vector size, for example)

Value	Description
	To correct: ensure that all the parameters being passed have valid values
CUSPARSE_STATUS_ARCH_MISMATCH	The function requires a feature absent from the device architecture **To correct:** compile and run the application on a device with appropriate compute capability
CUSPARSE_STATUS_EXECUTION_FAILED	The GPU program failed to execute. This is often caused by a launch failure of the kernel on the GPU, which can be caused by multiple reasons **To correct:** check that the hardware, an appropriate version of the driver, and the cuSPARSE library are correctly installed
CUSPARSE_STATUS_INTERNAL_ERROR	An internal cuSPARSE operation failed **To correct:** check that the hardware, an appropriate version of the driver, and the cuSPARSE library are correctly installed. Also, check that the memory passed as a parameter to the routine is not being deallocated prior to the routine completion
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	The matrix type is not supported by this function. This is usually caused by passing an invalid matrix descriptor to the function **To correct:** check that the fields in `cusparseMatDescr_t descrA` were set correctly
CUSPARSE_STATUS_NOT_SUPPORTED	The operation or data type combination is currently not supported by the function
CUSPARSE_STATUS_INSUFFICIENT_RESOURCES	The resources for the computation, such as GPU global or shared memory, are not sufficient to complete the operation. The error can also indicate that the current computation mode (e.g. bit size of sparse matrix indices) does not allow to handle the given input

4.3. cusparseHandle_t

This is a pointer type to an opaque cuSPARSE context, which the user must initialize by calling prior to calling `cusparseCreate()` any other library function. The handle created and returned by `cusparseCreate()` must be passed to every cuSPARSE function.

4.4. cusparsePointerMode_t

This type indicates whether the scalar values are passed by reference on the host or device. It is important to point out that if several scalar values are passed by reference in the function call, all of them will conform to the same single pointer mode. The pointer mode can be set and retrieved using `cusparseSetPointerMode()` and `cusparseGetPointerMode()` routines, respectively.

Value	Meaning
`CUSPARSE_POINTER_MODE_HOST`	the scalars are passed by reference on the host.
`CUSPARSE_POINTER_MODE_DEVICE`	the scalars are passed by reference on the device.

4.5. cusparseOperation_t

This type indicates which operations need to be performed with the sparse matrix.

Value	Meaning
`CUSPARSE_OPERATION_NON_TRANSPOSE`	the non-transpose operation is selected.
`CUSPARSE_OPERATION_TRANSPOSE`	the transpose operation is selected.
`CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE`	the conjugate transpose operation is selected.

4.6. cusparseAction_t

This type indicates whether the operation is performed only on indices or on data and indices.

Value	Meaning
`CUSPARSE_ACTION_SYMBOLIC`	the operation is performed only on indices.
`CUSPARSE_ACTION_NUMERIC`	the operation is performed on data and indices.

4.7. cusparseDirection_t

This type indicates whether the elements of a dense matrix should be parsed by rows or by columns (assuming column-major storage in memory of the dense matrix) in function cusparse[S|D|C|Z]nnz. Besides storage format of blocks in BSR format is also controlled by this type.

Value	Meaning
`CUSPARSE_DIRECTION_ROW`	the matrix should be parsed by rows.

Value	Meaning
CUSPARSE_DIRECTION_COLUMN	the matrix should be parsed by columns.

4.8. cusparseMatDescr_t

This structure is used to describe the shape and properties of a matrix.

```
typedef struct {
    cusparseMatrixType_t MatrixType;
    cusparseFillMode_t FillMode;
    cusparseDiagType_t DiagType;
    cusparseIndexBase_t IndexBase;
} cusparseMatDescr_t;
```

4.8.1. cusparseDiagType_t

This type indicates if the matrix diagonal entries are unity. The diagonal elements are always assumed to be present, but if `CUSPARSE_DIAG_TYPE_UNIT` is passed to an API routine, then the routine assumes that all diagonal entries are unity and will not read or modify those entries. Note that in this case the routine assumes the diagonal entries are equal to one, regardless of what those entries are actually set to in memory.

Value	Meaning
CUSPARSE_DIAG_TYPE_NON_UNIT	the matrix diagonal has non-unit elements.
CUSPARSE_DIAG_TYPE_UNIT	the matrix diagonal has unit elements.

4.8.2. cusparseFillMode_t

This type indicates if the lower or upper part of a matrix is stored in sparse storage.

Value	Meaning
CUSPARSE_FILL_MODE_LOWER	the lower triangular part is stored.
CUSPARSE_FILL_MODE_UPPER	the upper triangular part is stored.

4.8.3. cusparseIndexBase_t

This type indicates if the base of the matrix indices is zero or one.

Value	Meaning
CUSPARSE_INDEX_BASE_ZERO	the base index is zero.
CUSPARSE_INDEX_BASE_ONE	the base index is one.

4.8.4. cusparseMatrixType_t

This type indicates the type of matrix stored in sparse storage. Notice that for symmetric, Hermitian and triangular matrices only their lower or upper part is assumed to be stored.

The whole idea of matrix type and fill mode is to keep minimum storage for symmetric/Hermitian matrix, and also to take advantage of symmetric property on SpMV (Sparse Matrix Vector multiplication). To compute `y=A*x` when `A` is symmetric and only lower triangular part is stored, two steps are needed. First step is to compute `y=(L+D)*x` and second step is to compute `y=L^T*x + y`. Given the fact that the transpose operation `y=L^T*x` is 10x slower than non-transpose version `y=L*x`, the symmetric property does not show up any performance gain. It is better for the user to extend the symmetric matrix to a general matrix and apply `y=A*x` with matrix type `CUSPARSE_MATRIX_TYPE_GENERAL`.

In general, SpMV, preconditioners (incomplete Cholesky or incomplete LU) and triangular solver are combined together in iterative solvers, for example PCG and GMRES. If the user always uses general matrix (instead of symmetric matrix), there is no need to support other than general matrix in preconditioners. Therefore the new routines, `[bsr|csr]sv2` (triangular solver), `[bsr|csr]ilu02` (incomplete LU) and `[bsr|csr]ic02` (incomplete Cholesky), only support matrix type `CUSPARSE_MATRIX_TYPE_GENERAL`.

Value	Meaning
`CUSPARSE_MATRIX_TYPE_GENERAL`	the matrix is general.
`CUSPARSE_MATRIX_TYPE_SYMMETRIC`	the matrix is symmetric.
`CUSPARSE_MATRIX_TYPE_HERMITIAN`	the matrix is Hermitian.
`CUSPARSE_MATRIX_TYPE_TRIANGULAR`	the matrix is triangular.

4.9. cusparseColorInfo_t

This is a pointer type to an opaque structure holding the information used in `csrcolor()`.

4.10. cusparseSolvePolicy_t

This type indicates whether level information is generated and used in `csrsv2, csric02, csrilu02, bsrsv2, bsric02 and bsrilu02`.

Value	Meaning
`CUSPARSE_SOLVE_POLICY_NO_LEVEL`	no level information is generated and used.
`CUSPARSE_SOLVE_POLICY_USE_LEVEL`	generate and use level information.

4.11. bsric02Info_t

This is a pointer type to an opaque structure holding the information used in `bsric02_bufferSize()`, `bsric02_analysis()`, and `bsric02()`.

4.12. bsrilu02Info_t

This is a pointer type to an opaque structure holding the information used in `bsrilu02_bufferSize()`, `bsrilu02_analysis()`, and `bsrilu02()`.

4.13. bsrsm2Info_t

This is a pointer type to an opaque structure holding the information used in `bsrsm2_bufferSize()`, `bsrsm2_analysis()`, and `bsrsm2_solve()`.

4.14. bsrsv2Info_t

This is a pointer type to an opaque structure holding the information used in `bsrsv2_bufferSize()`, `bsrsv2_analysis()`, and `bsrsv2_solve()`.

4.15. csric02Info_t

This is a pointer type to an opaque structure holding the information used in `csric02_bufferSize()`, `csric02_analysis()`, and `csric02()`.

4.16. csrilu02Info_t

This is a pointer type to an opaque structure holding the information used in `csrilu02_bufferSize()`, `csrilu02_analysis()`, and `csrilu02()`.

Chapter 5. cuSPARSE Management Function Reference

The cuSPARSE functions for managing the library are described in this section.

5.1. cusparseCreate()

```
cusparseStatus_t
cusparseCreate(cusparseHandle_t *handle)
```

This function initializes the cuSPARSE library and creates a handle on the cuSPARSE context. It must be called before any other cuSPARSE API function is invoked. It allocates hardware resources necessary for accessing the GPU.

Param.	In/out	Meaning
`handle`	IN	The pointer to the handle to the cuSPARSE context

See `cusparseStatus_t` for the description of the return status

5.2. cusparseDestroy()

```
cusparseStatus_t
cusparseDestroy(cusparseHandle_t handle)
```

This function releases CPU-side resources used by the cuSPARSE library. The release of GPU-side resources may be deferred until the application shuts down.

Param.	In/out	Meaning
`handle`	IN	The handle to the cuSPARSE context

See `cusparseStatus_t` for the description of the return status

5.3. cusparseGetErrorName()

```
const char*
cusparseGetErrorString(cusparseStatus_t status)
```

The function returns the string representation of an error code enum name. If the error code is not recognized, "unrecognized error code" is returned.

Param.	In/out	Meaning
`status`	IN	Error code to convert to string
`const char*`	OUT	Pointer to a NULL-terminated string

5.4. cusparseGetErrorString()

```
const char*
cusparseGetErrorString(cusparseStatus_t status)
```

Returns the description string for an error code. If the error code is not recognized, "unrecognized error code" is returned.

Param.	In/out	Meaning
`status`	IN	Error code to convert to string
`const char*`	OUT	Pointer to a NULL-terminated string

5.5. cusparseGetProperty()

```
cusparseStatus_t
cusparseGetProperty(libraryPropertyType type,
                    int*                value)
```

The function returns the value of the requested property. Refer to `libraryPropertyType` for supported types.

Param.	In/out	Meaning
`type`	IN	Requested property
`value`	OUT	Value of the requested property

`libraryPropertyType` (defined in `library_types.h`):

Value	Meaning
`MAJOR_VERSION`	Enumerator to query the major version
`MINOR_VERSION`	Enumerator to query the minor version

Value	Meaning
PATCH_LEVEL	Number to identify the patch level

See cusparseStatus_t for the description of the return status

5.6. cusparseGetVersion()

```
cusparseStatus_t
cusparseGetVersion(cusparseHandle_t handle,
                   int*             version)
```

This function returns the version number of the cuSPARSE library.

Param.	In/out	Meaning
handle	IN	cuSPARSE handle
version	OUT	The version number of the library

See cusparseStatus_t for the description of the return status

5.7. cusparseGetPointerMode()

```
cusparseStatus_t
cusparseGetPointerMode(cusparseHandlet handle,
                       cusparsePointerMode_t *mode)
```

This function obtains the pointer mode used by the cuSPARSE library. Please see the section on the `cusparsePointerMode_t` type for more details.

Param.	In/out	Meaning
handle	IN	The handle to the cuSPARSE context
mode	OUT	One of the enumerated pointer mode types

See cusparseStatus_t for the description of the return status

5.8. cusparseSetPointerMode()

```
cusparseStatus_t
cusparseSetPointerMode(cusparseHandle_t handle,
                       cusparsePointerMode_t mode)
```

This function sets the pointer mode used by the cuSPARSE library. The *default* is for the values to be passed by reference on the host. Please see the section on the `cublasPointerMode_t` type for more details.

Param.	In/out	Meaning
handle	IN	The handle to the cuSPARSE context
mode	IN	One of the enumerated pointer mode types

See cusparseStatus_t for the description of the return status

5.9. cusparseGetStream()

```
cusparseStatus_t
cusparseGetStream(cusparseHandle_t handle, cudaStream_t *streamId)
```

This function gets the cuSPARSE library stream, which is being used to to execute all calls to the cuSPARSE library functions. If the cuSPARSE library stream is not set, all kernels use the default NULL stream.

Param.	In/out	Meaning
handle	IN	The handle to the cuSPARSE context
streamId	OUT	The stream used by the library

See cusparseStatus_t for the description of the return status

5.10. cusparseSetStream()

```
cusparseStatus_t
cusparseSetStream(cusparseHandle_t handle, cudaStream_t streamId)
```

This function sets the stream to be used by the cuSPARSE library to execute its routines.

Param.	In/out	Meaning
handle	IN	The handle to the cuSPARSE context
streamId	IN	The stream to be used by the library

See cusparseStatus_t for the description of the return status

Chapter 6. cuSPARSE Logging

cuSPARSE logging mechanism can be enabled by setting the following environment variables before launching the target application:

`CUSPARSE_LOG_LEVEL=<level>` - while level is one of the following levels:

- `0` - **Off** - logging is disabled (default)
- `1` - **Error** - only errors will be logged
- `2` - **Trace** - API calls that launch CUDA kernels will log their parameters and important information
- `3` - **Hints** - hints that can potentially improve the application's performance
- `4` - **Info** - provides general information about the library execution, may contain details about heuristic status
- `5` - **API Trace** - API calls will log their parameter and important information

`CUSPARSE_LOG_MASK=<mask>` - while mask is a combination of the following masks:

- `0` - **Off**
- `1` - **Error**
- `2` - **Trace**
- `4` - **Hints**
- `8` - **Info**
- `16` - **API Trace**

`CUSPARSE_LOG_FILE=<file_name>` - while file name is a path to a logging file. File name may contain %i, that will be replaced with the process id. E.g "<file_name>_%i.log".

If `CUSPARSE_LOG_FILE` is not defined, the log messages are printed to stdout.

Another option is to use the experimental cuSPARSE logging API. See:

- cusparseLoggerSetCallback()
- cusparseLoggerSetFile()

- cusparseLoggerOpenFile()
- cusparseLoggerSetLevel()
- cusparseLoggerSetMask()
- cusparseLoggerForceDisable()

Note: The logging mechanism is not available for the legacy APIs.

6.1. cusparseLoggerSetCallback()

```
cusparseStatus_t
cusparseLoggerSetCallback(cusparseLoggerCallback_t callback)
```

Experimental: The function sets the logging callback function.

Param.	In/out	Meaning
`callback`	IN	Pointer to a callback function

where `cusparseLoggerCallback_t` has the following signature:

```
void (*cusparseLoggerCallback_t)(int          logLevel,
                                 const char* functionName,
                                 const char* message)
```

Param.	In/out	Meaning
`logLevel`	IN	Selected log level
`functionName`	IN	The name of the API that logged this message
`message`	IN	The log message

See `cusparseStatus_t` for the description of the return status

6.2. cusparseLoggerSetFile()

```
cusparseStatus_t
cusparseLoggerSetFile(FILE* file)
```

Experimental: The function sets the logging output file. Note: once registered using this function call, the provided file handle must not be closed unless the function is called again to switch to a different file handle.

Param.	In/out	Meaning
`file`	IN	Pointer to an open file. File should have write permission

See cusparseStatus_t for the description of the return status

6.3. cusparseLoggerOpenFile()

```
cusparseStatus_t
cusparseLoggerOpenFile(const char* logFile)
```

Experimental: The function opens a logging output file in the given path.

Param.	In/out	Meaning
logFile	IN	Path of the logging output file

See cusparseStatus_t for the description of the return status

6.4. cusparseLoggerSetLevel()

```
cusparseStatus_t
cusparseLoggerSetLevel(int level)
```

Experimental: The function sets the value of the logging level. path.

Param.	In/out	Meaning
level	IN	Value of the logging level

See cusparseStatus_t for the description of the return status

6.5. cusparseLoggerSetMask()

```
cusparseStatus_t
cusparseLoggerSetMask(int mask)
```

Experimental: The function sets the value of the logging mask.

Param.	In/out	Meaning
mask	IN	Value of the logging mask

See cusparseStatus_t for the description of the return status

6.6. cublasLtLoggerForceDisable()

```
cusparseStatus_t
cublasLtLoggerForceDisable()
```

Experimental: The function disables logging for the entier run.

See `cusparseStatus_t` for the description of the return status

Chapter 7. cuSPARSE Helper Function Reference

The cuSPARSE helper functions are described in this section.

7.1. cusparseCreateColorInfo()

```
cusparseStatus_t
cusparseCreateColorInfo(cusparseColorInfo_t* info)
```

This function creates and initializes the `cusparseColorInfo_t` structure to *default* values.

Input

`info`	the pointer to the `cusparseColorInfo_t` structure

See `cusparseStatus_t` for the description of the return status

7.2. cusparseCreateMatDescr()

```
cusparseStatus_t
cusparseCreateMatDescr(cusparseMatDescr_t *descrA)
```

This function initializes the matrix descriptor. It sets the fields `MatrixType` and `IndexBase` to the *default* values `CUSPARSE_MATRIX_TYPE_GENERAL` and `CUSPARSE_INDEX_BASE_ZERO` , respectively, while leaving other fields uninitialized.

Input

`descrA`	the pointer to the matrix descriptor.

See `cusparseStatus_t` for the description of the return status

7.3. cusparseDestroyColorInfo()

```
cusparseStatus_t
cusparseDestroyColorInfo(cusparseColorInfo_t info)
```

This function destroys and releases any memory required by the structure.

Input

`info`	the pointer to the structure of `csrcolor()`

See `cusparseStatus_t` for the description of the return status

7.4. cusparseDestroyMatDescr()

```
cusparseStatus_t
cusparseDestroyMatDescr(cusparseMatDescr_t descrA)
```

This function releases the memory allocated for the matrix descriptor.

Input

`descrA`	the matrix descriptor.

See `cusparseStatus_t` for the description of the return status

7.5. cusparseGetMatDiagType()

```
cusparseDiagType_t
cusparseGetMatDiagType(const cusparseMatDescr_t descrA)
```

This function returns the `DiagType` field of the matrix descriptor `descrA`.

Input

`descrA`	the matrix descriptor.

Returned

	One of the enumerated diagType types.

7.6. cusparseGetMatFillMode()

```
cusparseFillMode_t
cusparseGetMatFillMode(const cusparseMatDescr_t descrA)
```

This function returns the `FillMode` field of the matrix descriptor `descrA`.

Input

`descrA`	the matrix descriptor.

Returned

	One of the enumerated fillMode types.

7.7. cusparseGetMatIndexBase()

```
cusparseIndexBase_t
cusparseGetMatIndexBase(const cusparseMatDescr_t descrA)
```

This function returns the `IndexBase` field of the matrix descriptor `descrA`.

Input

`descrA`	the matrix descriptor.

Returned

	One of the enumerated indexBase types.

7.8. cusparseGetMatType()

```
cusparseMatrixType_t
cusparseGetMatType(const cusparseMatDescr_t descrA)
```

This function returns the `MatrixType` field of the matrix descriptor `descrA`.

Input

`descrA`	the matrix descriptor.

Returned

	One of the enumerated matrix types.

7.9. cusparseSetMatDiagType()

```
cusparseStatus_t
cusparseSetMatDiagType(cusparseMatDescr_t descrA,
                       cusparseDiagType_t diagType)
```

This function sets the `DiagType` field of the matrix descriptor `descrA`.

Input

`diagType`	One of the enumerated diagType types.

Output

`descrA`	the matrix descriptor.

See `cusparseStatus_t` for the description of the return status

7.10. cusparseSetMatFillMode()

```
cusparseStatus_t
cusparseSetMatFillMode(cusparseMatDescr_t descrA,
                       cusparseFillMode_t fillMode)
```

This function sets the `FillMode` field of the matrix descriptor `descrA`.

Input

`fillMode`	One of the enumerated fillMode types.

Output

`descrA`	the matrix descriptor.

See `cusparseStatus_t` for the description of the return status

7.11. cusparseSetMatIndexBase()

```
cusparseStatus_t
cusparseSetMatIndexBase(cusparseMatDescr_t descrA,
                        cusparseIndexBase_t base)
```

This function sets the `IndexBase` field of the matrix descriptor `descrA`.

Input

`base`	One of the enumerated indexBase types.

Output

`descrA`	the matrix descriptor.

See `cusparseStatus_t` for the description of the return status

7.12. cusparseSetMatType()

```
cusparseStatus_t
cusparseSetMatType(cusparseMatDescr_t descrA, cusparseMatrixType_t type)
```

This function sets the `MatrixType` field of the matrix descriptor `descrA`.

Input

`type`	One of the enumerated matrix types.

Output

`descrA`	the matrix descriptor.

See cusparseStatus_t for the description of the return status

7.13. cusparseCreateCsric02Info()

```
cusparseStatus_t
cusparseCreateCsric02Info(csric02Info_t *info);
```

This function creates and initializes the solve and analysis structure of incomplete Cholesky to *default* values.

Input

info	the pointer to the solve and analysis structure of incomplete Cholesky.

See cusparseStatus_t for the description of the return status

7.14. cusparseDestroyCsric02Info()

```
cusparseStatus_t
cusparseDestroyCsric02Info(csric02Info_t info);
```

This function destroys and releases any memory required by the structure.

Input

info	the solve (csric02_solve) and analysis (csric02_analysis) structure.

See cusparseStatus_t for the description of the return status

7.15. cusparseCreateCsrilu02Info()

```
cusparseStatus_t
cusparseCreateCsrilu02Info(csrilu02Info_t *info);
```

This function creates and initializes the solve and analysis structure of incomplete LU to *default* values.

Input

info	the pointer to the solve and analysis structure of incomplete LU.

See cusparseStatus_t for the description of the return status

7.16. cusparseDestroyCsrilu02Info()

```
cusparseStatus_t
cusparseDestroyCsrilu02Info(csrilu02Info_t info);
```

This function destroys and releases any memory required by the structure.

Input

`info`	the solve `(csrilu02_solve)` and analysis `(csrilu02_analysis)` structure.

See `cusparseStatus_t` for the description of the return status

7.17. cusparseCreateBsrsv2Info()

```
cusparseStatus_t
cusparseCreateBsrsv2Info(bsrsv2Info_t *info);
```

This function creates and initializes the solve and analysis structure of bsrsv2 to *default* values.

Input

`info`	the pointer to the solve and analysis structure of bsrsv2.

See `cusparseStatus_t` for the description of the return status

7.18. cusparseDestroyBsrsv2Info()

```
cusparseStatus_t
cusparseDestroyBsrsv2Info(bsrsv2Info_t info);
```

This function destroys and releases any memory required by the structure.

Input

`info`	the solve `(bsrsv2_solve)` and analysis `(bsrsv2_analysis)` structure.

See `cusparseStatus_t` for the description of the return status

7.19. cusparseCreateBsrsm2Info()

```
cusparseStatus_t
cusparseCreateBsrsm2Info(bsrsm2Info_t *info);
```

This function creates and initializes the solve and analysis structure of bsrsm2 to *default* values.

Input

`info`	the pointer to the solve and analysis structure of bsrsm2.

See `cusparseStatus_t` for the description of the return status

7.20. cusparseDestroyBsrsm2Info()

```
cusparseStatus_t
cusparseDestroyBsrsm2Info(bsrsm2Info_t info);
```

This function destroys and releases any memory required by the structure.

Input

`info`	the solve `(bsrsm2_solve)` and analysis `(bsrsm2_analysis)` structure.

See `cusparseStatus_t` for the description of the return status

7.21. cusparseCreateBsric02Info()

```
cusparseStatus_t
cusparseCreateBsric02Info(bsric02Info_t *info);
```

This function creates and initializes the solve and analysis structure of block incomplete Cholesky to *default* values.

Input

`info`	the pointer to the solve and analysis structure of block incomplete Cholesky.

See `cusparseStatus_t` for the description of the return status

7.22. cusparseDestroyBsric02Info()

```
cusparseStatus_t
cusparseDestroyBsric02Info(bsric02Info_t info);
```

This function destroys and releases any memory required by the structure.

Input

`info`	the solve `(bsric02_solve)` and analysis `(bsric02_analysis)` structure.

See `cusparseStatus_t` for the description of the return status

7.23. cusparseCreateBsrilu02Info()

```
cusparseStatus_t
cusparseCreateBsrilu02Info(bsrilu02Info_t *info);
```

This function creates and initializes the solve and analysis structure of block incomplete LU to *default* values.

Input

`info`	the pointer to the solve and analysis structure of block incomplete LU.

See `cusparseStatus_t` for the description of the return status

7.24. cusparseDestroyBsrilu02Info()

```
cusparseStatus_t
cusparseDestroyBsrilu02Info(bsrilu02Info_t info);
```

This function destroys and releases any memory required by the structure.

Input

`info`	the solve `(bsrilu02_solve)` and analysis `(bsrilu02_analysis)` structure.

See `cusparseStatus_t` for the description of the return status

7.25. cusparseCreatePruneInfo()

```
cusparseStatus_t
cusparseCreatePruneInfo(pruneInfo_t *info);
```

This function creates and initializes structure of `prune` to *default* values.

Input

`info`	the pointer to the structure of `prune`.

See `cusparseStatus_t` for the description of the return status

7.26. cusparseDestroyPruneInfo()

```
cusparseStatus_t
cusparseDestroyPruneInfo(pruneInfo_t info);
```

This function destroys and releases any memory required by the structure.

Input

`info`	the structure of `prune`.

See `cusparseStatus_t` for the description of the return status

Chapter 8. cuSPARSE Level 2 Function Reference

This chapter describes the sparse linear algebra functions that perform operations between sparse matrices and dense vectors.

In particular, the solution of sparse triangular linear systems is implemented in two phases. First, during the analysis phase, the sparse triangular matrix is analyzed to determine the dependencies between its elements by calling the appropriate `csrsv2_analysis()` function. The analysis is specific to the sparsity pattern of the given matrix and to the selected `cusparseOperation_t` type. The information from the analysis phase is stored in the parameter of type `csrsv2Info_t` that has been initialized previously with a call to `cusparseCreateCsrsv2Info()`.

Second, during the solve phase, the given sparse triangular linear system is solved using the information stored in the `csrsv2Info_t` parameter by calling the appropriate `csrsv2_solve()` function. The solve phase may be performed multiple times with different right-hand sides, while the analysis phase needs to be performed only once. This is especially useful when a sparse triangular linear system must be solved for a set of different right-hand sides one at a time, while its coefficient matrix remains the same.

Finally, once all the solves have completed, the opaque data structure pointed to by the `csrsv2Info_t` parameter can be released by calling `cusparseDestroyCsrsv2Info()`

8.1. cusparse<t>bsrmv()

```
cusparseStatus_t
cusparseSbsrmv(cusparseHandle_t         handle,
               cusparseDirection_t      dir,
               cusparseOperation_t      trans,
               int                      mb,
               int                      nb,
               int                      nnzb,
               const float*             alpha,
               const cusparseMatDescr_t descr,
               const float*             bsrVal,
               const int*               bsrRowPtr,
               const int*               bsrColInd,
               int                      blockDim,
               const float*             x,
```

```
                 const float*              beta,
                 float*                    y)

cusparseStatus_t
cusparseDbsrmv(cusparseHandle_t          handle,
                 cusparseDirection_t       dir,
                 cusparseOperation_t       trans,
                 int                       mb,
                 int                       nb,
                 int                       nnzb,
                 const double*             alpha,
                 const cusparseMatDescr_t descr,
                 const double*             bsrVal,
                 const int*                bsrRowPtr,
                 const int*                bsrColInd,
                 int                       blockDim,
                 const double*             x,
                 const double*             beta,
                 double*                   y)

cusparseStatus_t
cusparseCbsrmv(cusparseHandle_t          handle,
                 cusparseDirection_t       dir,
                 cusparseOperation_t       trans,
                 int                       mb,
                 int                       nb,
                 int                       nnzb,
                 const cuComplex*          alpha,
                 const cusparseMatDescr_t descr,
                 const cuComplex*          bsrVal,
                 const int*                bsrRowPtr,
                 const int*                bsrColInd,
                 int                       blockDim,
                 const cuComplex*          x,
                 const cuComplex*          beta,
                 cuComplex*                y)

cusparseStatus_t
cusparseZbsrmv(cusparseHandle_t          handle,
                 cusparseDirection_t       dir,
                 cusparseOperation_t       trans,
                 int                       mb,
                 int                       nb,
                 int                       nnzb,
                 const cuDoubleComplex*    alpha,
                 const cusparseMatDescr_t descr,
                 const cuDoubleComplex*    bsrVal,
                 const int*                bsrRowPtr,
                 const int*                bsrColInd,
                 int                       blockDim,
                 const cuDoubleComplex*    x,
                 const cuDoubleComplex*    beta,
                 cuDoubleComplex*          y)
```

This function performs the matrix-vector operation

$$\mathrm{y} = \alpha * \mathrm{op}(A) * \mathrm{x} + \beta * \mathrm{y}$$

where A is an $(mb*blockDim)\times(nb*blockDim)$ sparse matrix that is defined in BSR storage format by the three arrays `bsrVal`, `bsrRowPtr`, and `bsrColInd`); `x` and `y` are vectors; α and β are scalars; and

$$\mathrm{op}(A)=\begin{cases} A & \text{if trans == CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans == CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans == CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

`bsrmv()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Several comments on `bsrmv()`:

- Only `blockDim > 1` is supported
- Only `CUSPARSE_OPERATION_NON_TRANSPOSE` is supported, that is

$$\mathrm{y}=\alpha*A*\mathrm{x}+\beta*\mathrm{y}$$

- Only `CUSPARSE_MATRIX_TYPE_GENERAL` is supported.
- The size of vector `x` should be $(nb*blockDim)$ at least, and the size of vector `y` should be $(mb*blockDim)$ at least; otherwise, the kernel may return `CUSPARSE_STATUS_EXECUTION_FAILED` because of an out-of-bounds array.

For example, suppose the user has a CSR format and wants to try `bsrmv()`, the following code demonstrates how to use `csr2bsr()` conversion and `bsrmv()` multiplication in single precision.

```
// Suppose that A is m x n sparse matrix represented by CSR format,
// hx is a host vector of size n, and hy is also a host vector of size m.
// m and n are not multiple of blockDim.
// step 1: transform CSR to BSR with column-major order
int base, nnz;
int nnzb;
cusparseDirection_t dirA = CUSPARSE_DIRECTION_COLUMN;
int mb = (m + blockDim-1)/blockDim;
int nb = (n + blockDim-1)/blockDim;
cudaMalloc((void**)&bsrRowPtrC, sizeof(int) *(mb+1));
cusparseXcsr2bsrNnz(handle, dirA, m, n,
        descrA, csrRowPtrA, csrColIndA, blockDim,
        descrC, bsrRowPtrC, &nnzb);
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzb);
cudaMalloc((void**)&bsrValC, sizeof(float)*(blockDim*blockDim)*nnzb);
cusparseScsr2bsr(handle, dirA, m, n,
        descrA, csrValA, csrRowPtrA, csrColIndA, blockDim,
        descrC, bsrValC, bsrRowPtrC, bsrColIndC);
// step 2: allocate vector x and vector y large enough for bsrmv
cudaMalloc((void**)&x, sizeof(float)*(nb*blockDim));
cudaMalloc((void**)&y, sizeof(float)*(mb*blockDim));
cudaMemcpy(x, hx, sizeof(float)*n, cudaMemcpyHostToDevice);
cudaMemcpy(y, hy, sizeof(float)*m, cudaMemcpyHostToDevice);
// step 3: perform bsrmv
cusparseSbsrmv(handle, dirA, transA, mb, nb, nnzb, &alpha,
   descrC, bsrValC, bsrRowPtrC, bsrColIndC, blockDim, x, &beta, y);
```

Input

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.
trans	the operation $\mathrm{op}(A)$. Only CUSPARSE_OPERATION_NON_TRANSPOSE is supported.
mb	number of block rows of matrix A.
nb	number of block columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
alpha	<type> scalar used for multiplication.
descr	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrVal	<type> array of nnz $($= csrRowPtrA(mb) $-$ csrRowPtrA(0) $)$ nonzero blocks of matrix A.
bsrRowPtr	integer array of mb $+1$ elements that contains the start of every block row and the end of the last block row plus one.
bsrColInd	integer array of nnz $($= csrRowPtrA(mb) $-$ csrRowPtrA(0) $)$ column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.
x	<type> vector of $nb * blockDim$ elements.
beta	<type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.
y	<type> vector of $mb * blockDim$ elements.

Output

y	<type> updated vector.

See cusparseStatus_t for the description of the return status.

8.2. cusparse<t>bsrxmv()

```
cusparseStatus_t
cusparseSbsrxmv(cusparseHandle_t         handle,
                cusparseDirection_t      dir,
                cusparseOperation_t      trans,
                int                      sizeOfMask,
                int                      mb,
```

```
                int                      nb,
                int                      nnzb,
                const float*             alpha,
                const cusparseMatDescr_t descr,
                const float*             bsrVal,
                const int*               bsrMaskPtr,
                const int*               bsrRowPtr,
                const int*               bsrEndPtr,
                const int*               bsrColInd,
                int                      blockDim,
                const float*             x,
                const float*             beta,
                float*                   y)

cusparseStatus_t
cusparseDbsrxmv(cusparseHandle_t         handle,
                cusparseDirection_t      dir,
                cusparseOperation_t      trans,
                int                      sizeOfMask,
                int                      mb,
                int                      nb,
                int                      nnzb,
                const double*            alpha,
                const cusparseMatDescr_t descr,
                const double*            bsrVal,
                const int*               bsrMaskPtr,
                const int*               bsrRowPtr,
                const int*               bsrEndPtr,
                const int*               bsrColInd,
                int                      blockDim,
                const double*            x,
                const double*            beta,
                double*                  y)

cusparseStatus_t
cusparseCbsrxmv(cusparseHandle_t         handle,
                cusparseDirection_t      dir,
                cusparseOperation_t      trans,
                int                      sizeOfMask,
                int                      mb,
                int                      nb,
                int                      nnzb,
                const cuComplex*         alpha,
                const cusparseMatDescr_t descr,
                const cuComplex*         bsrVal,
                const int*               bsrMaskPtr,
                const int*               bsrRowPtr,
                const int*               bsrEndPtr,
                const int*               bsrColInd,
                int                      blockDim,
                const cuComplex*         x,
                const cuComplex*         beta,
                cuComplex*               y)

cusparseStatus_t
cusparseZbsrxmv(cusparseHandle_t         handle,
                cusparseDirection_t      dir,
                cusparseOperation_t      trans,
                int                      sizeOfMask,
                int                      mb,
                int                      nb,
```

```
                 int                       nnzb,
                 const cuDoubleComplex*    alpha,
                 const cusparseMatDescr_t  descr,
                 const cuDoubleComplex*    bsrVal,
                 const int*                bsrMaskPtr,
                 const int*                bsrRowPtr,
                 const int*                bsrEndPtr,
                 const int*                bsrColInd,
                 int                       blockDim,
                 const cuDoubleComplex*    x,
                 const cuDoubleComplex*    beta,
                 cuDoubleComplex*          y)
```

This function performs a `bsrmv` and a mask operation

$$\mathrm{y(mask)} = (\alpha * \mathrm{op}(A) * \mathrm{x} + \beta * \mathrm{y})(\mathrm{mask})$$

where A is an $(mb*blockDim)\times(nb*blockDim)$ sparse matrix that is defined in BSRX storage format by the four arrays `bsrVal`, `bsrRowPtr`, `bsrEndPtr`, and `bsrColInd`); `x` and `y` are vectors; α and β are scalars; and

$$\mathrm{op}(A) = \begin{cases} A & \text{if trans == CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans == CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans == CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

The mask operation is defined by array `bsrMaskPtr` which contains updated block row indices of y. If row i is not specified in `bsrMaskPtr`, then `bsrxmv()` does not touch row block i of A and y.

For example, consider the 2×3 block matrix A:

$$A = \begin{bmatrix} A_{11} & A_{12} & O \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

and its one-based BSR format (three vector form) is

$$\begin{aligned} \text{bsrVal} &= [A_{11} \quad A_{12} \quad A_{21} \quad A_{22} \quad A_{23}] \\ \text{bsrRowPtr} &= [1 \quad 3 \quad 6] \\ \text{bsrColInd} &= [1 \quad 2 \quad 1 \quad 2 \quad 3] \end{aligned}$$

Suppose we want to do the following `bsrmv` operation on a matrix $\overline{A}$ which is slightly different from A.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} := alpha * \left(\tilde{A} = \begin{bmatrix} O & O & O \\ O & A_{22} & O \end{bmatrix}\right) * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ beta * y_2 \end{bmatrix}$$

We don't need to create another BSR format for the new matrix $\overline{A}$, all that we should do is to keep `bsrVal` and `bsrColInd` unchanged, but modify `bsrRowPtr` and add an additional array `bsrEndPtr` which points to the last nonzero elements per row of $\overline{A}$ plus 1.

For example, the following `bsrRowPtr` and `bsrEndPtr` can represent matrix $\overline{A}$:

$$\begin{aligned} \text{bsrRowPtr} &= [1 \quad 4] \\ \text{bsrEndPtr} &= [1 \quad 5] \end{aligned}$$

Further we can use a mask operator (specified by array `bsrMaskPtr`) to update particular block row indices of y only because y_1 is never changed. In this case, `bsrMaskPtr = [2]` and `sizeOfMask=1`.

The mask operator is equivalent to the following operation:

$$\begin{bmatrix} ? \\ y_2 \end{bmatrix} := alpha * \begin{bmatrix} ? & ? & ? \\ O & A_{22} & O \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + beta * \begin{bmatrix} ? \\ y_2 \end{bmatrix}$$

If a block row is not present in the `bsrMaskPtr`, then no calculation is performed on that row, and the corresponding value in `y` is unmodified. The question mark "?" is used to inidcate row blocks not in `bsrMaskPtr`.

In this case, first row block is not present in `bsrMaskPtr`, so `bsrRowPtr[0]` and `bsrEndPtr[0]` are not touched also.

$$\begin{array}{rl} \text{bsrRowPtr} = & [? \quad 4] \\ \text{bsrEndPtr} = & [? \quad 5] \end{array}$$

`bsrxmv()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

A couple of comments on `bsrxmv()`:

- Only `blockDim > 1` is supported
- Only `CUSPARSE_OPERATION_NON_TRANSPOSE` and `CUSPARSE_MATRIX_TYPE_GENERAL` are supported.
- Parameters `bsrMaskPtr`, `bsrRowPtr`, `bsrEndPtr` and `bsrColInd` are consistent with base index, either one-based or zero-based. The above example is one-based.

Input

`handle`	handle to the cuSPARSE library context.
`dir`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`trans`	the operation $\text{op}(A)$. Only `CUSPARSE_OPERATION_NON_TRANSPOSE` is supported.
`sizeOfMask`	number of updated block rows of y.
`mb`	number of block rows of matrix A.
`nb`	number of block columns of matrix A.
`nnzb`	number of nonzero blocks of matrix A.
`alpha`	<type> scalar used for multiplication.

descr	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrVal	<type> array of nnz nonzero blocks of matrix A.
bsrMaskPtr	integer array of sizeOfMask elements that contains the indices corresponding to updated block rows.
bsrRowPtr	integer array of mb elements that contains the start of every block row.
bsrEndPtr	integer array of mb elements that contains the end of the every block row plus one.
bsrColInd	integer array of nnzb column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.
x	<type> vector of $nb * blockDim$ elements.
beta	<type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.
y	<type> vector of $mb * blockDim$ elements.

See cusparseStatus_t for the description of the return status

8.3. cusparse<t>bsrsv2_bufferSize()

```
cusparseStatus_t
cusparseSbsrsv2_bufferSize(cusparseHandle_t         handle,
                           cusparseDirection_t      dirA,
                           cusparseOperation_t      transA,
                           int                      mb,
                           int                      nnzb,
                           const cusparseMatDescr_t descrA,
                           float*                   bsrValA,
                           const int*               bsrRowPtrA,
                           const int*               bsrColIndA,
                           int                      blockDim,
                           bsrsv2Info_t             info,
                           int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseDbsrsv2_bufferSize(cusparseHandle_t         handle,
                           cusparseDirection_t      dirA,
                           cusparseOperation_t      transA,
                           int                      mb,
                           int                      nnzb,
                           const cusparseMatDescr_t descrA,
                           double*                  bsrValA,
                           const int*               bsrRowPtrA,
                           const int*               bsrColIndA,
```

```
                                              int                       blockDim,
                                              bsrsv2Info_t              info,
                                              int*                      pBufferSizeInBytes)

cusparseStatus_t
cusparseCbsrsv2_bufferSize(cusparseHandle_t          handle,
                                              cusparseDirection_t       dirA,
                                              cusparseOperation_t       transA,
                                              int                       mb,
                                              int                       nnzb,
                                              const cusparseMatDescr_t descrA,
                                              cuComplex*                bsrValA,
                                              const int*                bsrRowPtrA,
                                              const int*                bsrColIndA,
                                              int                       blockDim,
                                              bsrsv2Info_t              info,
                                              int*                      pBufferSizeInBytes)

cusparseStatus_t
cusparseZbsrsv2_bufferSize(cusparseHandle_t          handle,
                                              cusparseDirection_t       dirA,
                                              cusparseOperation_t       transA,
                                              int                       mb,
                                              int                       nnzb,
                                              const cusparseMatDescr_t descrA,
                                              cuDoubleComplex*          bsrValA,
                                              const int*                bsrRowPtrA,
                                              const int*                bsrColIndA,
                                              int                       blockDim,
                                              bsrsv2Info_t              info,
                                              int*                      pBufferSizeInBytes)
```

This function returns size of the buffer used in `bsrsv2`, a new sparse triangular linear system `op(A)*y =` α `x`.

`A` is an `(mb*blockDim)x(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`); `x` and `y` are the right-hand-side and the solution vectors; α is a scalar; and

$$\text{op}(A) = \begin{cases} A & \text{if trans == CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans == CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans == CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

Although there are six combinations in terms of parameter `trans` and the upper (lower) triangular part of `A`, `bsrsv2_bufferSize()` returns the maximum size buffer among these combinations. The buffer size depends on the dimensions `mb`, `blockDim`, and the number of nonzero blocks of the matrix `nnzb`. If the user changes the matrix, it is necessary to call `bsrsv2_bufferSize()` again to have the correct buffer size; otherwise a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.

dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.
transA	the operation $\text{op}(A)$.
mb	number of block rows of matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
bsrValA	<type> array of nnzb $($= bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ nonzero blocks of matrix A.
bsrRowPtrA	integer array of mb $+1$ elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb $($= bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A; must be larger than zero.

Output

info	record of internal states based on different algorithms.
pBufferSizeInBytes	number of bytes of the buffer used in the bsrsv2_analysis() and bsrsv2_solve().

See cusparseStatus_t for the description of the return status.

8.4. cusparse<t>bsrsv2_analysis()

```
cusparseStatus_t
cusparseSbsrsv2_analysis(cusparseHandle_t         handle,
                         cusparseDirection_t      dirA,
                         cusparseOperation_t      transA,
                         int                      mb,
                         int                      nnzb,
                         const cusparseMatDescr_t descrA,
                         const float*             bsrValA,
                         const int*               bsrRowPtrA,
                         const int*               bsrColIndA,
                         int                      blockDim,
                         bsrsv2Info_t             info,
                         cusparseSolvePolicy_t    policy,
                         void*                    pBuffer)

cusparseStatus_t
```

```
cusparseDbsrsv2_analysis(cusparseHandle_t         handle,
                         cusparseDirection_t      dirA,
                         cusparseOperation_t      transA,
                         int                      mb,
                         int                      nnzb,
                         const cusparseMatDescr_t descrA,
                         const double*            bsrValA,
                         const int*               bsrRowPtrA,
                         const int*               bsrColIndA,
                         int                      blockDim,
                         bsrsv2Info_t             info,
                         cusparseSolvePolicy_t    policy,
                         void*                    pBuffer)

cusparseStatus_t
cusparseDbsrsv2_analysis(cusparseHandle_t         handle,
                         cusparseDirection_t      dirA,
                         cusparseOperation_t      transA,
                         int                      mb,
                         int                      nnzb,
                         const cusparseMatDescr_t descrA,
                         const cuComplex*         bsrValA,
                         const int*               bsrRowPtrA,
                         const int*               bsrColIndA,
                         int                      blockDim,
                         bsrsv2Info_t             info,
                         cusparseSolvePolicy_t    policy,
                         void*                    pBuffer)

cusparseStatus_t
cusparseZbsrsv2_analysis(cusparseHandle_t         handle,
                         cusparseDirection_t      dirA,
                         cusparseOperation_t      transA,
                         int                      mb,
                         int                      nnzb,
                         const cusparseMatDescr_t descrA,
                         const cuDoubleComplex*   bsrValA,
                         const int*               bsrRowPtrA,
                         const int*               bsrColIndA,
                         int                      blockDim,
                         bsrsv2Info_t             info,
                         cusparseSolvePolicy_t    policy,
                         void*                    pBuffer)
```

This function performs the analysis phase of `bsrsv2`, a new sparse triangular linear system `op(A)*y =` α `x`.

`A` is an `(mb*blockDim)x(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`); `x` and `y` are the right-hand side and the solution vectors; α is a scalar; and

$$\mathrm{op}(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

The block of BSR format is of size `blockDim*blockDim`, stored as column-major or row-major as determined by parameter `dirA`, which is either `CUSPARSE_DIRECTION_COLUMN` or `CUSPARSE_DIRECTION_ROW`. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, and the fill mode and diagonal type are ignored.

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires a buffer size returned by `bsrsv2_bufferSize()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `bsrsv2_analysis()` reports a structural zero and computes level information, which stored in the opaque structure `info`. The level information can extract more parallelism for a triangular solver. However `bsrsv2_solve()` can be done without level information. To disable level information, the user needs to specify the policy of the triangular solver as `CUSPARSE_SOLVE_POLICY_NO_LEVEL`.

Function `bsrsv2_analysis()` always reports the first structural zero, even when parameter `policy` is `CUSPARSE_SOLVE_POLICY_NO_LEVEL`. No structural zero is reported if `CUSPARSE_DIAG_TYPE_UNIT` is specified, even if block `A(j,j)` is missing for some `j`. The user needs to call `cusparseXbsrsv2_zeroPivot()` to know where the structural zero is.

It is the user's choice whether to call `bsrsv2_solve()` if `bsrsv2_analysis()` reports a structural zero. In this case, the user can still call `bsrsv2_solve()`, which will return a numerical zero at the same position as a structural zero. However the result `x` is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`transA`	the operation $\text{op}(A)$.
`mb`	number of block rows of matrix `A`.
`nnzb`	number of nonzero blocks of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`, while the supported diagonal types are `CUSPARSE_DIAG_TYPE_UNIT` and `CUSPARSE_DIAG_TYPE_NON_UNIT`.
`bsrValA`	<type> array of `nnzb` $($= `bsrRowPtrA(mb)` − `bsrRowPtrA(0)` $)$ nonzero blocks of matrix `A`.
`bsrRowPtrA`	integer array of `mb` $+1$ elements that contains the start of every block row and the end of the last block row plus one.
`bsrColIndA`	integer array of `nnzb` $($= `bsrRowPtrA(mb)` − `bsrRowPtrA(0)` $)$ column indices of the nonzero blocks of matrix `A`.

`blockDim`	block dimension of sparse matrix A, larger than zero.
`info`	structure initialized using `cusparseCreateBsrsv2Info()`.
`policy`	the supported policies are `CUSPARSE_SOLVE_POLICY_NO_LEVEL` and `CUSPARSE_SOLVE_POLICY_USE_LEVEL`.
`pBuffer`	buffer allocated by the user, the size is return by `bsrsv2_bufferSize()`.

Output

`info`	structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged).

See `cusparseStatus_t` for the description of the return status.

8.5. cusparse<t>bsrsv2_solve()

```
cusparseStatus_t
cusparseSbsrsv2_solve(cusparseHandle_t         handle,
                      cusparseDirection_t      dirA,
                      cusparseOperation_t      transA,
                      int                      mb,
                      int                      nnzb,
                      const float*             alpha,
                      const cusparseMatDescr_t descrA,
                      const float*             bsrValA,
                      const int*               bsrRowPtrA,
                      const int*               bsrColIndA,
                      int                      blockDim,
                      bsrsv2Info_t             info,
                      const float*             x,
                      float*                   y,
                      cusparseSolvePolicy_t    policy,
                      void*                    pBuffer)

cusparseStatus_t
cusparseDbsrsv2_solve(cusparseHandle_t         handle,
                      cusparseDirection_t      dirA,
                      cusparseOperation_t      transA,
                      int                      mb,
                      int                      nnzb,
                      const double*            alpha,
                      const cusparseMatDescr_t descrA,
                      const double*            bsrValA,
                      const int*               bsrRowPtrA,
                      const int*               bsrColIndA,
                      int                      blockDim,
                      bsrsv2Info_t             info,
                      const double*            x,
                      double*                  y,
                      cusparseSolvePolicy_t    policy,
```

```
                        void*                         pBuffer)

cusparseStatus_t
cusparseCbsrsv2_solve(cusparseHandle_t          handle,
                      cusparseDirection_t       dirA,
                      cusparseOperation_t       transA,
                      int                       mb,
                      int                       nnzb,
                      const cuComplex*          alpha,
                      const cusparseMatDescr_t  descrA,
                      const cuComplex*          bsrValA,
                      const int*                bsrRowPtrA,
                      const int*                bsrColIndA,
                      int                       blockDim,
                      bsrsv2Info_t              info,
                      const cuComplex*          x,
                      cuComplex*                y,
                      cusparseSolvePolicy_t     policy,
                      void*                     pBuffer)

cusparseStatus_t
cusparseZbsrsv2_solve(cusparseHandle_t          handle,
                      cusparseDirection_t       dirA,
                      cusparseOperation_t       transA,
                      int                       mb,
                      int                       nnzb,
                      const cuDoubleComplex*    alpha,
                      const cusparseMatDescr_t  descrA,
                      const cuDoubleComplex*    bsrValA,
                      const int*                bsrRowPtrA,
                      const int*                bsrColIndA,
                      int                       blockDim,
                      bsrsv2Info_t              info,
                      const cuDoubleComplex*    x,
                      cuDoubleComplex*          y,
                      cusparseSolvePolicy_t     policy,
                      void*                     pBuffer)
```

This function performs the solve phase of `bsrsv2`, a new sparse triangular linear system `op(A)*y =` α `x`.

`A` is an `(mb*blockDim)x(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`); `x` and `y` are the right-hand-side and the solution vectors; α is a scalar; and

$$\text{op}(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

The block in BSR format is of size `blockDim*blockDim`, stored as column-major or row-major as determined by parameter `dirA`, which is either `CUSPARSE_DIRECTION_COLUMN` or `CUSPARSE_DIRECTION_ROW`. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, and the fill mode and diagonal type are ignored. Function `bsrsv02_solve()` can support an arbitrary `blockDim`.

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires a buffer size returned by `bsrsv2_bufferSize()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Although `bsrsv2_solve()` can be done without level information, the user still needs to be aware of consistency. If `bsrsv2_analysis()` is called with policy `CUSPARSE_SOLVE_POLICY_USE_LEVEL`, `bsrsv2_solve()` can be run with or without levels. On the other hand, if `bsrsv2_analysis()` is called with `CUSPARSE_SOLVE_POLICY_NO_LEVEL`, `bsrsv2_solve()` can only accept `CUSPARSE_SOLVE_POLICY_NO_LEVEL`; otherwise, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

The level information may not improve the performance, but may spend extra time doing analysis. For example, a tridiagonal matrix has no parallelism. In this case, `CUSPARSE_SOLVE_POLICY_NO_LEVEL` performs better than `CUSPARSE_SOLVE_POLICY_USE_LEVEL`. If the user has an iterative solver, the best approach is to do `bsrsv2_analysis()` with `CUSPARSE_SOLVE_POLICY_USE_LEVEL` once. Then do `bsrsv2_solve()` with `CUSPARSE_SOLVE_POLICY_NO_LEVEL` in the first run, and with `CUSPARSE_SOLVE_POLICY_USE_LEVEL` in the second run, and pick the fastest one to perform the remaining iterations.

Function `bsrsv02_solve()` has the same behavior as `csrsv02_solve()`. That is, `bsr2csr(bsrsv02(A)) = csrsv02(bsr2csr(A))`. The numerical zero of `csrsv02_solve()` means there exists some zero `A(j,j)`. The numerical zero of `bsrsv02_solve()` means there exists some block `A(j,j)` that is not invertible.

Function `bsrsv2_solve()` reports the first numerical zero, including a structural zero. No numerical zero is reported if `CUSPARSE_DIAG_TYPE_UNIT` is specified, even if `A(j,j)` is not invertible for some `j`. The user needs to call `cusparseXbsrsv2_zeroPivot()` to know where the numerical zero is.

The function supports the following properties if `pBuffer != NULL`

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

For example, suppose L is a lower triangular matrix with unit diagonal, then the following code solves `L*y=x` by level information.

```
// Suppose that L is m x m sparse matrix represented by BSR format,
// The number of block rows/columns is mb, and
// the number of nonzero blocks is nnzb.
// L is lower triangular with unit diagonal.
// Assumption:
// - dimension of matrix L is m(=mb*blockDim),
// - matrix L has nnz(=nnzb*blockDim*blockDim) nonzero elements,
// - handle is already created by cusparseCreate(),
// - (d_bsrRowPtr, d_bsrColInd, d_bsrVal) is BSR of L on device memory,
// - d_x is right hand side vector on device memory.
// - d_y is solution vector on device memory.
// - d_x and d_y are of size m.
cusparseMatDescr_t descr = 0;
bsrsv2Info_t info = 0;
int pBufferSize;
void *pBuffer = 0;
```

```
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;

// step 1: create a descriptor which contains
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has unit diagonal, specified by parameter CUSPARSE_DIAG_TYPE_UNIT
//   (L may not have all diagonal elements.)
cusparseCreateMatDescr(&descr);
cusparseSetMatIndexBase(descr, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatFillMode(descr, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr, CUSPARSE_DIAG_TYPE_UNIT);

// step 2: create a empty info structure
cusparseCreateBsrsv2Info(&info);

// step 3: query how much memory used in bsrsv2, and allocate the buffer
cusparseDbsrsv2_bufferSize(handle, dir, trans, mb, nnzb, descr,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, &pBufferSize);

// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis
cusparseDbsrsv2_analysis(handle, dir, trans, mb, nnzb, descr,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim,
    info, policy, pBuffer);
// L has unit diagonal, so no structural zero is reported.
status = cusparseXbsrsv2_zeroPivot(handle, info, &structural_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
   printf("L(%d,%d) is missing\n", structural_zero, structural_zero);
}

// step 5: solve L*y = x
cusparseDbsrsv2_solve(handle, dir, trans, mb, nnzb, &alpha, descr,
   d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info,
   d_x, d_y, policy, pBuffer);
// L has unit diagonal, so no numerical zero is reported.
status = cusparseXbsrsv2_zeroPivot(handle, info, &numerical_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
   printf("L(%d,%d) is zero\n", numerical_zero, numerical_zero);
}

// step 6: free resources
cudaFree(pBuffer);
cusparseDestroyBsrsv2Info(info);
cusparseDestroyMatDescr(descr);
cusparseDestroy(handle);
```

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`transA`	the operation $\text{op}(A)$.
`mb`	number of block rows and block columns of matrix `A`.
`alpha`	<type> scalar used for multiplication.

descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
bsrValA	<type> array of nnzb $($= bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ nonzero blocks of matrix A.
bsrRowPtrA	integer array of mb $+1$ elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb $($= bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
x	<type> right-hand-side vector of size m.
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user, the size is returned by bsrsv2_bufferSize().

Output

y	<type> solution vector of size m.

See cusparseStatus_t for the description of the return status.

8.6. cusparseXbsrsv2_zeroPivot()

```
cusparseStatus_t
cusparseXbsrsv2_zeroPivot(cusparseHandle_t handle,
                          bsrsv2Info_t     info,
                          int*             position)
```

If the returned error code is CUSPARSE_STATUS_ZERO_PIVOT, position=j means A(j,j) is either structural zero or numerical zero (singular block). Otherwise position=-1.

The position can be 0-based or 1-based, the same as the matrix.

Function cusparseXbsrsv2_zeroPivot() is a blocking call. It calls cudaDeviceSynchronize() to make sure all previous kernels are done.

The position can be in the host memory or device memory. The user can set the proper mode with cusparseSetPointerMode().

- The routine requires no extra storage
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`info`	`info` contains a structural zero or numerical zero if the user already called `bsrsv2_analysis()` or `bsrsv2_solve()`.

Output

`position`	if no structural or numerical zero, `position` is -1; otherwise if `A(j,j)` is missing or `U(j,j)` is zero, `position=j`.

See `cusparseStatus_t` for the description of the return status

8.7. cusparse<t>gemvi()

```
cusparseStatus_t
cusparseSgemvi_bufferSize(cusparseHandle_t    handle,
                          cusparseOperation_t transA,
                          int                 m,
                          int                 n,
                          int                 nnz,
                          int*                pBufferSize)

cusparseStatus_t
cusparseDgemvi_bufferSize(cusparseHandle_t    handle,
                          cusparseOperation_t transA,
                          int                 m,
                          int                 n,
                          int                 nnz,
                          int*                pBufferSize)

cusparseStatus_t
cusparseCgemvi_bufferSize(cusparseHandle_t    handle,
                          cusparseOperation_t transA,
                          int                 m,
                          int                 n,
                          int                 nnz,
                          int*                pBufferSize)

cusparseStatus_t
cusparseZgemvi_bufferSize(cusparseHandle_t    handle,
                          cusparseOperation_t transA,
                          int                 m,
                          int                 n,
                          int                 nnz,
```

```
                           int*                    pBufferSize)
```

```
cusparseStatus_t
cusparseSgemvi(cusparseHandle_t     handle,
               cusparseOperation_t  transA,
               int                  m,
               int                  n,
               const float*         alpha,
               const float*         A,
               int                  lda,
               int                  nnz,
               const float*         x,
               const int*           xInd,
               const float*         beta,
               float*               y,
               cusparseIndexBase_t  idxBase,
               void*                pBuffer)

cusparseStatus_t
cusparseDgemvi(cusparseHandle_t     handle,
               cusparseOperation_t  transA,
               int                  m,
               int                  n,
               const double*        alpha,
               const double*        A,
               int                  lda,
               int                  nnz,
               const double*        x,
               const int*           xInd,
               const double*        beta,
               double*              y,
               cusparseIndexBase_t  idxBase,
               void*                pBuffer)

cusparseStatus_t
cusparseCgemvi(cusparseHandle_t     handle,
               cusparseOperation_t  transA,
               int                  m,
               int                  n,
               const cuComplex*     alpha,
               const cuComplex*     A,
               int                  lda,
               int                  nnz,
               const cuComplex*     x,
               const int*           xInd,
               const cuComplex*     beta,
               cuComplex*           y,
               cusparseIndexBase_t  idxBase,
               void*                pBuffer)

cusparseStatus_t
cusparseZgemvi(cusparseHandle_t       handle,
               cusparseOperation_t    transA,
               int                    m,
               int                    n,
               const cuDoubleComplex* alpha,
               const cuDoubleComplex* A,
               int                    lda,
               int                    nnz,
               const cuDoubleComplex* x,
```

```
                   const int*             xInd,
                   const cuDoubleComplex* beta,
                   cuDoubleComplex*       y,
                   cusparseIndexBase_t    idxBase,
                   void*                  pBuffer)
```

This function performs the matrix-vector operation

$$y = \alpha * \mathrm{op}(A) * x + \beta * y$$

`A` is an `m×n` dense matrix and a sparse vector x that is defined in a sparse storage format by the two arrays `xVal`, `xInd` of length `nnz`, and `y` is a dense vector; α and β are scalars; and

$$\mathrm{op}(A) = \begin{cases} A & \text{if trans == CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans == CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans == CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

To simplify the implementation, we have not (yet) optimized the transpose multiple case. We recommend the following for users interested in this case.

1. Convert the matrix from CSR to CSC format using one of the `csr2csc()` functions. Notice that by interchanging the rows and columns of the result you are implicitly transposing the matrix.

2. Call the `gemvi()` function with the `cusparseOperation_t` parameter set to `CUSPARSE_OPERATION_NON_TRANSPOSE` and with the interchanged rows and columns of the matrix stored in CSC format. This (implicitly) multiplies the vector by the transpose of the matrix in the original CSR format.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

The function `cusparse<t>gemvi_bufferSize()` returns size of buffer used in `cusparse<t>gemvi()`

Input

`handle`	handle to the cuSPARSE library context.
`trans`	the operation $\mathrm{op}(A)$.
`m`	number of rows of matrix `A`.
`n`	number of columns of matrix `A`.
`alpha`	<type> scalar used for multiplication.
`A`	the pointer to dense matrix `A`.
`lda`	size of the leading dimension of `A`.
`nnz`	number of nonzero elements of vector `x`.
`x`	<type> sparse vector of `nnz` elements of size `n` if $\mathrm{op}(A) = A$, and size `m` if $\mathrm{op}(A) = A^T$ or $\mathrm{op}(A) = A^H$
`xInd`	Indices of non-zero values in `x`
`beta`	<type> scalar used for multiplication. If `beta` is zero, `y` does not have to be a valid input.

`y`	<type> dense vector of `m` elements if $\text{op}(A) = A$, and `n` elements if $\text{op}(A) = A^T$ or $\text{op}(A) = A^H$
`idxBase`	0 or 1, for 0 based or 1 based indexing, respectively
`pBufferSize`	number of elements needed the buffer used in `cusparse<t>gemvi()`.
`pBuffer`	working space buffer

Output

`y`	<type> updated dense vector.

See `cusparseStatus_t` for the description of the return status

Chapter 9. cuSPARSE Level 3 Function Reference

This chapter describes sparse linear algebra functions that perform operations between sparse and (usually tall) dense matrices.

In particular, the solution of sparse triangular linear systems with multiple right-hand sides is implemented in two phases. First, during the analysis phase, the sparse triangular matrix is analyzed to determine the dependencies between its elements by calling the appropriate `csrsm2_analysis()` function. The analysis is specific to the sparsity pattern of the given matrix and to the selected `cusparseOperation_t` type. The information from the analysis phase is stored in the parameter of type `csrsm2Info_t` that has been initialized previously with a call to `cusparseCreateCsrsm2Info()`.

Second, during the solve phase, the given sparse triangular linear system is solved using the information stored in the `csrsm2Info_t` parameter by calling the appropriate `csrsm2_solve()` function. The solve phase may be performed multiple times with different multiple right-hand sides, while the analysis phase needs to be performed only once. This is especially useful when a sparse triangular linear system must be solved for different sets of multiple right-hand sides one at a time, while its coefficient matrix remains the same.

Finally, once all the solves have completed, the opaque data structure pointed to by the `csrsm2Info_t` parameter can be released by calling `cusparseDestroyCsrsm2Info()`.

9.1. cusparse<t>bsrmm()

```
cusparseStatus_t
cusparseSbsrmm(cusparseHandle_t         handle,
               cusparseDirection_t      dirA,
               cusparseOperation_t      transA,
               cusparseOperation_t      transB,
               int                      mb,
               int                      n,
               int                      kb,
               int                      nnzb,
               const float*             alpha,
               const cusparseMatDescr_t descrA,
               const float*             bsrValA,
               const int*               bsrRowPtrA,
               const int*               bsrColIndA,
```

```
                int                         blockDim,
                const float*                B,
                int                         ldb,
                const float*                beta,
                float*                      C,
                int                         ldc)

cusparseStatus_t
cusparseDbsrmm(cusparseHandle_t             handle,
               cusparseDirection_t          dirA,
               cusparseOperation_t          transA,
               cusparseOperation_t          transB,
               int                          mb,
               int                          n,
               int                          kb,
               int                          nnzb,
               const double*                alpha,
               const cusparseMatDescr_t     descrA,
               const double*                bsrValA,
               const int*                   bsrRowPtrA,
               const int*                   bsrColIndA,
               int                          blockDim,
               const double*                B,
               int                          ldb,
               const double*                beta,
               double*                      C,
               int                          ldc)

cusparseStatus_t
cusparseCbsrmm(cusparseHandle_t             handle,
               cusparseDirection_t          dirA,
               cusparseOperation_t          transA,
               cusparseOperation_t          transB,
               int                          mb,
               int                          n,
               int                          kb,
               int                          nnzb,
               const cuComplex*             alpha,
               const cusparseMatDescr_t     descrA,
               const cuComplex*             bsrValA,
               const int*                   bsrRowPtrA,
               const int*                   bsrColIndA,
               int                          blockDim,
               const cuComplex*             B,
               int                          ldb,
               const cuComplex*             beta,
               cuComplex*                   C,
               int                          ldc)

cusparseStatus_t
cusparseZbsrmm(cusparseHandle_t             handle,
               cusparseDirection_t          dirA,
               cusparseOperation_t          transA,
               cusparseOperation_t          transB,
               int                          mb,
               int                          n,
               int                          kb,
               int                          nnzb,
               const cuDoubleComplex*       alpha,
               const cusparseMatDescr_t     descrA,
               const cuDoubleComplex*       bsrValA,
```

```
                    const int*                  bsrRowPtrA,
                    const int*                  bsrColIndA,
                    int                         blockDim,
                    const cuDoubleComplex*      B,
                    int                         ldb,
                    const cuDoubleComplex*      beta,
                    cuDoubleComplex*            C,
                    int                         ldc)
```

This function performs one of the following matrix-matrix operations:

$$C = \alpha * \mathrm{op}(A) * \mathrm{op}(B) + \beta * C$$

`A` is an `mb×kb` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`; `B` and `C` are dense matrices; α and β are scalars; and

$$\mathrm{op}(A) = \begin{cases} A & \text{if transA == CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if transA == CUSPARSE\_OPERATION\_TRANSPOSE (not supported)} \\ A^H & \text{if transA == CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE (not supported)} \end{cases}$$

and

$$\mathrm{op}(B) = \begin{cases} B & \text{if transB == CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ B^T & \text{if transB == CUSPARSE\_OPERATION\_TRANSPOSE} \\ B^H & \text{if transB == CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE (not supported)} \end{cases}$$

The function has the following limitations:

- Only `CUSPARSE_MATRIX_TYPE_GENERAL` matrix type is supported
- Only `blockDim > 1` is supported
- if `blockDim` ≤ 4, then max(mb)/max(n) = 524,272
- if 4 < `blockDim` ≤ 8, then max(mb) = 524,272, max(n) = 262,136
- if `blockDim` > 8, then m < 65,535 and max(n) = 262,136

The motivation of `transpose(B)` is to improve memory access of matrix `B`. The computational pattern of `A*transpose(B)` with matrix `B` in column-major order is equivalent to `A*B` with matrix `B` in row-major order.

In practice, no operation in an iterative solver or eigenvalue solver uses `A*transpose(B)`. However, we can perform `A*transpose(transpose(B))` which is the same as `A*B`. For example, suppose `A` is `mb*kb`, `B` is `k*n` and `C` is `m*n`, the following code shows usage of `cusparseDbsrmm()`.

```
// A is mb*kb, B is k*n and C is m*n
    const int m = mb*blockSize;
    const int k = kb*blockSize;
    const int ldb_B = k; // leading dimension of B
    const int ldc   = m; // leading dimension of C
// perform C:=alpha*A*B + beta*C
    cusparseSetMatType(descrA, CUSPARSE_MATRIX_TYPE_GENERAL );
    cusparseDbsrmm(cusparse_handle,
               CUSPARSE_DIRECTION_COLUMN,
               CUSPARSE_OPERATION_NON_TRANSPOSE,
               CUSPARSE_OPERATION_NON_TRANSPOSE,
```

```
                mb, n, kb, nnzb, alpha,
                descrA, bsrValA, bsrRowPtrA, bsrColIndA, blockSize,
                B, ldb_B,
                beta, C, ldc);
```

Instead of using `A*B`, our proposal is to transpose `B` to `Bt` by first calling `cublas<t>geam()`, and then to perform `A*transpose(Bt)`.

```
// step 1: Bt := transpose(B)
    const int m = mb*blockSize;
    const int k = kb*blockSize;
    double *Bt;
    const int ldb_Bt = n; // leading dimension of Bt
    cudaMalloc((void**)&Bt, sizeof(double)*ldb_Bt*k);
    double one  = 1.0;
    double zero = 0.0;
    cublasSetPointerMode(cublas_handle, CUBLAS_POINTER_MODE_HOST);
    cublasDgeam(cublas_handle, CUBLAS_OP_T, CUBLAS_OP_T,
        n, k, &one, B, int ldb_B, &zero, B, int ldb_B, Bt, ldb_Bt);

// step 2: perform C:=alpha*A*transpose(Bt) + beta*C
    cusparseDbsrmm(cusparse_handle,
                CUSPARSE_DIRECTION_COLUMN,
                CUSPARSE_OPERATION_NON_TRANSPOSE,
                CUSPARSE_OPERATION_TRANSPOSE,
                mb, n, kb, nnzb, alpha,
                descrA, bsrValA, bsrRowPtrA, bsrColIndA, blockSize,
                Bt, ldb_Bt,
                beta, C, ldc);
```

`bsrmm()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`dir`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`transA`	the operation `op(A)`.
`transB`	the operation `op(B)`.
`mb`	number of block rows of sparse matrix `A`.
`n`	number of columns of dense matrix `op(B)` and `A`.
`kb`	number of block columns of sparse matrix `A`.
`nnzb`	number of non-zero blocks of sparse matrix `A`.
`alpha`	<type> scalar used for multiplication.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are

	CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<type> array of nnzb $(=$ bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ nonzero blocks of matrix A.
bsrRowPtrA	integer array of mb $+1$ elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb $(=$ bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.
B	array of dimensions (ldb, n) if op(B)=B and (ldb, k) otherwise.
ldb	leading dimension of B. If op(B)=B, it must be at least $\max(1, k)$ If op(B) != B, it must be at least max(1, n).
beta	<type> scalar used for multiplication. If beta is zero, C does not have to be a valid input.
C	array of dimensions (ldc, n).
ldc	leading dimension of C. It must be at least $\max(1, m)$ if op(A)=A and at least $\max(1, k)$ otherwise.

Output

C	<type> updated array of dimensions (ldc, n).

See cusparseStatus_t for the description of the return status

9.2. cusparse<t>bsrsm2_bufferSize()

```
cusparseStatus_t
cusparseSbsrsm2_bufferSize(cusparseHandle_t         handle,
                           cusparseDirection_t      dirA,
                           cusparseOperation_t      transA,
                           cusparseOperation_t      transX,
                           int                      mb,
                           int                      n,
                           int                      nnzb,
                           const cusparseMatDescr_t descrA,
                           float*                   bsrSortedValA,
                           const int*               bsrSortedRowPtrA,
                           const int*               bsrSortedColIndA,
                           int                      blockDim,
                           bsrsm2Info_t             info,
                           int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseDbsrsm2_bufferSize(cusparseHandle_t         handle,
```

```
                             cusparseDirection_t       dirA,
                             cusparseOperation_t       transA,
                             cusparseOperation_t       transX,
                             int                       mb,
                             int                       n,
                             int                       nnzb,
                             const cusparseMatDescr_t descrA,
                             double*                   bsrSortedValA,
                             const int*                bsrSortedRowPtrA,
                             const int*                bsrSortedColIndA,
                             int                       blockDim,
                             bsrsm2Info_t              info,
                             int*                      pBufferSizeInBytes)

cusparseStatus_t
cusparseCbsrsm2_bufferSize(cusparseHandle_t          handle,
                             cusparseDirection_t       dirA,
                             cusparseOperation_t       transA,
                             cusparseOperation_t       transX,
                             int                       mb,
                             int                       n,
                             int                       nnzb,
                             const cusparseMatDescr_t descrA,
                             cuComplex*                bsrSortedValA,
                             const int*                bsrSortedRowPtrA,
                             const int*                bsrSortedColIndA,
                             int                       blockDim,
                             bsrsm2Info_t              info,
                             int*                      pBufferSizeInBytes)

cusparseStatus_t
cusparseZbsrsm2_bufferSize(cusparseHandle_t          handle,
                             cusparseDirection_t       dirA,
                             cusparseOperation_t       transA,
                             cusparseOperation_t       transX,
                             int                       mb,
                             int                       n,
                             int                       nnzb,
                             const cusparseMatDescr_t descrA,
                             cuDoubleComplex*          bsrSortedValA,
                             const int*                bsrSortedRowPtrA,
                             const int*                bsrSortedColIndA,
                             int                       blockDim,
                             bsrsm2Info_t              info,
                             int*                      pBufferSizeInBytes)
```

This function returns size of buffer used in `bsrsm2()`, a new sparse triangular linear system `op(A)*op(X)=` α `op(B)`.

`A` is an `(mb*blockDim)x(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`); `B` and `X` are the right-hand-side and the solution matrices; α is a scalar; and

$$\text{op}(A) = \begin{cases} A & \text{if trans == CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans == CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans == CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

Although there are six combinations in terms of parameter `trans` and the upper (and lower) triangular part of `A`, `bsrsm2_bufferSize()` returns the maximum size of the buffer among these combinations. The buffer size depends on dimension `mb,blockDim` and the number of nonzeros of the matrix, `nnzb`. If the user changes the matrix, it is necessary to call `bsrsm2_bufferSize()` again to get the correct buffer size, otherwise a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`transA`	the operation `op(A)`.
`transX`	the operation `op(X)`.
`mb`	number of block rows of matrix `A`.
`n`	number of columns of matrix `op(B)` and `op(X)`.
`nnzb`	number of nonzero blocks of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`, while the supported diagonal types are `CUSPARSE_DIAG_TYPE_UNIT` and `CUSPARSE_DIAG_TYPE_NON_UNIT`.
`bsrValA`	<type> array of `nnzb` (= `bsrRowPtrA(mb)` − `bsrRowPtrA(0)`) nonzero blocks of matrix `A`.
`bsrRowPtrA`	integer array of `mb` $+1$ elements that contains the start of every block row and the end of the last block row plus one.
`bsrColIndA`	integer array of `nnzb` (= `bsrRowPtrA(mb)` − `bsrRowPtrA(0)`) column indices of the nonzero blocks of matrix `A`.
`blockDim`	block dimension of sparse matrix `A`; larger than zero.

Output

`info`	record internal states based on different algorithms.
`pBufferSizeInBytes`	number of bytes of the buffer used in `bsrsm2_analysis()` and `bsrsm2_solve()`.

See `cusparseStatus_t` for the description of the return status

9.3. cusparse<t>bsrsm2_analysis()

```
cusparseStatus_t
cusparseSbsrsm2_analysis(cusparseHandle_t         handle,
                         cusparseDirection_t      dirA,
                         cusparseOperation_t      transA,
                         cusparseOperation_t      transX,
                         int                      mb,
                         int                      n,
                         int                      nnzb,
                         const cusparseMatDescr_t descrA,
                         const float*             bsrSortedVal,
                         const int*               bsrSortedRowPtr,
                         const int*               bsrSortedColInd,
                         int                      blockDim,
                         bsrsm2Info_t             info,
                         cusparseSolvePolicy_t    policy,
                         void*                    pBuffer)

cusparseStatus_t
cusparseDbsrsm2_analysis(cusparseHandle_t         handle,
                         cusparseDirection_t      dirA,
                         cusparseOperation_t      transA,
                         cusparseOperation_t      transX,
                         int                      mb,
                         int                      n,
                         int                      nnzb,
                         const cusparseMatDescr_t descrA,
                         const double*            bsrSortedVal,
                         const int*               bsrSortedRowPtr,
                         const int*               bsrSortedColInd,
                         int                      blockDim,
                         bsrsm2Info_t             info,
                         cusparseSolvePolicy_t    policy,
                         void*                    pBuffer)

cusparseStatus_t
cusparseCbsrsm2_analysis(cusparseHandle_t         handle,
                         cusparseDirection_t      dirA,
                         cusparseOperation_t      transA,
                         cusparseOperation_t      transX,
                         int                      mb,
                         int                      n,
                         int                      nnzb,
                         const cusparseMatDescr_t descrA,
                         const cuComplex*         bsrSortedVal,
                         const int*               bsrSortedRowPtr,
                         const int*               bsrSortedColInd,
                         int                      blockDim,
                         bsrsm2Info_t             info,
                         cusparseSolvePolicy_t    policy,
                         void*                    pBuffer)

cusparseStatus_t
cusparseZbsrsm2_analysis(cusparseHandle_t         handle,
                         cusparseDirection_t      dirA,
                         cusparseOperation_t      transA,
```

```
                            cusparseOperation_t      transX,
                            int                      mb,
                            int                      n,
                            int                      nnzb,
                            const cusparseMatDescr_t descrA,
                            const cuDoubleComplex*   bsrSortedVal,
                            const int*               bsrSortedRowPtr,
                            const int*               bsrSortedColInd,
                            int                      blockDim,
                            bsrsm2Info_t             info,
                            cusparseSolvePolicy_t    policy,
                            void*                    pBuffer)
```

This function performs the analysis phase of `bsrsm2()`, a new sparse triangular linear system `op(A)*op(X) =` α `op(B)`.

`A` is an `(mb*blockDim)x(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`); `B` and `X` are the right-hand-side and the solution matrices; α is a scalar; and

$$\text{op}(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

and

$$\text{op}(X) = \begin{cases} X & \text{if transX} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ X^T & \text{if transX} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ X^H & \text{if transX} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE (not supported)} \end{cases}$$

and `op(B)` and `op(X)` are equal.

The block of BSR format is of size `blockDim*blockDim`, stored in column-major or row-major as determined by parameter `dirA`, which is either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, and the fill mode and diagonal type are ignored.

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires the buffer size returned by `bsrsm2_bufferSize()`. The address of `pBuffer` must be multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `bsrsm2_analysis()` reports a structural zero and computes the level information stored in opaque structure `info`. The level information can extract more parallelism during a triangular solver. However `bsrsm2_solve()` can be done without level information. To disable level information, the user needs to specify the policy of the triangular solver as `CUSPARSE_SOLVE_POLICY_NO_LEVEL`.

Function `bsrsm2_analysis()` always reports the first structural zero, even if the parameter `policy` is `CUSPARSE_SOLVE_POLICY_NO_LEVEL`. Besides, no structural zero is reported if `CUSPARSE_DIAG_TYPE_UNIT` is specified, even if block `A(j,j)` is missing for some `j`. The user must call `cusparseXbsrsm2_query_zero_pivot()` to know where the structural zero is.

If `bsrsm2_analysis()` reports a structural zero, the solve will return a numerical zero in the same position as the structural zero but this result `X` is meaningless.

- This function requires temporary extra storage that is allocated internally

- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`transA`	the operation `op(A)`.
`transX`	the operation `op(B)` and `op(X)`.
`mb`	number of block rows of matrix `A`.
`n`	number of columns of matrix `op(B)` and `op(X)`.
`nnzb`	number of non-zero blocks of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`, while the supported diagonal types are `CUSPARSE_DIAG_TYPE_UNIT` and `CUSPARSE_DIAG_TYPE_NON_UNIT`.
`bsrValA`	<type> array of `nnzb` ($=$ `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)`) nonzero blocks of matrix `A`.
`bsrRowPtrA`	integer array of `mb` $+1$ elements that contains the start of every block row and the end of the last block row plus one.
`bsrColIndA`	integer array of `nnzb` ($=$ `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)`) column indices of the nonzero blocks of matrix `A`.
`blockDim`	block dimension of sparse matrix `A`; larger than zero.
`info`	structure initialized using `cusparseCreateBsrsm2Info`.
`policy`	The supported policies are `CUSPARSE_SOLVE_POLICY_NO_LEVEL` and `CUSPARSE_SOLVE_POLICY_USE_LEVEL`.
`pBuffer`	buffer allocated by the user; the size is return by `bsrsm2_bufferSize()`.

Output

`info`	structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged).

See `cusparseStatus_t` for the description of the return status

9.4. cusparse<t>bsrsm2_solve()

```
cusparseStatus_t
cusparseSbsrsm2_solve(cusparseHandle_t         handle,
                      cusparseDirection_t      dirA,
                      cusparseOperation_t      transA,
                      cusparseOperation_t      transX,
                      int                      mb,
                      int                      n,
                      int                      nnzb,
                      const float*             alpha,
                      const cusparseMatDescr_t descrA,
                      const float*             bsrSortedVal,
                      const int*               bsrSortedRowPtr,
                      const int*               bsrSortedColInd,
                      int                      blockDim,
                      bsrsm2Info_t             info,
                      const float*             B,
                      int                      ldb,
                      float*                   X,
                      int                      ldx,
                      cusparseSolvePolicy_t    policy,
                      void*                    pBuffer)

cusparseStatus_t
cusparseDbsrsm2_solve(cusparseHandle_t         handle,
                      cusparseDirection_t      dirA,
                      cusparseOperation_t      transA,
                      cusparseOperation_t      transX,
                      int                      mb,
                      int                      n,
                      int                      nnzb,
                      const double*            alpha,
                      const cusparseMatDescr_t descrA,
                      const double*            bsrSortedVal,
                      const int*               bsrSortedRowPtr,
                      const int*               bsrSortedColInd,
                      int                      blockDim,
                      bsrsm2Info_t             info,
                      const double*            B,
                      int                      ldb,
                      double*                  X,
                      int                      ldx,
                      cusparseSolvePolicy_t    policy,
                      void*                    pBuffer)

cusparseStatus_t
cusparseCbsrsm2_solve(cusparseHandle_t         handle,
                      cusparseDirection_t      dirA,
                      cusparseOperation_t      transA,
                      cusparseOperation_t      transX,
                      int                      mb,
                      int                      n,
                      int                      nnzb,
                      const cuComplex*         alpha,
                      const cusparseMatDescr_t descrA,
                      const cuComplex*         bsrSortedVal,
```

```
                      const int*               bsrSortedRowPtr,
                      const int*               bsrSortedColInd,
                      int                      blockDim,
                      bsrsm2Info_t             info,
                      const cuComplex*         B,
                      int                      ldb,
                      cuComplex*               X,
                      int                      ldx,
                      cusparseSolvePolicy_t    policy,
                      void*                    pBuffer)

cusparseStatus_t
cusparseZbsrsm2_solve(cusparseHandle_t         handle,
                      cusparseDirection_t      dirA,
                      cusparseOperation_t      transA,
                      cusparseOperation_t      transX,
                      int                      mb,
                      int                      n,
                      int                      nnzb,
                      const cuDoubleComplex*   alpha,
                      const cusparseMatDescr_t descrA,
                      const cuDoubleComplex*   bsrSortedVal,
                      const int*               bsrSortedRowPtr,
                      const int*               bsrSortedColInd,
                      int                      blockDim,
                      bsrsm2Info_t             info,
                      const cuDoubleComplex*   B,
                      int                      ldb,
                      cuDoubleComplex*         X,
                      int                      ldx,
                      cusparseSolvePolicy_t    policy,
                      void*                    pBuffer)
```

This function performs the solve phase of the solution of a sparse triangular linear system:

$$\mathrm{op}(A) * \mathrm{op}(\mathrm{X}) = \alpha * \mathrm{op}(\mathrm{B})$$

`A` is an `(mb*blockDim)x(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`); `B` and `X` are the right-hand-side and the solution matrices; α is a scalar, and

$$\mathrm{op}(A) = \begin{cases} A & \text{if transA} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if transA} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if transA} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

and

$$\mathrm{op}(X) = \begin{cases} X & \text{if transX} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ X^T & \text{if transX} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ X^H & \text{not supported} \end{cases}$$

Only `op(A)=A` is supported.

`op(B)` and `op(X)` must be performed in the same way. In other words, if `op(B)=B`, `op(X)=X`.

The block of BSR format is of size `blockDim*blockDim`, stored as column-major or row-major as determined by parameter `dirA`, which is either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, and the fill mode and diagonal type are ignored. Function `bsrsm02_solve()` can support an arbitrary `blockDim`.

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires the buffer size returned by `bsrsm2_bufferSize()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Although `bsrsm2_solve()` can be done without level information, the user still needs to be aware of consistency. If `bsrsm2_analysis()` is called with policy `CUSPARSE_SOLVE_POLICY_USE_LEVEL`, `bsrsm2_solve()` can be run with or without levels. On the other hand, if `bsrsm2_analysis()` is called with `CUSPARSE_SOLVE_POLICY_NO_LEVEL`, `bsrsm2_solve()` can only accept `CUSPARSE_SOLVE_POLICY_NO_LEVEL`; otherwise, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `bsrsm02_solve()` has the same behavior as `bsrsv02_solve()`, reporting the first numerical zero, including a structural zero. The user must call `cusparseXbsrsm2_query_zero_pivot()` to know where the numerical zero is.

The motivation of `transpose(X)` is to improve the memory access of matrix `X`. The computational pattern of `transpose(X)` with matrix `X` in column-major order is equivalent to `X` with matrix `X` in row-major order.

In-place is supported and requires that `B` and `X` point to the same memory block, and `ldb=ldx`.

The function supports the following properties if `pBuffer != NULL`:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`transA`	the operation `op(A)`.
`transX`	the operation `op(B)` and `op(X)`.
`mb`	number of block rows of matrix `A`.
`n`	number of columns of matrix `op(B)` and `op(X)`.
`nnzb`	number of non-zero blocks of matrix `A`.
`alpha`	<type> scalar used for multiplication.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`, while the supported diagonal types are `CUSPARSE_DIAG_TYPE_UNIT` and `CUSPARSE_DIAG_TYPE_NON_UNIT`.
`bsrValA`	<type> array of `nnzb` ($=$ `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)`) non-zero blocks of matrix `A`.

bsrRowPtrA	integer array of mb $+1$ elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb ($=$ bsrRowPtrA(mb) $-$ bsrRowPtrA(0)) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A; larger than zero.
info	structure initialized using cusparseCreateBsrsm2Info().
B	<type> right-hand-side array.
ldb	leading dimension of B. If op(B)=B, ldb >= (mb*blockDim); otherwise, ldb >= n.
ldx	leading dimension of X. If op(X)=X, then ldx >= (mb*blockDim). otherwise ldx >= n.
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is returned by bsrsm2_bufferSize().

Output

X	<type> solution array with leading dimensions ldx.

See cusparseStatus_t for the description of the return status.

9.5. cusparseXbsrsm2_zeroPivot()

```
cusparseStatus_t
cusparseXbsrsm2_zeroPivot(cusparseHandle_t handle,
                          bsrsm2Info_t     info,
                          int*             position)
```

If the returned error code is CUSPARSE_STATUS_ZERO_PIVOT, position=j means A(j,j) is either a structural zero or a numerical zero (singular block). Otherwise position=-1.

The position can be 0-base or 1-base, the same as the matrix.

Function cusparseXbsrsm2_zeroPivot() is a blocking call. It calls cudaDeviceSynchronize() to make sure all previous kernels are done.

The position can be in the host memory or device memory. The user can set the proper mode with cusparseSetPointerMode().

- The routine requires no extra storage

- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`info`	`info` contains a structural zero or a numerical zero if the user already called `bsrsm2_analysis()` or `bsrsm2_solve()`.

Output

`position`	if no structural or numerical zero, `position` is -1; otherwise, if `A(j,j)` is missing or `U(j,j)` is zero, `position=j`.

See `cusparseStatus_t` for the description of the return status.

Chapter 10. cuSPARSE Extra Function Reference

This chapter describes the extra routines used to manipulate sparse matrices.

10.1. cusparse<t>csrgeam2()

```
cusparseStatus_t
cusparseScsrgeam2_bufferSizeExt(cusparseHandle_t         handle,
                                int                      m,
                                int                      n,
                                const float*             alpha,
                                const cusparseMatDescr_t descrA,
                                int                      nnzA,
                                const float*             csrSortedValA,
                                const int*               csrSortedRowPtrA,
                                const int*               csrSortedColIndA,
                                const float*             beta,
                                const cusparseMatDescr_t descrB,
                                int                      nnzB,
                                const float*             csrSortedValB,
                                const int*               csrSortedRowPtrB,
                                const int*               csrSortedColIndB,
                                const cusparseMatDescr_t descrC,
                                const float*             csrSortedValC,
                                const int*               csrSortedRowPtrC,
                                const int*               csrSortedColIndC,
                                size_t*                  pBufferSizeInBytes)

cusparseStatus_t
cusparseDcsrgeam2_bufferSizeExt(cusparseHandle_t         handle,
                                int                      m,
                                int                      n,
                                const double*            alpha,
                                const cusparseMatDescr_t descrA,
                                int                      nnzA,
                                const double*            csrSortedValA,
                                const int*               csrSortedRowPtrA,
                                const int*               csrSortedColIndA,
                                const double*            beta,
                                const cusparseMatDescr_t descrB,
                                int                      nnzB,
                                const double*            csrSortedValB,
```

```
                              const int*             csrSortedRowPtrB,
                              const int*             csrSortedColIndB,
                              const cusparseMatDescr_t descrC,
                              const double*          csrSortedValC,
                              const int*             csrSortedRowPtrC,
                              const int*             csrSortedColIndC,
                              size_t*                pBufferSizeInBytes)

cusparseStatus_t
cusparseCcsrgeam2_bufferSizeExt(cusparseHandle_t         handle,
                              int                    m,
                              int                    n,
                              const cuComplex*       alpha,
                              const cusparseMatDescr_t descrA,
                              int                    nnzA,
                              const cuComplex*       csrSortedValA,
                              const int*             csrSortedRowPtrA,
                              const int*             csrSortedColIndA,
                              const cuComplex*       beta,
                              const cusparseMatDescr_t descrB,
                              int                    nnzB,
                              const cuComplex*       csrSortedValB,
                              const int*             csrSortedRowPtrB,
                              const int*             csrSortedColIndB,
                              const cusparseMatDescr_t descrC,
                              const cuComplex*       csrSortedValC,
                              const int*             csrSortedRowPtrC,
                              const int*             csrSortedColIndC,
                              size_t*                pBufferSizeInBytes)

cusparseStatus_t
cusparseZcsrgeam2_bufferSizeExt(cusparseHandle_t         handle,
                              int                    m,
                              int                    n,
                              const cuDoubleComplex*   alpha,
                              const cusparseMatDescr_t descrA,
                              int                    nnzA,
                              const cuDoubleComplex*   csrSortedValA,
                              const int*             csrSortedRowPtrA,
                              const int*             csrSortedColIndA,
                              const cuDoubleComplex*   beta,
                              const cusparseMatDescr_t descrB,
                              int                    nnzB,
                              const cuDoubleComplex*   csrSortedValB,
                              const int*             csrSortedRowPtrB,
                              const int*             csrSortedColIndB,
                              const cusparseMatDescr_t descrC,
                              const cuDoubleComplex*   csrSortedValC,
                              const int*             csrSortedRowPtrC,
                              const int*             csrSortedColIndC,
                              size_t*                pBufferSizeInBytes)

cusparseStatus_t
cusparseXcsrgeam2Nnz(cusparseHandle_t         handle,
                    int                      m,
                    int                      n,
                    const cusparseMatDescr_t descrA,
                    int                      nnzA,
                    const int*               csrSortedRowPtrA,
                    const int*               csrSortedColIndA,
                    const cusparseMatDescr_t descrB,
```

```
                    int                       nnzB,
                    const int*                csrSortedRowPtrB,
                    const int*                csrSortedColIndB,
                    const cusparseMatDescr_t descrC,
                    int*                      csrSortedRowPtrC,
                    int*                      nnzTotalDevHostPtr,
                    void*                     workspace)
```

```
cusparseStatus_t
cusparseScsrgeam2(cusparseHandle_t         handle,
                  int                      m,
                  int                      n,
                  const float*             alpha,
                  const cusparseMatDescr_t descrA,
                  int                      nnzA,
                  const float*             csrSortedValA,
                  const int*               csrSortedRowPtrA,
                  const int*               csrSortedColIndA,
                  const float*             beta,
                  const cusparseMatDescr_t descrB,
                  int                      nnzB,
                  const float*             csrSortedValB,
                  const int*               csrSortedRowPtrB,
                  const int*               csrSortedColIndB,
                  const cusparseMatDescr_t descrC,
                  float*                   csrSortedValC,
                  int*                     csrSortedRowPtrC,
                  int*                     csrSortedColIndC,
                  void*                    pBuffer)

cusparseStatus_t
cusparseDcsrgeam2(cusparseHandle_t         handle,
                  int                      m,
                  int                      n,
                  const double*            alpha,
                  const cusparseMatDescr_t descrA,
                  int                      nnzA,
                  const double*            csrSortedValA,
                  const int*               csrSortedRowPtrA,
                  const int*               csrSortedColIndA,
                  const double*            beta,
                  const cusparseMatDescr_t descrB,
                  int                      nnzB,
                  const double*            csrSortedValB,
                  const int*               csrSortedRowPtrB,
                  const int*               csrSortedColIndB,
                  const cusparseMatDescr_t descrC,
                  double*                  csrSortedValC,
                  int*                     csrSortedRowPtrC,
                  int*                     csrSortedColIndC,
                  void*                    pBuffer)

cusparseStatus_t
cusparseCcsrgeam2(cusparseHandle_t         handle,
                  int                      m,
                  int                      n,
                  const cuComplex*         alpha,
                  const cusparseMatDescr_t descrA,
                  int                      nnzA,
                  const cuComplex*         csrSortedValA,
```

```
                    const int*                  csrSortedRowPtrA,
                    const int*                  csrSortedColIndA,
                    const cuComplex*            beta,
                    const cusparseMatDescr_t descrB,
                    int                         nnzB,
                    const cuComplex*            csrSortedValB,
                    const int*                  csrSortedRowPtrB,
                    const int*                  csrSortedColIndB,
                    const cusparseMatDescr_t descrC,
                    cuComplex*                  csrSortedValC,
                    int*                        csrSortedRowPtrC,
                    int*                        csrSortedColIndC,
                    void*                       pBuffer)

cusparseStatus_t
cusparseZcsrgeam2(cusparseHandle_t              handle,
                    int                         m,
                    int                         n,
                    const cuDoubleComplex*      alpha,
                    const cusparseMatDescr_t descrA,
                    int                         nnzA,
                    const cuDoubleComplex*      csrSortedValA,
                    const int*                  csrSortedRowPtrA,
                    const int*                  csrSortedColIndA,
                    const cuDoubleComplex*      beta,
                    const cusparseMatDescr_t descrB,
                    int                         nnzB,
                    const cuDoubleComplex*      csrSortedValB,
                    const int*                  csrSortedRowPtrB,
                    const int*                  csrSortedColIndB,
                    const cusparseMatDescr_t descrC,
                    cuDoubleComplex*            csrSortedValC,
                    int*                        csrSortedRowPtrC,
                    int*                        csrSortedColIndC,
                    void*                       pBuffer)
```

This function performs following matrix-matrix operation

$$C = \alpha * A + \beta * B$$

where `A`, `B`, and `C` are `m×n` sparse matrices (defined in CSR storage format by the three arrays `csrValA|csrValB|csrValC`, `csrRowPtrA|csrRowPtrB|csrRowPtrC`, and `csrColIndA|csrColIndB|csrcolIndC` respectively), and α and β are scalars. Since `A` and `B` have different sparsity patterns, cuSPARSE adopts a two-step approach to complete sparse matrix `C`. In the first step, the user allocates `csrRowPtrC` of `m+1`elements and uses function `cusparseXcsrgeam2Nnz()` to determine `csrRowPtrC` and the total number of nonzero elements. In the second step, the user gathers `nnzC` (number of nonzero elements of matrix `C`) from either `(nnzC=*nnzTotalDevHostPtr)` or `(nnzC=csrRowPtrC(m)-csrRowPtrC(0))` and allocates `csrValC`, `csrColIndC` of `nnzC` elements respectively, then finally calls function `cusparse[S|D|C|Z]csrgeam2()` to complete matrix `C`.

The general procedure is as follows:

```
int baseC, nnzC;
/* alpha, nnzTotalDevHostPtr points to host memory */
size_t BufferSizeInBytes;
char *buffer = NULL;
int *nnzTotalDevHostPtr = &nnzC;
cusparseSetPointerMode(handle, CUSPARSE_POINTER_MODE_HOST);
```

```
cudaMalloc((void**)&csrRowPtrC, sizeof(int)*(m+1));
/* prepare buffer */
cusparseScsrgeam2_bufferSizeExt(handle, m, n,
    alpha,
    descrA, nnzA,
    csrValA, csrRowPtrA, csrColIndA,
    beta,
    descrB, nnzB,
    csrValB, csrRowPtrB, csrColIndB,
    descrC,
    csrValC, csrRowPtrC, csrColIndC
    &bufferSizeInBytes
    );
cudaMalloc((void**)&buffer, sizeof(char)*bufferSizeInBytes);
cusparseXcsrgeam2Nnz(handle, m, n,
        descrA, nnzA, csrRowPtrA, csrColIndA,
        descrB, nnzB, csrRowPtrB, csrColIndB,
        descrC, csrRowPtrC, nnzTotalDevHostPtr,
        buffer);
if (NULL != nnzTotalDevHostPtr){
    nnzC = *nnzTotalDevHostPtr;
}else{
    cudaMemcpy(&nnzC, csrRowPtrC+m, sizeof(int), cudaMemcpyDeviceToHost);
    cudaMemcpy(&baseC, csrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
    nnzC -= baseC;
}
cudaMalloc((void**)&csrColIndC, sizeof(int)*nnzC);
cudaMalloc((void**)&csrValC, sizeof(float)*nnzC);
cusparseScsrgeam2(handle, m, n,
        alpha,
        descrA, nnzA,
        csrValA, csrRowPtrA, csrColIndA,
        beta,
        descrB, nnzB,
        csrValB, csrRowPtrB, csrColIndB,
        descrC,
        csrValC, csrRowPtrC, csrColIndC
        buffer);
```

Several comments on `csrgeam2()`:

- The other three combinations, NT, TN, and TT, are not supported by cuSPARSE. In order to do any one of the three, the user should use the routine `csr2csc()` to convert $A|B$ to $A^T|B^T$.
- Only `CUSPARSE_MATRIX_TYPE_GENERAL` is supported. If either `A` or `B` is symmetric or Hermitian, then the user must extend the matrix to a full one and reconfigure the `MatrixType` field of the descriptor to `CUSPARSE_MATRIX_TYPE_GENERAL`.
- If the sparsity pattern of matrix `C` is known, the user can skip the call to function `cusparseXcsrgeam2Nnz()`. For example, suppose that the user has an iterative algorithm which would update `A` and `B` iteratively but keep the sparsity patterns. The user can call function `cusparseXcsrgeam2Nnz()` once to set up the sparsity pattern of `C`, then call function `cusparse[S|D|C|Z]geam()` only for each iteration.
- The pointers `alpha` and `beta` must be valid.
- When `alpha` or `beta` is zero, it is not considered a special case by cuSPARSE. The sparsity pattern of `C` is independent of the value of `alpha` and `beta`. If the user wants $C = 0 \times A + 1 \times B^T$, then `csr2csc()` is better than `csrgeam2()`.

- `csrgeam2()` is the same as `csrgeam()` except `csrgeam2()` needs explicit buffer where `csrgeam()` allocates the buffer internally.
- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`m`	number of rows of sparse matrix `A,B,C`.
`n`	number of columns of sparse matrix `A,B,C`.
`alpha`	<type> scalar used for multiplication.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL` only.
`nnzA`	number of nonzero elements of sparse matrix `A`.
`csrValA`	<type> array of `nnzA` $($= `csrRowPtrA(m)` − `csrRowPtrA(0)` $)$ nonzero elements of matrix `A`.
`csrRowPtrA`	integer array of `m` $+1$ elements that contains the start of every row and the end of the last row plus one.
`csrColIndA`	integer array of `nnzA` $($= `csrRowPtrA(m)` − `csrRowPtrA(0)` $)$ column indices of the nonzero elements of matrix `A`.
`beta`	<type> scalar used for multiplication. If `beta` is zero, `y` does not have to be a valid input.
`descrB`	the descriptor of matrix `B`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL` only.
`nnzB`	number of nonzero elements of sparse matrix `B`.
`csrValB`	<type> array of `nnzB` $($= `csrRowPtrB(m)` − `csrRowPtrB(0)` $)$ nonzero elements of matrix `B`.
`csrRowPtrB`	integer array of `m` $+1$ elements that contains the start of every row and the end of the last row plus one.
`csrColIndB`	integer array of `nnzB` $($= `csrRowPtrB(m)` − `csrRowPtrB(0)` $)$ column indices of the nonzero elements of matrix `B`.
`descrC`	the descriptor of matrix `C`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL` only.

Output

`csrValC`	<type> array of `nnzC` $($= `csrRowPtrC(m)` − `csrRowPtrC(0)` $)$ nonzero elements of matrix `C`.

`csrRowPtrC`	integer array of `m` $+1$ elements that contains the start of every row and the end of the last row plus one.
`csrColIndC`	integer array of `nnzC` $(=$ `csrRowPtrC(m)` - `csrRowPtrC(0)` $)$ column indices of the nonzero elements of matrix`C`.
`nnzTotalDevHostPtr`	total number of nonzero elements in device or host memory. It is equal to `(csrRowPtrC(m)-csrRowPtrC(0))`.

See `cusparseStatus_t` for the description of the return status

Chapter 11. cuSPARSE Preconditioners Reference

This chapter describes the routines that implement different preconditioners.

11.1. Incomplete Cholesky Factorization: level 0

Different algorithms for ic0 are discussed in this section.

11.1.1. cusparse<t>csric02_bufferSize()

```
cusparseStatus_t
cusparseScsric02_bufferSize(cusparseHandle_t         handle,
                            int                      m,
                            int                      nnz,
                            const cusparseMatDescr_t descrA,
                            float*                   csrValA,
                            const int*               csrRowPtrA,
                            const int*               csrColIndA,
                            csric02Info_t            info,
                            int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseDcsric02_bufferSize(cusparseHandle_t         handle,
                            int                      m,
                            int                      nnz,
                            const cusparseMatDescr_t descrA,
                            double*                  csrValA,
                            const int*               csrRowPtrA,
                            const int*               csrColIndA,
                            csric02Info_t            info,
                            int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseCcsric02_bufferSize(cusparseHandle_t         handle,
                            int                      m,
                            int                      nnz,
                            const cusparseMatDescr_t descrA,
                            cuComplex*               csrValA,
```

```
                              const int*                   csrRowPtrA,
                              const int*                   csrColIndA,
                              csric02Info_t                info,
                              int*                         pBufferSizeInBytes)

cusparseStatus_t
cusparseZcsric02_bufferSize(cusparseHandle_t         handle,
                              int                          m,
                              int                          nnz,
                              const cusparseMatDescr_t descrA,
                              cuDoubleComplex*             csrValA,
                              const int*                   csrRowPtrA,
                              const int*                   csrColIndA,
                              csric02Info_t                info,
                              int*                         pBufferSizeInBytes)
```

This function returns size of buffer used in computing the incomplete-Cholesky factorization with 0 fill-in and no pivoting:

$$A \approx LL^H$$

`A` is an `m×m` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`.

The buffer size depends on dimension `m` and `nnz`, the number of nonzeros of the matrix. If the user changes the matrix, it is necessary to call `csric02_bufferSize()` again to have the correct buffer size; otherwise, a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`m`	number of rows and columns of matrix `A`.
`nnz`	number of nonzeros of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrValA`	<type> array of `nnz` $($= `csrRowPtrA(m)` − `csrRowPtrA(0)` $)$ nonzero elements of matrix `A`.
`csrRowPtrA`	integer array of `m` $+1$ elements that contains the start of every row and the end of the last row plus one.
`csrColIndA`	integer array of `nnz` $($= `csrRowPtrA(m)` − `csrRowPtrA(0)` $)$ column indices of the nonzero elements of matrix `A`.

Output

info	record internal states based on different algorithms
pBufferSizeInBytes	number of bytes of the buffer used in csric02_analysis() and csric02()

See cusparseStatus_t for the description of the return status.

11.1.2. cusparse<t>csric02_analysis()

```
cusparseStatus_t
cusparseScsric02_analysis(cusparseHandle_t         handle,
                          int                      m,
                          int                      nnz,
                          const cusparseMatDescr_t descrA,
                          const float*             csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          csric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)

cusparseStatus_t
cusparseDcsric02_analysis(cusparseHandle_t         handle,
                          int                      m,
                          int                      nnz,
                          const cusparseMatDescr_t descrA,
                          const double*            csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          csric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)

cusparseStatus_t
cusparseCcsric02_analysis(cusparseHandle_t         handle,
                          int                      m,
                          int                      nnz,
                          const cusparseMatDescr_t descrA,
                          const cuComplex*         csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          csric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)

cusparseStatus_t
cusparseZcsric02_analysis(cusparseHandle_t         handle,
                          int                      m,
                          int                      nnz,
                          const cusparseMatDescr_t descrA,
                          const cuDoubleComplex*   csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          csric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)
```

This function performs the analysis phase of the incomplete-Cholesky factorization with 0 fill-in and no pivoting:

$$A \approx LL^H$$

`A` is an `m×m` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`.

This function requires a buffer size returned by `csric02_bufferSize()`. The address of `pBuffer` must be multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `csric02_analysis()` reports a structural zero and computes level information stored in the opaque structure `info`. The level information can extract more parallelism during incomplete Cholesky factorization. However `csric02()` can be done without level information. To disable level information, the user must specify the policy of `csric02_analysis()` and `csric02()` as `CUSPARSE_SOLVE_POLICY_NO_LEVEL`.

Function `csric02_analysis()` always reports the first structural zero, even if the policy is `CUSPARSE_SOLVE_POLICY_NO_LEVEL`. The user needs to call `cusparseXcsric02_zeroPivot()` to know where the structural zero is.

It is the user's choice whether to call `csric02()` if `csric02_analysis()` reports a structural zero. In this case, the user can still call `csric02()`, which will return a numerical zero at the same position as the structural zero. However the result is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`m`	number of rows and columns of matrix `A`.
`nnz`	number of nonzeros of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrValA`	<type> array of `nnz` ($=$ `csrRowPtrA(m)` $-$ `csrRowPtrA(0)`) nonzero elements of matrix `A`.
`csrRowPtrA`	integer array of `m` $+1$ elements that contains the start of every row and the end of the last row plus one.
`csrColIndA`	integer array of `nnz` ($=$ `csrRowPtrA(m)` $-$ `csrRowPtrA(0)`) column indices of the nonzero elements of matrix `A`.

info	structure initialized using `cusparseCreateCsric02Info()`.
policy	the supported policies are `CUSPARSE_SOLVE_POLICY_NO_LEVEL` and `CUSPARSE_SOLVE_POLICY_USE_LEVEL`.
pBuffer	buffer allocated by the user; the size is returned by `csric02_bufferSize()`.

Output

info	number of bytes of the buffer used in `csric02_analysis()` and `csric02()`

See cusparseStatus_t for the description of the return status.

11.1.3. cusparse<t>csric02()

```
cusparseStatus_t
cusparseScsric02(cusparseHandle_t         handle,
                 int                      m,
                 int                      nnz,
                 const cusparseMatDescr_t descrA,
                 float*                   csrValA_valM,
                 const int*               csrRowPtrA,
                 const int*               csrColIndA,
                 csric02Info_t            info,
                 cusparseSolvePolicy_t    policy,
                 void*                    pBuffer)

cusparseStatus_t
cusparseDcsric02(cusparseHandle_t         handle,
                 int                      m,
                 int                      nnz,
                 const cusparseMatDescr_t descrA,
                 double*                  csrValA_valM,
                 const int*               csrRowPtrA,
                 const int*               csrColIndA,
                 csric02Info_t            info,
                 cusparseSolvePolicy_t    policy,
                 void*                    pBuffer)

cusparseStatus_t
cusparseCcsric02(cusparseHandle_t         handle,
                 int                      m,
                 int                      nnz,
                 const cusparseMatDescr_t descrA,
                 cuComplex*               csrValA_valM,
                 const int*               csrRowPtrA,
                 const int*               csrColIndA,
                 csric02Info_t            info,
                 cusparseSolvePolicy_t    policy,
                 void*                    pBuffer)

cusparseStatus_t
cusparseZcsric02(cusparseHandle_t         handle,
                 int                      m,
```

```
                   int                         nnz,
                   const cusparseMatDescr_t descrA,
                   cuDoubleComplex*            csrValA_valM,
                   const int*                  csrRowPtrA,
                   const int*                  csrColIndA,
                   csric02Info_t               info,
                   cusparseSolvePolicy_t       policy,
                   void*                       pBuffer)
```

This function performs the solve phase of the computing the incomplete-Cholesky factorization with 0 fill-in and no pivoting:

$$A \approx LL^H$$

This function requires a buffer size returned by `csric02_bufferSize()`. The address of `pBuffer` must be a multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Although `csric02()` can be done without level information, the user still needs to be aware of consistency. If `csric02_analysis()` is called with policy `CUSPARSE_SOLVE_POLICY_USE_LEVEL`, `csric02()` can be run with or without levels. On the other hand, if `csric02_analysis()` is called with `CUSPARSE_SOLVE_POLICY_NO_LEVEL`, `csric02()` can only accept `CUSPARSE_SOLVE_POLICY_NO_LEVEL`; otherwise, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `csric02()` reports the first numerical zero, including a structural zero. The user must call `cusparseXcsric02_zeroPivot()` to know where the numerical zero is.

Function `csric02()` only takes the lower triangular part of matrix `A` to perform factorization. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, the fill mode and diagonal type are ignored, and the strictly upper triangular part is ignored and never touched. It does not matter if `A` is Hermitian or not. In other words, from the point of view of `csric02()` `A` is Hermitian and only the lower triangular part is provided.

Note: In practice, a positive definite matrix may not have incomplete cholesky factorization. To the best of our knowledge, only matrix `M` can guarantee the existence of incomplete cholesky factorization. If `csric02()` failed cholesky factorization and reported a numerical zero, it is possible that incomplete cholesky factorization does not exist.

For example, suppose `A` is a real `m × m` matrix, the following code solves the precondition system `M*y = x` where `M` is the product of Cholesky factorization `L` and its transpose.

$$M = LL^H$$

```
// Suppose that A is m x m sparse matrix represented by CSR format,
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_csrRowPtr, d_csrColInd, d_csrVal) is CSR of A on device memory,
// - d_x is right hand side vector on device memory,
// - d_y is solution vector on device memory.
// - d_z is intermediate result on device memory.

cusparseMatDescr_t descr_M = 0;
cusparseMatDescr_t descr_L = 0;
csric02Info_t info_M  = 0;
csrsv2Info_t  info_L  = 0;
csrsv2Info_t  info_Lt = 0;
```

```
int pBufferSize_M;
int pBufferSize_L;
int pBufferSize_Lt;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy_M  = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_L  = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_Lt = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans_L  = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_Lt = CUSPARSE_OPERATION_TRANSPOSE;

// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has non-unit diagonal
cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_M, CUSPARSE_MATRIX_TYPE_GENERAL);

cusparseCreateMatDescr(&descr_L);
cusparseSetMatIndexBase(descr_L, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_NON_UNIT);

// step 2: create a empty info structure
// we need one info for csric02 and two info's for csrsv2
cusparseCreateCsric02Info(&info_M);
cusparseCreateCsrsv2Info(&info_L);
cusparseCreateCsrsv2Info(&info_Lt);

// step 3: query how much memory used in csric02 and csrsv2, and allocate the buffer
cusparseDcsric02_bufferSize(handle, m, nnz,
    descr_M, d_csrVal, d_csrRowPtr, d_csrColInd, info_M, &bufferSize_M);
cusparseDcsrsv2_bufferSize(handle, trans_L, m, nnz,
    descr_L, d_csrVal, d_csrRowPtr, d_csrColInd, info_L, &pBufferSize_L);
cusparseDcsrsv2_bufferSize(handle, trans_Lt, m, nnz,
    descr_L, d_csrVal, d_csrRowPtr, d_csrColInd, info_Lt,&pBufferSize_Lt);

pBufferSize = max(bufferSize_M, max(pBufferSize_L, pBufferSize_Lt));

// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis of incomplete Cholesky on M
//         perform analysis of triangular solve on L
//         perform analysis of triangular solve on L'
// The lower triangular part of M has the same sparsity pattern as L, so
// we can do analysis of csric02 and csrsv2 simultaneously.

cusparseDcsric02_analysis(handle, m, nnz, descr_M,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_M,
    policy_M, pBuffer);
status = cusparseXcsric02_zeroPivot(handle, info_M, &structural_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
   printf("A(%d,%d) is missing\n", structural_zero, structural_zero);
}

cusparseDcsrsv2_analysis(handle, trans_L, m, nnz, descr_L,
    d_csrVal, d_csrRowPtr, d_csrColInd,
    info_L, policy_L, pBuffer);

cusparseDcsrsv2_analysis(handle, trans_Lt, m, nnz, descr_L,
```

```
    d_csrVal, d_csrRowPtr, d_csrColInd,
    info_Lt, policy_Lt, pBuffer);

// step 5: M = L * L'
cusparseDcsric02(handle, m, nnz, descr_M,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_M, policy_M, pBuffer);
status = cusparseXcsric02_zeroPivot(handle, info_M, &numerical_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
   printf("L(%d,%d) is zero\n", numerical_zero, numerical_zero);
}

// step 6: solve L*z = x
cusparseDcsrsv2_solve(handle, trans_L, m, nnz, &alpha, descr_L,
   d_csrVal, d_csrRowPtr, d_csrColInd, info_L,
   d_x, d_z, policy_L, pBuffer);

// step 7: solve L'*y = z
cusparseDcsrsv2_solve(handle, trans_Lt, m, nnz, &alpha, descr_L,
   d_csrVal, d_csrRowPtr, d_csrColInd, info_Lt,
   d_z, d_y, policy_Lt, pBuffer);

// step 6: free resources
cudaFree(pBuffer);
cusparseDestroyMatDescr(descr_M);
cusparseDestroyMatDescr(descr_L);
cusparseDestroyCsric02Info(info_M);
cusparseDestroyCsrsv2Info(info_L);
cusparseDestroyCsrsv2Info(info_Lt);
cusparseDestroy(handle);
```

The function supports the following properties if `pBuffer != NULL`

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`m`	number of rows and columns of matrix `A`.
`nnz`	number of nonzeros of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrValA_valM`	<type> array of `nnz` (= `csrRowPtrA(m)` − `csrRowPtrA(0)`) nonzero elements of matrix `A`.
`csrRowPtrA`	integer array of `m` $+1$ elements that contains the start of every row and the end of the last row plus one.
`csrColIndA`	integer array of `nnz` (= `csrRowPtrA(m)` − `csrRowPtrA(0)`) column indices of the nonzero elements of matrix `A`.

`info`	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
`policy`	the supported policies are `CUSPARSE_SOLVE_POLICY_NO_LEVEL` and `CUSPARSE_SOLVE_POLICY_USE_LEVEL`.
`pBuffer`	buffer allocated by the user; the size is returned by `csric02_bufferSize()`.

Output

`csrValA_valM`	<type> matrix containing the incomplete-Cholesky lower triangular factor.

See `cusparseStatus_t` for the description of the return status.

11.1.4. cusparseXcsric02_zeroPivot()

```
cusparseStatus_t
cusparseXcsric02_zeroPivot(cusparseHandle_t handle,
                           csric02Info_t    info,
                           int*             position)
```

If the returned error code is `CUSPARSE_STATUS_ZERO_PIVOT`, `position=j` means `A(j,j)` has either a structural zero or a numerical zero; otherwise, `position=-1`.

The `position` can be 0-based or 1-based, the same as the matrix.

Function `cusparseXcsric02_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The `position` can be in the host memory or device memory. The user can set proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`info`	`info` contains structural zero or numerical zero if the user already called `csric02_analysis()` or `csric02()`.

Output

position	if no structural or numerical zero, position is -1; otherwise, if A(j,j) is missing or L(j,j) is zero, position=j.

See cusparseStatus_t for the description of the return status.

11.1.5. cusparse<t>bsric02_bufferSize()

```
cusparseStatus_t
cusparseSbsric02_bufferSize(cusparseHandle_t         handle,
                            cusparseDirection_t      dirA,
                            int                      mb,
                            int                      nnzb,
                            const cusparseMatDescr_t descrA,
                            float*                   bsrValA,
                            const int*               bsrRowPtrA,
                            const int*               bsrColIndA,
                            int                      blockDim,
                            bsric02Info_t            info,
                            int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseDbsric02_bufferSize(cusparseHandle_t         handle,
                            cusparseDirection_t      dirA,
                            int                      mb,
                            int                      nnzb,
                            const cusparseMatDescr_t descrA,
                            double*                  bsrValA,
                            const int*               bsrRowPtrA,
                            const int*               bsrColIndA,
                            int                      blockDim,
                            bsric02Info_t            info,
                            int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseCbsric02_bufferSize(cusparseHandle_t         handle,
                            cusparseDirection_t      dirA,
                            int                      mb,
                            int                      nnzb,
                            const cusparseMatDescr_t descrA,
                            cuComplex*               bsrValA,
                            const int*               bsrRowPtrA,
                            const int*               bsrColIndA,
                            int                      blockDim,
                            bsric02Info_t            info,
                            int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseZbsric02_bufferSize(cusparseHandle_t         handle,
                            cusparseDirection_t      dirA,
                            int                      mb,
                            int                      nnzb,
                            const cusparseMatDescr_t descrA,
                            cuDoubleComplex*         bsrValA,
                            const int*               bsrRowPtrA,
                            const int*               bsrColIndA,
                            int                      blockDim,
                            bsric02Info_t            info,
```

```
                                int*                  pBufferSizeInBytes)
```

This function returns the size of a buffer used in computing the incomplete-Cholesky factorization with 0 fill-in and no pivoting

$$A \approx LL^H$$

`A` is an `(mb*blockDim)*(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`.

The buffer size depends on the dimensions of `mb`, `blockDim`, and the number of nonzero blocks of the matrix `nnzb`. If the user changes the matrix, it is necessary to call `bsric02_bufferSize()` again to have the correct buffer size; otherwise, a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`mb`	number of block rows and block columns of matrix `A`.
`nnzb`	number of nonzero blocks of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`bsrValA`	<type> array of `nnzb` $($= `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)` $)$ nonzero blocks of matrix `A`.
`bsrRowPtrA`	integer array of `mb` $+1$ elements that contains the start of every block row and the end of the last block row plus one.
`bsrColIndA`	integer array of `nnzb` $($= `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)` $)$ column indices of the nonzero blocks of matrix `A`.
`blockDim`	block dimension of sparse matrix A, larger than zero.

Output

`info`	record internal states based on different algorithms.

pBufferSizeInBytes	number of bytes of the buffer used in `bsric02_analysis()` and `bsric02()`.

See `cusparseStatus_t` for the description of the return status.

11.1.6. cusparse<t>bsric02_analysis()

```
cusparseStatus_t
cusparseSbsric02_analysis(cusparseHandle_t        handle,
                          cusparseDirection_t     dirA,
                          int                      mb,
                          int                      nnzb,
                          const cusparseMatDescr_t descrA,
                          const float*             bsrValA,
                          const int*               bsrRowPtrA,
                          const int*               bsrColIndA,
                          int                      blockDim,
                          bsric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)

cusparseStatus_t
cusparseDbsric02_analysis(cusparseHandle_t        handle,
                          cusparseDirection_t     dirA,
                          int                      mb,
                          int                      nnzb,
                          const cusparseMatDescr_t descrA,
                          const double*            bsrValA,
                          const int*               bsrRowPtrA,
                          const int*               bsrColIndA,
                          int                      blockDim,
                          bsric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)

cusparseStatus_t
cusparseCbsric02_analysis(cusparseHandle_t        handle,
                          cusparseDirection_t     dirA,
                          int                      mb,
                          int                      nnzb,
                          const cusparseMatDescr_t descrA,
                          const cuComplex*         bsrValA,
                          const int*               bsrRowPtrA,
                          const int*               bsrColIndA,
                          int                      blockDim,
                          bsric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)

cusparseStatus_t
cusparseZbsric02_analysis(cusparseHandle_t        handle,
                          cusparseDirection_t     dirA,
                          int                      mb,
                          int                      nnzb,
                          const cusparseMatDescr_t descrA,
                          const cuDoubleComplex*   bsrValA,
                          const int*               bsrRowPtrA,
                          const int*               bsrColIndA,
```

```
                             int                       blockDim,
                             bsric02Info_t             info,
                             cusparseSolvePolicy_t     policy,
                             void*                     pBuffer)
```

This function performs the analysis phase of the incomplete-Cholesky factorization with 0 fill-in and no pivoting

$$A \approx LL^H$$

`A` is an `(mb*blockDim)x(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`. The block in BSR format is of size `blockDim*blockDim`, stored as column-major or row-major as determined by parameter `dirA`, which is either `CUSPARSE_DIRECTION_COLUMN` or `CUSPARSE_DIRECTION_ROW`. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, and the fill mode and diagonal type are ignored.

This function requires a buffer size returned by `bsric02_bufferSize90`. The address of `pBuffer` must be a multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function`bsric02_analysis()` reports structural zero and computes level information stored in the opaque structure `info`. The level information can extract more parallelism during incomplete Cholesky factorization. However `bsric02()` can be done without level information. To disable level information, the user needs to specify the parameter `policy` of `bsric02[_analysis| ]` as `CUSPARSE_SOLVE_POLICY_NO_LEVEL`.

Function `bsric02_analysis` always reports the first structural zero, even when parameter `policy` is `CUSPARSE_SOLVE_POLICY_NO_LEVEL`. The user must call `cusparseXbsric02_zeroPivot()` to know where the structural zero is.

It is the user's choice whether to call `bsric02()` if `bsric02_analysis()` reports a structural zero. In this case, the user can still call `bsric02()`, which returns a numerical zero in the same position as the structural zero. However the result is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`mb`	number of block rows and block columns of matrix `A`.
`nnzb`	number of nonzero blocks of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`.

	Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`bsrValA`	<type> array of `nnzb` $($= `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)` $)$ nonzero blocks of matrix `A`.
`bsrRowPtrA`	integer array of `mb` $+1$ elements that contains the start of every block row and the end of the last block row plus one.
`bsrColIndA`	integer array of `nnzb` $($= `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)` $)$ column indices of the nonzero blocks of matrix `A`.
`blockDim`	block dimension of sparse matrix A; must be larger than zero.
`info`	structure initialized using `cusparseCreateBsric02Info()`.
`policy`	the supported policies are `CUSPARSE_SOLVE_POLICY_NO_LEVEL` and `CUSPARSE_SOLVE_POLICY_USE_LEVEL`.
`pBuffer`	buffer allocated by the user; the size is returned by `bsric02_bufferSize()`.

Output

`info`	Structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged).

See `cusparseStatus_t` for the description of the return status.

11.1.7. cusparse<t>bsric02()

```
cusparseStatus_t
cusparseSbsric02(cusparseHandle_t         handle,
                 cusparseDirection_t      dirA,
                 int                      mb,
                 int                      nnzb,
                 const cusparseMatDescr_t descrA,
                 float*                   bsrValA,
                 const int*               bsrRowPtrA,
                 const int*               bsrColIndA,
                 int                      blockDim,
                 bsric02Info_t            info,
                 cusparseSolvePolicy_t    policy,
                 void*                    pBuffer)

cusparseStatus_t
cusparseDbsric02(cusparseHandle_t         handle,
                 cusparseDirection_t      dirA,
                 int                      mb,
                 int                      nnzb,
                 const cusparseMatDescr_t descrA,
                 double*                  bsrValA,
```

```
                  const int*                bsrRowPtrA,
                  const int*                bsrColIndA,
                  int                       blockDim,
                  bsric02Info_t             info,
                  cusparseSolvePolicy_t     policy,
                  void*                     pBuffer)

cusparseStatus_t
cusparseCbsric02(cusparseHandle_t           handle,
                  cusparseDirection_t       dirA,
                  int                       mb,
                  int                       nnzb,
                  const cusparseMatDescr_t  descrA,
                  cuComplex*                bsrValA,
                  const int*                bsrRowPtrA,
                  const int*                bsrColIndA,
                  int                       blockDim,
                  bsric02Info_t             info,
                  cusparseSolvePolicy_t     policy,
                  void*                     pBuffer)

cusparseStatus_t
cusparseZbsric02(cusparseHandle_t           handle,
                  cusparseDirection_t       dirA,
                  int                       mb,
                  int                       nnzb,
                  const cusparseMatDescr_t  descrA,
                  cuDoubleComplex*          bsrValA,
                  const int*                bsrRowPtrA,
                  const int*                bsrColIndA,
                  int                       blockDim,
                  bsric02Info_t             info,
                  cusparseSolvePolicy_t     policy,
                  void*                     pBuffer)
```

This function performs the solve phase of the incomplete-Cholesky factorization with 0 fill-in and no pivoting

$$A \approx LL^H$$

`A` is an `(mb*blockDim)`×`(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`. The block in BSR format is of size `blockDim*blockDim`, stored as column-major or row-major as determined by parameter `dirA`, which is either `CUSPARSE_DIRECTION_COLUMN` or `CUSPARSE_DIRECTION_ROW`. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, and the fill mode and diagonal type are ignored.

This function requires a buffer size returned by `bsric02_bufferSize()`. The address of `pBuffer` must be a multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Although `bsric02()` can be done without level information, the user must be aware of consistency. If `bsric02_analysis()` is called with policy `CUSPARSE_SOLVE_POLICY_USE_LEVEL`, `bsric02()` can be run with or without levels. On the other hand, if `bsric02_analysis()` is called with `CUSPARSE_SOLVE_POLICY_NO_LEVEL`, `bsric02()` can only accept `CUSPARSE_SOLVE_POLICY_NO_LEVEL`; otherwise, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `bsric02()` has the same behavior as `csric02()`. That is, `bsr2csr(bsric02(A)) = csric02(bsr2csr(A))`. The numerical zero of `csric02()` means there exists some zero `L(j,j)`. The numerical zero of `bsric02()` means there exists some block `Lj,j)` that is not invertible.

Function `bsric02` reports the first numerical zero, including a structural zero. The user must call `cusparseXbsric02_zeroPivot()` to know where the numerical zero is.

The `bsric02()` function only takes the lower triangular part of matrix `A` to perform factorization. The strictly upper triangular part is ignored and never touched. It does not matter if `A` is Hermitian or not. In other words, from the point of view of `bsric02()`, `A` is Hermitian and only the lower triangular part is provided. Moreover, the imaginary part of diagonal elements of diagonal blocks is ignored.

For example, suppose `A` is a real m-by-m matrix, where `m=mb*blockDim`. The following code solves precondition system `M*y = x`, where `M` is the product of Cholesky factorization `L` and its transpose.

$$M = LL^H$$

```
// Suppose that A is m x m sparse matrix represented by BSR format,
// The number of block rows/columns is mb, and
// the number of nonzero blocks is nnzb.
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_bsrRowPtr, d_bsrColInd, d_bsrVal) is BSR of A on device memory,
// - d_x is right hand side vector on device memory,
// - d_y is solution vector on device memory.
// - d_z is intermediate result on device memory.
// - d_x, d_y and d_z are of size m.
cusparseMatDescr_t descr_M = 0;
cusparseMatDescr_t descr_L = 0;
bsric02Info_t info_M  = 0;
bsrsv2Info_t  info_L  = 0;
bsrsv2Info_t  info_Lt = 0;
int pBufferSize_M;
int pBufferSize_L;
int pBufferSize_Lt;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy_M  = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_L  = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_Lt = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans_L  = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_Lt = CUSPARSE_OPERATION_TRANSPOSE;
const cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;

// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has non-unit diagonal
cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_M, CUSPARSE_MATRIX_TYPE_GENERAL);

cusparseCreateMatDescr(&descr_L);
cusparseSetMatIndexBase(descr_L, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
```

```
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_NON_UNIT);

// step 2: create a empty info structure
// we need one info for bsric02 and two info's for bsrsv2
cusparseCreateBsric02Info(&info_M);
cusparseCreateBsrsv2Info(&info_L);
cusparseCreateBsrsv2Info(&info_Lt);

// step 3: query how much memory used in bsric02 and bsrsv2, and allocate the buffer
cusparseDbsric02_bufferSize(handle, dir, mb, nnzb,
    descr_M, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_M, &bufferSize_M);
cusparseDbsrsv2_bufferSize(handle, dir, trans_L, mb, nnzb,
    descr_L, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_L, &pBufferSize_L);
cusparseDbsrsv2_bufferSize(handle, dir, trans_Lt, mb, nnzb,
    descr_L, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_Lt,
 &pBufferSize_Lt);

pBufferSize = max(bufferSize_M, max(pBufferSize_L, pBufferSize_Lt));

// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis of incomplete Cholesky on M
//         perform analysis of triangular solve on L
//         perform analysis of triangular solve on L'
// The lower triangular part of M has the same sparsity pattern as L, so
// we can do analysis of bsric02 and bsrsv2 simultaneously.

cusparseDbsric02_analysis(handle, dir, mb, nnzb, descr_M,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_M,
    policy_M, pBuffer);
status = cusparseXbsric02_zeroPivot(handle, info_M, &structural_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
   printf("A(%d,%d) is missing\n", structural_zero, structural_zero);
}

cusparseDbsrsv2_analysis(handle, dir, trans_L, mb, nnzb, descr_L,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim,
    info_L, policy_L, pBuffer);

cusparseDbsrsv2_analysis(handle, dir, trans_Lt, mb, nnzb, descr_L,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim,
    info_Lt, policy_Lt, pBuffer);

// step 5: M = L * L'
cusparseDbsric02_solve(handle, dir, mb, nnzb, descr_M,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_M, policy_M, pBuffer);
status = cusparseXbsric02_zeroPivot(handle, info_M, &numerical_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
   printf("L(%d,%d) is not positive definite\n", numerical_zero, numerical_zero);
}

// step 6: solve L*z = x
cusparseDbsrsv2_solve(handle, dir, trans_L, mb, nnzb, &alpha, descr_L,
   d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_L,
   d_x, d_z, policy_L, pBuffer);

// step 7: solve L'*y = z
cusparseDbsrsv2_solve(handle, dir, trans_Lt, mb, nnzb, &alpha, descr_L,
   d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_Lt,
   d_z, d_y, policy_Lt, pBuffer);

// step 6: free resources
cudaFree(pBuffer);
cusparseDestroyMatDescr(descr_M);
cusparseDestroyMatDescr(descr_L);
```

```
cusparseDestroyBsric02Info(info_M);
cusparseDestroyBsrsv2Info(info_L);
cusparseDestroyBsrsv2Info(info_Lt);
cusparseDestroy(handle);
```

The function supports the following properties if `pBuffer != NULL`

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`mb`	number of block rows and block columns of matrix `A`.
`nnzb`	number of nonzero blocks of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`bsrValA`	<type> array of `nnzb` $($= `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)` $)$ nonzero blocks of matrix `A`.
`bsrRowPtrA`	integer array of `mb` $+1$ elements that contains the start of every block row and the end of the last block row plus one.
`bsrColIndA`	integer array of `nnzb` $($= `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)` $)$ column indices of the nonzero blocks of matrix `A`.
`blockDim`	block dimension of sparse matrix A, larger than zero.
`info`	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
`policy`	the supported policies are `CUSPARSE_SOLVE_POLICY_NO_LEVEL` and `CUSPARSE_SOLVE_POLICY_USE_LEVEL`.
`pBuffer`	buffer allocated by the user, the size is returned by `bsric02_bufferSize()`.

Output

`bsrValA`	<type> matrix containing the incomplete-Cholesky lower triangular factor.

See cusparseStatus_t for the description of the return status.

11.1.8. cusparseXbsric02_zeroPivot()

```
cusparseStatus_t
cusparseXbsric02_zeroPivot(cusparseHandle_t handle,
                           bsric02Info_t    info,
                           int*             position)
```

If the returned error code is `CUSPARSE_STATUS_ZERO_PIVOT`, `position=j` means `A(j,j)` has either a structural zero or a numerical zero (the block is not positive definite). Otherwise `position=-1`.

The `position` can be 0-based or 1-based, the same as the matrix.

Function `cusparseXbsric02_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The `position` can be in the host memory or device memory. The user can set the proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`info`	`info` contains a structural zero or a numerical zero if the user already called `bsric02_analysis()` or `bsric02()`.

Output

`position`	If no structural or numerical zero, `position` is -1, otherwise if `A(j,j)` is missing or `L(j,j)` is not positive definite, `position=j`.

See cusparseStatus_t for the description of the return status.

11.2. Incomplete LU Factorization: level 0

Different algorithms for ilu0 are discussed in this section.

11.2.1. cusparse<t>csrilu02_numericBoost()

```
cusparseStatus_t
```

```
cusparseScsrilu02_numericBoost(cusparseHandle_t handle,
                               csrilu02Info_t   info,
                               int              enable_boost,
                               double*          tol,
                               float*           boost_val)

cusparseStatus_t
cusparseDcsrilu02_numericBoost(cusparseHandle_t handle,
                               csrilu02Info_t   info,
                               int              enable_boost,
                               double*          tol,
                               double*          boost_val)

cusparseStatus_t
cusparseCcsrilu02_numericBoost(cusparseHandle_t handle,
                               csrilu02Info_t   info,
                               int              enable_boost,
                               double*          tol,
                               cuComplex*       boost_val)

cusparseStatus_t
cusparseZcsrilu02_numericBoost(cusparseHandle_t handle,
                               csrilu02Info_t   info,
                               int              enable_boost,
                               double*          tol,
                               cuDoubleComplex* boost_val)
```

The user can use a boost value to replace a numerical value in incomplete LU factorization. The `tol` is used to determine a numerical zero, and the `boost_val` is used to replace a numerical zero. The behavior is

if `tol >= fabs(A(j,j))`, then `A(j,j)=boost_val`.

To enable a boost value, the user has to set parameter `enable_boost` to 1 before calling `csrilu02()`. To disable a boost value, the user can call `csrilu02_numericBoost()` again with parameter `enable_boost=0`.

If `enable_boost=0`, `tol` and `boost_val` are ignored.

Both `tol` and `boost_val` can be in the host memory or device memory. The user can set the proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context
`info`	structure initialized using `cusparseCreateCsrilu02Info()`
`enable_boost`	disable boost by `enable_boost=0`; otherwise, boost is enabled
`tol`	tolerance to determine a numerical zero
`boost_val`	boost value to replace a numerical zero

See cusparseStatus_t for the description of the return status.

11.2.2. cusparse<t>csrilu02_bufferSize()

```
cusparseStatus_t
cusparseScsrilu02_bufferSize(cusparseHandle_t         handle,
                             int                      m,
                             int                      nnz,
                             const cusparseMatDescr_t descrA,
                             float*                   csrValA,
                             const int*               csrRowPtrA,
                             const int*               csrColIndA,
                             csrilu02Info_t           info,
                             int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseDcsrilu02_bufferSize(cusparseHandle_t         handle,
                             int                      m,
                             int                      nnz,
                             const cusparseMatDescr_t descrA,
                             double*                  csrValA,
                             const int*               csrRowPtrA,
                             const int*               csrColIndA,
                             csrilu02Info_t           info,
                             int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseCcsrilu02_bufferSize(cusparseHandle_t         handle,
                             int                      m,
                             int                      nnz,
                             const cusparseMatDescr_t descrA,
                             cuComplex*               csrValA,
                             const int*               csrRowPtrA,
                             const int*               csrColIndA,
                             csrilu02Info_t           info,
                             int*                     pBufferSizeInBytes)

cusparseStatus_t
cusparseZcsrilu02_bufferSize(cusparseHandle_t         handle,
                             int                      m,
                             int                      nnz,
                             const cusparseMatDescr_t descrA,
                             cuDoubleComplex*         csrValA,
                             const int*               csrRowPtrA,
                             const int*               csrColIndA,
                             csrilu02Info_t           info,
                             int*                     pBufferSizeInBytes)
```

This function returns size of the buffer used in computing the incomplete-LU factorization with 0 fill-in and no pivoting:

$$A \approx LU$$

`A` is an `m×m` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`.

The buffer size depends on the dimension `m` and `nnz`, the number of nonzeros of the matrix. If the user changes the matrix, it is necessary to call `csrilu02_bufferSize()` again to have the correct buffer size; otherwise, a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`m`	number of rows and columns of matrix `A`.
`nnz`	number of nonzeros of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrValA`	<type> array of `nnz` (= `csrRowPtrA(m)` − `csrRowPtrA(0)`) nonzero elements of matrix `A`.
`csrRowPtrA`	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.
`csrColIndA`	integer array of `nnz` (= `csrRowPtrA(m)` − `csrRowPtrA(0)`) column indices of the nonzero elements of matrix `A`.

Output

`info`	record internal states based on different algorithms
`pBufferSizeInBytes`	number of bytes of the buffer used in `csrilu02_analysis()` and `csrilu02()`

See `cusparseStatus_t` for the description of the return status.

11.2.3. cusparse<t>csrilu02_analysis()

```
cusparseStatus_t
cusparseScsrilu02_analysis(cusparseHandle_t         handle,
                           int                      m,
                           int                      nnz,
                           const cusparseMatDescr_t descrA,
                           const float*             csrValA,
                           const int*               csrRowPtrA,
                           const int*               csrColIndA,
                           csrilu02Info_t           info,
                           cusparseSolvePolicy_t    policy,
                           void*                    pBuffer)
```

```
cusparseStatus_t
cusparseDcsrilu02_analysis(cusparseHandle_t         handle,
                           int                      m,
                           int                      nnz,
                           const cusparseMatDescr_t descrA,
                           const double*            csrValA,
                           const int*               csrRowPtrA,
                           const int*               csrColIndA,
                           csrilu02Info_t           info,
                           cusparseSolvePolicy_t    policy,
                           void*                    pBuffer)

cusparseStatus_t
cusparseCcsrilu02_analysis(cusparseHandle_t         handle,
                           int                      m,
                           int                      nnz,
                           const cusparseMatDescr_t descrA,
                           const cuComplex*         csrValA,
                           const int*               csrRowPtrA,
                           const int*               csrColIndA,
                           csrilu02Info_t           info,
                           cusparseSolvePolicy_t    policy,
                           void*                    pBuffer)

cusparseStatus_t
cusparseZcsrilu02_analysis(cusparseHandle_t         handle,
                           int                      m,
                           int                      nnz,
                           const cusparseMatDescr_t descrA,
                           const cuDoubleComplex*   csrValA,
                           const int*               csrRowPtrA,
                           const int*               csrColIndA,
                           csrilu02Info_t           info,
                           cusparseSolvePolicy_t    policy,
                           void*                    pBuffer)
```

This function performs the analysis phase of the incomplete-LU factorization with 0 fill-in and no pivoting:

$$A \approx LU$$

`A` is an `m×m` sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`.

This function requires the buffer size returned by `csrilu02_bufferSize()`. The address of `pBuffer` must be a multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `csrilu02_analysis()` reports a structural zero and computes level information stored in the opaque structure `info`. The level information can extract more parallelism during incomplete LU factorization; however `csrilu02()` can be done without level information. To disable level information, the user must specify the policy of `csrilu02()` as `CUSPARSE_SOLVE_POLICY_NO_LEVEL`.

It is the user's choice whether to call `csrilu02()` if `csrilu02_analysis()` reports a structural zero. In this case, the user can still call `csrilu02()`, which will return a numerical zero at the same position as the structural zero. However the result is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

handle	handle to the cuSPARSE library context.
m	number of rows and columns of matrix A.
nnz	number of nonzeros of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<type> array of nnz $($= csrRowPtrA(m) $-$ csrRowPtrA(0) $)$ nonzero elements of matrix A.
csrRowPtrA	integer array of m $+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz $($= csrRowPtrA(m) $-$ csrRowPtrA(0) $)$ column indices of the nonzero elements of matrix A.
info	structure initialized using cusparseCreateCsrilu02Info().
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user, the size is returned by csrilu02_bufferSize().

Output

info	Structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged).

See cusparseStatus_t for the description of the return status.

11.2.4. cusparse<t>csrilu02()

```
cusparseStatus_t
cusparseScsrilu02(cusparseHandle_t         handle,
                  int                      m,
                  int                      nnz,
                  const cusparseMatDescr_t descrA,
                  float*                   csrValA_valM,
```

```
                     const int*                  csrRowPtrA,
                     const int*                  csrColIndA,
                     csrilu02Info_t              info,
                     cusparseSolvePolicy_t       policy,
                     void*                       pBuffer)

cusparseStatus_t
cusparseDcsrilu02(cusparseHandle_t          handle,
                  int                       m,
                  int                       nnz,
                  const cusparseMatDescr_t  descrA,
                  double*                   csrValA_valM,
                  const int*                csrRowPtrA,
                  const int*                csrColIndA,
                  csrilu02Info_t            info,
                  cusparseSolvePolicy_t     policy,
                  void*                     pBuffer)

cusparseStatus_t
cusparseCcsrilu02(cusparseHandle_t          handle,
                  int                       m,
                  int                       nnz,
                  const cusparseMatDescr_t  descrA,
                  cuComplex*                csrValA_valM,
                  const int*                csrRowPtrA,
                  const int*                csrColIndA,
                  csrilu02Info_t            info,
                  cusparseSolvePolicy_t     policy,
                  void*                     pBuffer)

cusparseStatus_t
cusparseZcsrilu02(cusparseHandle_t          handle,
                  int                       m,
                  int                       nnz,
                  const cusparseMatDescr_t  descrA,
                  cuDoubleComplex*          csrValA_valM,
                  const int*                csrRowPtrA,
                  const int*                csrColIndA,
                  csrilu02Info_t            info,
                  cusparseSolvePolicy_t     policy,
                  void*                     pBuffer)
```

This function performs the solve phase of the incomplete-LU factorization with 0 fill-in and no pivoting:

$$A \approx LU$$

`A` is an `m×m` sparse matrix that is defined in CSR storage format by the three arrays `csrValA_valM`, `csrRowPtrA`, and `csrColIndA`.

This function requires a buffer size returned by `csrilu02_bufferSize()`. The address of `pBuffer` must be a multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`. The fill mode and diagonal type are ignored.

Although `csrilu02()` can be done without level information, the user still needs to be aware of consistency. If `csrilu02_analysis()` is called with policy

CUSPARSE_SOLVE_POLICY_USE_LEVEL, csrilu02() can be run with or without levels. On the other hand, if csrilu02_analysis() is called with CUSPARSE_SOLVE_POLICY_NO_LEVEL, csrilu02() can only accept CUSPARSE_SOLVE_POLICY_NO_LEVEL; otherwise, CUSPARSE_STATUS_INVALID_VALUE is returned.

Function csrilu02() reports the first numerical zero, including a structural zero. The user must call cusparseXcsrilu02_zeroPivot() to know where the numerical zero is.

For example, suppose A is a real m × m matrix, the following code solves precondition system M*y = x where M is the product of LU factors L and U.

```
// Suppose that A is m x m sparse matrix represented by CSR format,
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_csrRowPtr, d_csrColInd, d_csrVal) is CSR of A on device memory,
// - d_x is right hand side vector on device memory,
// - d_y is solution vector on device memory.
// - d_z is intermediate result on device memory.

cusparseMatDescr_t descr_M = 0;
cusparseMatDescr_t descr_L = 0;
cusparseMatDescr_t descr_U = 0;
csrilu02Info_t info_M  = 0;
csrsv2Info_t  info_L  = 0;
csrsv2Info_t  info_U  = 0;
int pBufferSize_M;
int pBufferSize_L;
int pBufferSize_U;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy_M = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_L = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_U = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans_L  = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_U  = CUSPARSE_OPERATION_NON_TRANSPOSE;

// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has unit diagonal
// - matrix U is base-1
// - matrix U is upper triangular
// - matrix U has non-unit diagonal
cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_M, CUSPARSE_MATRIX_TYPE_GENERAL);

cusparseCreateMatDescr(&descr_L);
cusparseSetMatIndexBase(descr_L, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_UNIT);

cusparseCreateMatDescr(&descr_U);
cusparseSetMatIndexBase(descr_U, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_U, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_U, CUSPARSE_FILL_MODE_UPPER);
cusparseSetMatDiagType(descr_U, CUSPARSE_DIAG_TYPE_NON_UNIT);

// step 2: create a empty info structure
// we need one info for csrilu02 and two info's for csrsv2
```

```
cusparseCreateCsrilu02Info(&info_M);
cusparseCreateCsrsv2Info(&info_L);
cusparseCreateCsrsv2Info(&info_U);

// step 3: query how much memory used in csrilu02 and csrsv2, and allocate the
 buffer
cusparseDcsrilu02_bufferSize(handle, m, nnz,
    descr_M, d_csrVal, d_csrRowPtr, d_csrColInd, info_M, &pBufferSize_M);
cusparseDcsrsv2_bufferSize(handle, trans_L, m, nnz,
    descr_L, d_csrVal, d_csrRowPtr, d_csrColInd, info_L, &pBufferSize_L);
cusparseDcsrsv2_bufferSize(handle, trans_U, m, nnz,
    descr_U, d_csrVal, d_csrRowPtr, d_csrColInd, info_U, &pBufferSize_U);

pBufferSize = max(pBufferSize_M, max(pBufferSize_L, pBufferSize_U));

// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis of incomplete Cholesky on M
//         perform analysis of triangular solve on L
//         perform analysis of triangular solve on U
// The lower(upper) triangular part of M has the same sparsity pattern as L(U),
// we can do analysis of csrilu0 and csrsv2 simultaneously.

cusparseDcsrilu02_analysis(handle, m, nnz, descr_M,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_M,
    policy_M, pBuffer);
status = cusparseXcsrilu02_zeroPivot(handle, info_M, &structural_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
   printf("A(%d,%d) is missing\n", structural_zero, structural_zero);
}

cusparseDcsrsv2_analysis(handle, trans_L, m, nnz, descr_L,
    d_csrVal, d_csrRowPtr, d_csrColInd,
    info_L, policy_L, pBuffer);

cusparseDcsrsv2_analysis(handle, trans_U, m, nnz, descr_U,
    d_csrVal, d_csrRowPtr, d_csrColInd,
    info_U, policy_U, pBuffer);

// step 5: M = L * U
cusparseDcsrilu02(handle, m, nnz, descr_M,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_M, policy_M, pBuffer);
status = cusparseXcsrilu02_zeroPivot(handle, info_M, &numerical_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
   printf("U(%d,%d) is zero\n", numerical_zero, numerical_zero);
}

// step 6: solve L*z = x
cusparseDcsrsv2_solve(handle, trans_L, m, nnz, &alpha, descr_L,
   d_csrVal, d_csrRowPtr, d_csrColInd, info_L,
   d_x, d_z, policy_L, pBuffer);

// step 7: solve U*y = z
cusparseDcsrsv2_solve(handle, trans_U, m, nnz, &alpha, descr_U,
   d_csrVal, d_csrRowPtr, d_csrColInd, info_U,
   d_z, d_y, policy_U, pBuffer);

// step 6: free resources
cudaFree(pBuffer);
cusparseDestroyMatDescr(descr_M);
cusparseDestroyMatDescr(descr_L);
cusparseDestroyMatDescr(descr_U);
cusparseDestroyCsrilu02Info(info_M);
cusparseDestroyCsrsv2Info(info_L);
cusparseDestroyCsrsv2Info(info_U);
cusparseDestroy(handle);
```

The function supports the following properties if `pBuffer != NULL`

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

handle	handle to the cuSPARSE library context.
m	number of rows and columns of matrix A.
nnz	number of nonzeros of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA_valM	<type> array of nnz ($=$ csrRowPtrA(m) − csrRowPtrA(0)) nonzero elements of matrix A.
csrRowPtrA	integer array of m $+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz ($=$ csrRowPtrA(m) − csrRowPtrA(0)) column indices of the nonzero elements of matrix A.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is returned by csrilu02_bufferSize().

Output

csrValA_valM	<type> matrix containing the incomplete-LU lower and upper triangular factors.

See cusparseStatus_t for the description of the return status.

11.2.5. cusparseXcsrilu02_zeroPivot()

```
cusparseStatus_t
cusparseXcsrilu02_zeroPivot(cusparseHandle_t handle,
                            csrilu02Info_t   info,
                            int*             position)
```

If the returned error code is `CUSPARSE_STATUS_ZERO_PIVOT`, `position=j` means `A(j,j)` has either a structural zero or a numerical zero; otherwise, `position=-1`.

The `position` can be 0-based or 1-based, the same as the matrix.

Function `cusparseXcsrilu02_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The `position` can be in the host memory or device memory. The user can set proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	Handle to the cuSPARSE library context.
`info`	`info` contains structural zero or numerical zero if the user already called `csrilu02_analysis()` or `csrilu02()`.

Output

`position`	If no structural or numerical zero, `position` is -1; otherwise if `A(j,j)` is missing or `U(j,j)` is zero, `position=j`.

See `cusparseStatus_t` for the description of the return status.

11.2.6. cusparse<t>bsrilu02_numericBoost()

```
cusparseStatus_t
cusparseSbsrilu02_numericBoost(cusparseHandle_t handle,
                               bsrilu02Info_t   info,
                               int              enable_boost,
                               double*          tol,
                               float*           boost_val)

cusparseStatus_t
cusparseDbsrilu02_numericBoost(cusparseHandle_t handle,
                               bsrilu02Info_t   info,
                               int              enable_boost,
                               double*          tol,
                               double*          boost_val)

cusparseStatus_t
cusparseCbsrilu02_numericBoost(cusparseHandle_t handle,
                               bsrilu02Info_t   info,
                               int              enable_boost,
                               double*          tol,
                               cuComplex*       boost_val)
```

```
cusparseStatus_t
cusparseZbsrilu02_numericBoost(cusparseHandle_t handle,
                               bsrilu02Info_t   info,
                               int              enable_boost,
                               double*          tol,
                               cuDoubleComplex* boost_val)
```

The user can use a boost value to replace a numerical value in incomplete LU factorization. Parameter `tol` is used to determine a numerical zero, and `boost_val` is used to replace a numerical zero. The behavior is as follows:

if `tol >= fabs(A(j,j))`, then reset each diagonal element of block `A(j,j)` by `boost_val`.

To enable a boost value, the user sets parameter `enable_boost` to 1 before calling `bsrilu02()`. To disable the boost value, the user can call `bsrilu02_numericBoost()` with parameter `enable_boost=0`.

If `enable_boost=0`, `tol` and `boost_val` are ignored.

Both `tol` and `boost_val` can be in host memory or device memory. The user can set the proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`info`	structure initialized using `cusparseCreateBsrilu02Info()`.
`enable_boost`	disable boost by setting `enable_boost=0`. Otherwise, boost is enabled.
`tol`	tolerance to determine a numerical zero.
`boost_val`	boost value to replace a numerical zero.

See `cusparseStatus_t` for the description of the return status.

11.2.7. cusparse<t>bsrilu02_bufferSize()

```
cusparseStatus_t
cusparseSbsrilu02_bufferSize(cusparseHandle_t handle,
                             cusparseDirection_t dirA,
                             int mb,
                             int nnzb,
                             const cusparseMatDescr_t descrA,
                             float *bsrValA,
                             const int *bsrRowPtrA,
                             const int *bsrColIndA,
                             int blockDim,
                             bsrilu02Info_t info,
                             int *pBufferSizeInBytes);
```

```
cusparseStatus_t
cusparseDbsrilu02_bufferSize(cusparseHandle_t handle,
                             cusparseDirection_t dirA,
                             int mb,
                             int nnzb,
                             const cusparseMatDescr_t descrA,
                             double *bsrValA,
                             const int *bsrRowPtrA,
                             const int *bsrColIndA,
                             int blockDim,
                             bsrilu02Info_t info,
                             int *pBufferSizeInBytes);

cusparseStatus_t
cusparseCbsrilu02_bufferSize(cusparseHandle_t handle,
                             cusparseDirection_t dirA,
                             int mb,
                             int nnzb,
                             const cusparseMatDescr_t descrA,
                             cuComplex *bsrValA,
                             const int *bsrRowPtrA,
                             const int *bsrColIndA,
                             int blockDim,
                             bsrilu02Info_t info,
                             int *pBufferSizeInBytes);

cusparseStatus_t
cusparseZbsrilu02_bufferSize(cusparseHandle_t handle,
                             cusparseDirection_t dirA,
                             int mb,
                             int nnzb,
                             const cusparseMatDescr_t descrA,
                             cuDoubleComplex *bsrValA,
                             const int *bsrRowPtrA,
                             const int *bsrColIndA,
                             int blockDim,
                             bsrilu02Info_t info,
                             int *pBufferSizeInBytes);
```

This function returns the size of the buffer used in computing the incomplete-LU factorization with 0 fill-in and no pivoting.

$$A \approx LU$$

`A` is an `(mb*blockDim)*(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`.

The buffer size depends on the dimensions of `mb`, `blockDim`, and the number of nonzero blocks of the matrix `nnzb`. If the user changes the matrix, it is necessary to call `bsrilu02_bufferSize()` again to have the correct buffer size; otherwise, a segmentation fault may occur.

Input

`handle`	handle to the cuSPARSE library context.

dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows and columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<type> array of nnzb $($= bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ nonzero blocks of matrix A.
bsrRowPtrA	integer array of mb $+1$ elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb $($= bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.

Output

info	record internal states based on different algorithms.
pBufferSizeInBytes	number of bytes of the buffer used in bsrilu02_analysis() and bsrilu02().

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0), base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

11.2.8. cusparse<t>bsrilu02_analysis()

```
cusparseStatus_t
cusparseSbsrilu02_analysis(cusparseHandle_t         handle,
                           cusparseDirection_t      dirA,
                           int                      mb,
                           int                      nnzb,
                           const cusparseMatDescr_t descrA,
```

```
                           float*                   bsrValA,
                           const int*               bsrRowPtrA,
                           const int*               bsrColIndA,
                           int                      blockDim,
                           bsrilu02Info_t           info,
                           cusparseSolvePolicy_t    policy,
                           void*                    pBuffer)

cusparseStatus_t
cusparseDbsrilu02_analysis(cusparseHandle_t         handle,
                           cusparseDirection_t      dirA,
                           int                      mb,
                           int                      nnzb,
                           const cusparseMatDescr_t descrA,
                           double*                  bsrValA,
                           const int*               bsrRowPtrA,
                           const int*               bsrColIndA,
                           int                      blockDim,
                           bsrilu02Info_t           info,
                           cusparseSolvePolicy_t    policy,
                           void*                    pBuffer)

cusparseStatus_t
cusparseCbsrilu02_analysis(cusparseHandle_t         handle,
                           cusparseDirection_t      dirA,
                           int                      mb,
                           int                      nnzb,
                           const cusparseMatDescr_t descrA,
                           cuComplex*               bsrValA,
                           const int*               bsrRowPtrA,
                           const int*               bsrColIndA,
                           int                      blockDim,
                           bsrilu02Info_t           info,
                           cusparseSolvePolicy_t    policy,
                           void*                    pBuffer)

cusparseStatus_t
cusparseZbsrilu02_analysis(cusparseHandle_t         handle,
                           cusparseDirection_t      dirA,
                           int                      mb,
                           int                      nnzb,
                           const cusparseMatDescr_t descrA,
                           cuDoubleComplex*         bsrValA,
                           const int*               bsrRowPtrA,
                           const int*               bsrColIndA,
                           int                      blockDim,
                           bsrilu02Info_t           info,
                           cusparseSolvePolicy_t    policy,
                           void*                    pBuffer)
```

This function performs the analysis phase of the incomplete-LU factorization with 0 fill-in and no pivoting.

$$A \approx LU$$

`A` is an `(mb*blockDim)` × `(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`. The block in BSR format is of size `blockDim*blockDim`, stored as column-major or row-major as determined by parameter `dirA`, which is either `CUSPARSE_DIRECTION_COLUMN` or `CUSPARSE_DIRECTION_ROW`. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, and the fill mode and diagonal type are ignored.

This function requires a buffer size returned by `bsrilu02_bufferSize()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `bsrilu02_analysis()` reports a structural zero and computes level information stored in the opaque structure `info`. The level information can extract more parallelism during incomplete LU factorization. However `bsrilu02()` can be done without level information. To disable level information, the user needs to specify the parameter `policy` of `bsrilu02[_analysis| ]` as `CUSPARSE_SOLVE_POLICY_NO_LEVEL`.

Function `bsrilu02_analysis()` always reports the first structural zero, even with parameter `policy` is `CUSPARSE_SOLVE_POLICY_NO_LEVEL`. The user must call `cusparseXbsrilu02_zeroPivot()` to know where the structural zero is.

It is the user's choice whether to call `bsrilu02()` if `bsrilu02_analysis()` reports a structural zero. In this case, the user can still call `bsrilu02()`, which will return a numerical zero at the same position as the structural zero. However the result is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`mb`	number of block rows and block columns of matrix `A`.
`nnzb`	number of nonzero blocks of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`bsrValA`	<type> array of `nnzb` $($= `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)` $)$ nonzero blocks of matrix `A`.
`bsrRowPtrA`	integer array of `mb` $+1$ elements that contains the start of every block row and the end of the last block row plus one.
`bsrColIndA`	integer array of `nnzb` $($= `bsrRowPtrA(mb)` $-$ `bsrRowPtrA(0)` $)$ column indices of the nonzero blocks of matrix `A`.

`blockDim`	block dimension of sparse matrix A, larger than zero.
`info`	structure initialized using `cusparseCreateBsrilu02Info()`.
`policy`	the supported policies are `CUSPARSE_SOLVE_POLICY_NO_LEVEL` and `CUSPARSE_SOLVE_POLICY_USE_LEVEL`.
`pBuffer`	buffer allocated by the user, the size is returned by `bsrilu02_bufferSize()`.

Output

`info`	structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged)

See `cusparseStatus_t` for the description of the return status.

11.2.9. cusparse<t>bsrilu02()

```
cusparseStatus_t
cusparseSbsrilu02(cusparseHandle_t         handle,
                  cusparseDirection_t      dirA,
                  int                      mb,
                  int                      nnzb,
                  const cusparseMatDescr_t descry,
                  float*                   bsrValA,
                  const int*               bsrRowPtrA,
                  const int*               bsrColIndA,
                  int                      blockDim,
                  bsrilu02Info_t           info,
                  cusparseSolvePolicy_t    policy,
                  void*                    pBuffer)

cusparseStatus_t
cusparseDbsrilu02(cusparseHandle_t         handle,
                  cusparseDirection_t      dirA,
                  int                      mb,
                  int                      nnzb,
                  const cusparseMatDescr_t descry,
                  double*                  bsrValA,
                  const int*               bsrRowPtrA,
                  const int*               bsrColIndA,
                  int                      blockDim,
                  bsrilu02Info_t           info,
                  cusparseSolvePolicy_t    policy,
                  void*                    pBuffer)

cusparseStatus_t
cusparseCbsrilu02(cusparseHandle_t         handle,
                  cusparseDirection_t      dirA,
                  int                      mb,
                  int                      nnzb,
                  const cusparseMatDescr_t descry,
                  cuComplex*               bsrValA,
```

```
                    const int*                 bsrRowPtrA,
                    const int*                 bsrColIndA,
                    int                        blockDim,
                    bsrilu02Info_t             info,
                    cusparseSolvePolicy_t      policy,
                    void*                      pBuffer)

cusparseStatus_t
cusparseZbsrilu02(cusparseHandle_t             handle,
                  cusparseDirection_t          dirA,
                  int                          mb,
                  int                          nnzb,
                  const cusparseMatDescr_t     descry,
                  cuDoubleComplex*             bsrValA,
                  const int*                   bsrRowPtrA,
                  const int*                   bsrColIndA,
                  int                          blockDim,
                  bsrilu02Info_t               info,
                  cusparseSolvePolicy_t        policy,
                  void*                        pBuffer)
```

This function performs the solve phase of the incomplete-LU factorization with 0 fill-in and no pivoting.

$$A \approx LU$$

`A` is an `(mb*blockDim)`×`(mb*blockDim)` sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`. The block in BSR format is of size `blockDim*blockDim`, stored as column-major or row-major determined by parameter `dirA`, which is either `CUSPARSE_DIRECTION_COLUMN` or `CUSPARSE_DIRECTION_ROW`. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, and the fill mode and diagonal type are ignored. Function `bsrilu02()` supports an arbitrary `blockDim`.

This function requires a buffer size returned by `bsrilu02_bufferSize()`. The address of `pBuffer` must be a multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Although `bsrilu02()` can be used without level information, the user must be aware of consistency. If `bsrilu02_analysis()` is called with policy `CUSPARSE_SOLVE_POLICY_USE_LEVEL`, `bsrilu02()` can be run with or without levels. On the other hand, if `bsrilu02_analysis()` is called with `CUSPARSE_SOLVE_POLICY_NO_LEVEL`, `bsrilu02()` can only accept `CUSPARSE_SOLVE_POLICY_NO_LEVEL`; otherwise, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `bsrilu02()` has the same behavior as `csrilu02()`. That is, `bsr2csr(bsrilu02(A)) = csrilu02(bsr2csr(A))`. The numerical zero of `csrilu02()` means there exists some zero `U(j,j)`. The numerical zero of `bsrilu02()` means there exists some block `U(j,j)` that is not invertible.

Function `bsrilu02` reports the first numerical zero, including a structural zero. The user must call `cusparseXbsrilu02_zeroPivot()` to know where the numerical zero is.

For example, suppose `A` is a real m-by-m matrix where `m=mb*blockDim`. The following code solves precondition system `M*y = x`, where `M` is the product of LU factors `L` and `U`.

```
// Suppose that A is m x m sparse matrix represented by BSR format,
```

```
// The number of block rows/columns is mb, and
// the number of nonzero blocks is nnzb.
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_bsrRowPtr, d_bsrColInd, d_bsrVal) is BSR of A on device memory,
// - d_x is right hand side vector on device memory.
// - d_y is solution vector on device memory.
// - d_z is intermediate result on device memory.
// - d_x, d_y and d_z are of size m.
cusparseMatDescr_t descr_M = 0;
cusparseMatDescr_t descr_L = 0;
cusparseMatDescr_t descr_U = 0;
bsrilu02Info_t info_M = 0;
bsrsv2Info_t   info_L = 0;
bsrsv2Info_t   info_U = 0;
int pBufferSize_M;
int pBufferSize_L;
int pBufferSize_U;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy_M = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_L = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_U = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans_L  = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_U  = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;

// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has unit diagonal
// - matrix U is base-1
// - matrix U is upper triangular
// - matrix U has non-unit diagonal
cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_M, CUSPARSE_MATRIX_TYPE_GENERAL);

cusparseCreateMatDescr(&descr_L);
cusparseSetMatIndexBase(descr_L, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_UNIT);

cusparseCreateMatDescr(&descr_U);
cusparseSetMatIndexBase(descr_U, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_U, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_U, CUSPARSE_FILL_MODE_UPPER);
cusparseSetMatDiagType(descr_U, CUSPARSE_DIAG_TYPE_NON_UNIT);

// step 2: create a empty info structure
// we need one info for bsrilu02 and two info's for bsrsv2
cusparseCreateBsrilu02Info(&info_M);
cusparseCreateBsrsv2Info(&info_L);
cusparseCreateBsrsv2Info(&info_U);

// step 3: query how much memory used in bsrilu02 and bsrsv2, and allocate the
 buffer
cusparseDbsrilu02_bufferSize(handle, dir, mb, nnzb,
    descr_M, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_M, &pBufferSize_M);
cusparseDbsrsv2_bufferSize(handle, dir, trans_L, mb, nnzb,
    descr_L, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_L, &pBufferSize_L);
cusparseDbsrsv2_bufferSize(handle, dir, trans_U, mb, nnzb,
```

```
    descr_U, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_U, &pBufferSize_U);

pBufferSize = max(pBufferSize_M, max(pBufferSize_L, pBufferSize_U));

// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis of incomplete LU factorization on M
//         perform analysis of triangular solve on L
//         perform analysis of triangular solve on U
// The lower(upper) triangular part of M has the same sparsity pattern as L(U),
// we can do analysis of bsrilu0 and bsrsv2 simultaneously.

cusparseDbsrilu02_analysis(handle, dir, mb, nnzb, descr_M,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_M,
    policy_M, pBuffer);
status = cusparseXbsrilu02_zeroPivot(handle, info_M, &structural_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == statuss){
   printf("A(%d,%d) is missing\n", structural_zero, structural_zero);
}

cusparseDbsrsv2_analysis(handle, dir, trans_L, mb, nnzb, descr_L,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim,
    info_L, policy_L, pBuffer);

cusparseDbsrsv2_analysis(handle, dir, trans_U, mb, nnzb, descr_U,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim,
    info_U, policy_U, pBuffer);

// step 5: M = L * U
cusparseDbsrilu02(handle, dir, mb, nnzb, descr_M,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_M, policy_M, pBuffer);
status = cusparseXbsrilu02_zeroPivot(handle, info_M, &numerical_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == statuss){
   printf("block U(%d,%d) is not invertible\n", numerical_zero, numerical_zero);
}

// step 6: solve L*z = x
cusparseDbsrsv2_solve(handle, dir, trans_L, mb, nnzb, &alpha, descr_L,
   d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_L,
   d_x, d_z, policy_L, pBuffer);

// step 7: solve U*y = z
cusparseDbsrsv2_solve(handle, dir, trans_U, mb, nnzb, &alpha, descr_U,
   d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_U,
   d_z, d_y, policy_U, pBuffer);

// step 6: free resources
cudaFree(pBuffer);
cusparseDestroyMatDescr(descr_M);
cusparseDestroyMatDescr(descr_L);
cusparseDestroyMatDescr(descr_U);
cusparseDestroyBsrilu02Info(info_M);
cusparseDestroyBsrsv2Info(info_L);
cusparseDestroyBsrsv2Info(info_U);
cusparseDestroy(handle);
```

The function supports the following properties if `pBuffer != NULL`

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks: either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows and block columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<type> array of nnzb $($= bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ nonzero blocks of matrix A.
bsrRowPtrA	integer array of mb $+1$ elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb $($= bsrRowPtrA(mb) $-$ bsrRowPtrA(0) $)$ column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A; must be larger than zero.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is returned by bsrilu02_bufferSize().

Output

bsrValA	<type> matrix containing the incomplete-LU lower and upper triangular factors

See cusparseStatus_t for the description of the return status.

11.2.10. cusparseXbsrilu02_zeroPivot()

```
cusparseStatus_t
cusparseXbsrilu02_zeroPivot(cusparseHandle_t handle,
                            bsrilu02Info_t   info,
                            int*             position)
```

If the returned error code is `CUSPARSE_STATUS_ZERO_PIVOT`, `position=j` means `A(j,j)` has either a structural zero or a numerical zero (the block is not invertible). Otherwise `position=-1`.

The `position` can be 0-based or 1-based, the same as the matrix.

Function `cusparseXbsrilu02_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The `position` can be in the host memory or device memory. The user can set proper the mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`info`	`info` contains structural zero or numerical zero if the user already called `bsrilu02_analysis()` or `bsrilu02()`.

Output

`position`	if no structural or numerical zero, `position` is -1; otherwise if `A(j,j)` is missing or `U(j,j)` is not invertible, `position=j`.

See `cusparseStatus_t` for the description of the return status.

11.3. Tridiagonal Solve

Different algorithms for tridiagonal solve are discussed in this section.

11.3.1. cusparse<t>gtsv2_buffSizeExt()

```
cusparseStatus_t
cusparseSgtsv2_bufferSizeExt(cusparseHandle_t handle,
                             int              m,
                             int              n,
                             const float*     dl,
                             const float*     d,
                             const float*     du,
                             const float*     B,
                             int              ldb,
                             size_t*          bufferSizeInBytes)

cusparseStatus_t
```

```
cusparseDgtsv2_bufferSizeExt(cusparseHandle_t handle,
                             int              m,
                             int              n,
                             const double*    dl,
                             const double*    d,
                             const double*    du,
                             const double*    B,
                             int              ldb,
                             size_t*          bufferSizeInBytes)

cusparseStatus_t
cusparseCgtsv2_bufferSizeExt(cusparseHandle_t handle,
                             int              m,
                             int              n,
                             const cuComplex* dl,
                             const cuComplex* d,
                             const cuComplex* du,
                             const cuComplex* B,
                             int              ldb,
                             size_t*          bufferSizeInBytes)

cusparseStatus_t
cusparseZgtsv2_bufferSizeExt(cusparseHandle_t       handle,
                             int                    m,
                             int                    n,
                             const cuDoubleComplex* dl,
                             const cuDoubleComplex* d,
                             const cuDoubleComplex* du,
                             const cuDoubleComplex* B,
                             int                    ldb,
                             size_t*                bufferSizeInBytes)
```

This function returns the size of the buffer used in `gtsv2` which computes the solution of a tridiagonal linear system with multiple right-hand sides.

$$A * X = B$$

The coefficient matrix `A` of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (`dl`), main (`d`), and upper (`du`) matrix diagonals; the right-hand sides are stored in the dense matrix `B`. Notice that solution `X` overwrites right-hand-side matrix `B` on exit.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`m`	the size of the linear system (must be ≥ 3).
`n`	number of right-hand sides, columns of matrix `B`.
`dl`	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.

d	<type> dense array containing the main diagonal of the tri-diagonal linear system.
du	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.
B	<type> dense right-hand-side array of dimensions (ldb, n).
ldb	leading dimension of B (that is $\geq \max(1, m)$).

Output

pBufferSizeInBytes	number of bytes of the buffer used in the gtsv2.

See cusparseStatus_t for the description of the return status.

11.3.2. cusparse<t>gtsv2()

```
cusparseStatus_t
cusparseSgtsv2(cusparseHandle_t handle,
               int              m,
               int              n,
               const float*     dl,
               const float*     d,
               const float*     du,
               float*           B,
               int              ldb,
               void             pBuffer)

cusparseStatus_t
cusparseDgtsv2(cusparseHandle_t handle,
               int              m,
               int              n,
               const double*    dl,
               const double*    d,
               const double*    du,
               double*          B,
               int              ldb,
               void             pBuffer)

cusparseStatus_t
cusparseCgtsv2(cusparseHandle_t handle,
               int              m,
               int              n,
               const cuComplex* dl,
               const cuComplex* d,
               const cuComplex* du,
               cuComplex*       B,
               int              ldb,
               void             pBuffer)

cusparseStatus_t
cusparseZgtsv2(cusparseHandle_t       handle,
               int                    m,
               int                    n,
               const cuDoubleComplex* dl,
               const cuDoubleComplex* d,
```

```
                 const cuDoubleComplex* du,
                 cuDoubleComplex*       B,
                 int                    ldb,
                 void                   pBuffer)
```

This function computes the solution of a tridiagonal linear system with multiple right-hand sides:

$$A * X = B$$

The coefficient matrix `A` of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (`dl`), main (`d`), and upper (`du`) matrix diagonals; the right-hand sides are stored in the dense matrix `B`. Notice that solution `X` overwrites right-hand-side matrix `B` on exit.

Assuming `A` is of size `m` and base-1, `dl`, `d` and `du` are defined by the following formula:

`dl(i) := A(i, i-1)` for `i=1,2,...,m`

The first element of dl is out-of-bound (`dl(1) := A(1,0)`), so `dl(1) = 0`.

`d(i) = A(i,i)` for `i=1,2,...,m`

`du(i) = A(i,i+1)` for `i=1,2,...,m`

The last element of du is out-of-bound (`du(m) := A(m,m+1)`), so `du(m) = 0`.

The routine does perform pivoting, which usually results in more accurate and more stable results than `cusparse<t>gtsv_nopivot()` or `cusparse<t>gtsv2_nopivot()` at the expense of some execution time.

This function requires a buffer size returned by `gtsv2_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`m`	the size of the linear system (must be ≥ 3).
`n`	number of right-hand sides, columns of matrix `B`.
`dl`	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.
`d`	<type> dense array containing the main diagonal of the tri-diagonal linear system.
`du`	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.

B	<type> dense right-hand-side array of dimensions (ldb, n).
ldb	leading dimension of B (that is $\geq \max(1, m)$).
pBuffer	buffer allocated by the user, the size is return by gtsv2_bufferSizeExt.

Output

B	<type> dense solution array of dimensions (ldb, n).

See cusparseStatus_t for the description of the return status.

11.3.3. cusparse<t>gtsv2_nopivot_bufferSizeExt()

```
cusparseStatus_t
cusparseSgtsv2_nopivot_bufferSizeExt(cusparseHandle_t handle,
                                     int              m,
                                     int              n,
                                     const float*     dl,
                                     const float*     d,
                                     const float*     du,
                                     const float*     B,
                                     int              ldb,
                                     size_t*          bufferSizeInBytes)

cusparseStatus_t
cusparseDgtsv2_nopivot_bufferSizeExt(cusparseHandle_t handle,
                                     int              m,
                                     int              n,
                                     const double*    dl,
                                     const double*    d,
                                     const double*    du,
                                     const double*    B,
                                     int              ldb,
                                     size_t*          bufferSizeInBytes)

cusparseStatus_t
cusparseCgtsv2_nopivot_bufferSizeExt(cusparseHandle_t handle,
                                     int              m,
                                     int              n,
                                     const cuComplex* dl,
                                     const cuComplex* d,
                                     const cuComplex* du,
                                     const cuComplex* B,
                                     int              ldb,
                                     size_t*          bufferSizeInBytes)

cusparseStatus_t
cusparseZgtsv2_nopivot_bufferSizeExt(cusparseHandle_t       handle,
                                     int                    m,
                                     int                    n,
                                     const cuDoubleComplex* dl,
                                     const cuDoubleComplex* d,
                                     const cuDoubleComplex* du,
                                     const cuDoubleComplex* B,
```

```
                                              int                    ldb,
                                              size_t*
 bufferSizeInBytes)
```

This function returns the size of the buffer used in `gtsv2_nopivot` which computes the solution of a tridiagonal linear system with multiple right-hand sides.

$$A * X = B$$

The coefficient matrix `A` of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (`dl`), main (`d`), and upper (`du`) matrix diagonals; the right-hand sides are stored in the dense matrix `B`. Notice that solution `X` overwrites right-hand-side matrix `B` on exit.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`m`	the size of the linear system (must be ≥ 3).
`n`	number of right-hand sides, columns of matrix `B`.
`dl`	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.
`d`	<type> dense array containing the main diagonal of the tri-diagonal linear system.
`du`	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.
`B`	<type> dense right-hand-side array of dimensions `(ldb, n)`.
`ldb`	leading dimension of `B`. (that is ≥ $\max(1, m)$).

Output

`pBufferSizeInBytes`	number of bytes of the buffer used in the `gtsv2_nopivot`.

See `cusparseStatus_t` for the description of the return status.

11.3.4. cusparse<t>gtsv2_nopivot()

```
cusparseStatus_t
cusparseSgtsv2_nopivot(cusparseHandle_t handle,
                       int              m,
                       int              n,
                       const float*     dl,
```

```
                          const float*     d,
                          const float*     du,
                          float*           B,
                          int              ldb,
                          void*            pBuffer)

cusparseStatus_t
cusparseDgtsv2_nopivot(cusparseHandle_t handle,
                          int              m,
                          int              n,
                          const double*    dl,
                          const double*    d,
                          const double*    du,
                          double*          B,
                          int              ldb,
                          void*            pBuffer)

cusparseStatus_t
cusparseCgtsv2_nopivot(cusparseHandle_t handle,
                          int              m,
                          int              n,
                          const cuComplex* dl,
                          const cuComplex* d,
                          const cuComplex* du,
                          cuComplex*       B,
                          int              ldb,
                          void*            pBuffer)

cusparseStatus_t
cusparseZgtsv2_nopivot(cusparseHandle_t       handle,
                          int                    m,
                          int                    n,
                          const cuDoubleComplex* dl,
                          const cuDoubleComplex* d,
                          const cuDoubleComplex* du,
                          cuDoubleComplex*       B,
                          int                    ldb,
                          void*                  pBuffer)
```

This function computes the solution of a tridiagonal linear system with multiple right-hand sides:

$$A * X = B$$

The coefficient matrix `A` of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (`dl`), main (`d`), and upper (`du`) matrix diagonals; the right-hand sides are stored in the dense matrix `B`. Notice that solution `X` overwrites right-hand-side matrix `B` on exit.

The routine does not perform any pivoting and uses a combination of the Cyclic Reduction (CR) and the Parallel Cyclic Reduction (PCR) algorithms to find the solution. It achieves better performance when `m` is a power of 2.

This function requires a buffer size returned by `gtsv2_nopivot_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

- The routine requires no extra storage

- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`m`	the size of the linear system (must be ≥ 3).
`n`	number of right-hand sides, columns of matrix `B`.
`dl`	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.
`d`	<type> dense array containing the main diagonal of the tri-diagonal linear system.
`du`	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.
`B`	<type> dense right-hand-side array of dimensions `(ldb, n)`.
`ldb`	leading dimension of `B`. (that is ≥ $\max(1, m)$).
`pBuffer`	buffer allocated by the user, the size is return by `gtsv2_nopivot_bufferSizeExt`.

Output

`B`	<type> dense solution array of dimensions `(ldb, n)`.

See `cusparseStatus_t` for the description of the return status.

11.4. Batched Tridiagonal Solve

Different algorithms for batched tridiagonal solve are discussed in this section.

11.4.1. cusparse<t>gtsv2StridedBatch_bufferSizeExt()

```
cusparseStatus_t
cusparseSgtsv2StridedBatch_bufferSizeExt(cusparseHandle_t handle,
                                         int              m,
                                         const float*     dl,
                                         const float*     d,
                                         const float*     du,
                                         const float*     x,
                                         int              batchCount,
                                         int              batchStride,
                                         size_t*          bufferSizeInBytes)

cusparseStatus_t
cusparseDgtsv2StridedBatch_bufferSizeExt(cusparseHandle_t handle,
                                         int              m,
```

```
                                              const double*     dl,
                                              const double*     d,
                                              const double*     du,
                                              const double*     x,
                                              int               batchCount,
                                              int               batchStride,
                                              size_t*           bufferSizeInBytes)

cusparseStatus_t
cusparseCgtsv2StridedBatch_bufferSizeExt(cusparseHandle_t handle,
                                              int               m,
                                              const cuComplex* dl,
                                              const cuComplex* d,
                                              const cuComplex* du,
                                              const cuComplex* x,
                                              int               batchCount,
                                              int               batchStride,
                                              size_t*           bufferSizeInBytes)

cusparseStatus_t
cusparseZgtsv2StridedBatch_bufferSizeExt(cusparseHandle_t       handle,
                                              int                     m,
                                              const cuDoubleComplex* dl,
                                              const cuDoubleComplex* d,
                                              const cuDoubleComplex* du,
                                              const cuDoubleComplex* x,
                                              int                     batchCount,
                                              int                     batchStride,
                                              size_t*
 bufferSizeInBytes)
```

This function returns the size of the buffer used in `gtsv2StridedBatch` which computes the solution of multiple tridiagonal linear systems for *i*=0,…,`batchCount`:

$$A^{(i)} * \mathrm{y}^{(i)} = \mathrm{x}^{(i)}$$

The coefficient matrix `A` of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (`dl`), main (`d`), and upper (`du`) matrix diagonals; the right-hand sides are stored in the dense matrix `X`. Notice that solution `Y` overwrites right-hand-side matrix `X` on exit. The different matrices are assumed to be of the same size and are stored with a fixed `batchStride` in memory.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`n`	the size of the linear system (must be ≥ 3).
`dl`	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The lower diagonal $dl^{(i)}$ that corresponds to the i^{th} linear system starts at location `dl+batchStride×i` in

	memory. Also, the first element of each lower diagonal must be zero.
`d`	<type> dense array containing the main diagonal of the tri-diagonal linear system. The main diagonal $d^{(i)}$ that corresponds to the i^{th} linear system starts at location `d+batchStride×i` in memory.
`du`	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The upper diagonal $du^{(i)}$ that corresponds to the i^{th} linear system starts at location `du+batchStride×i` in memory. Also, the last element of each upper diagonal must be zero.
`x`	<type> dense array that contains the right-hand-side of the tri-diagonal linear system. The right-hand-side $x^{(i)}$ that corresponds to the i^{th} linear system starts at location `x+batchStride×i`in memory.
`batchCount`	number of systems to solve.
`batchStride`	stride (number of elements) that separates the vectors of every system (must be at least `m`).

Output

`pBufferSizeInBytes`	number of bytes of the buffer used in the `gtsv2StridedBatch`.

See `cusparseStatus_t` for the description of the return status.

11.4.2. cusparse<t>gtsv2StridedBatch()

```
cusparseStatus_t
cusparseSgtsv2StridedBatch(cusparseHandle_t handle,
                           int              m,
                           const float*     dl,
                           const float*     d,
                           const float*     du,
                           float*           x,
                           int              batchCount,
                           int              batchStride,
                           void*            pBuffer)

cusparseStatus_t
cusparseDgtsv2StridedBatch(cusparseHandle_t handle,
                           int              m,
                           const double*    dl,
                           const double*    d,
                           const double*    du,
                           double*          x,
                           int              batchCount,
                           int              batchStride,
                           void*            pBuffer)
```

```
cusparseStatus_t
cusparseCgtsv2StridedBatch(cusparseHandle_t handle,
                           int              m,
                           const cuComplex* dl,
                           const cuComplex* d,
                           const cuComplex* du,
                           cuComplex*       x,
                           int              batchCount,
                           int              batchStride,
                           void*            pBuffer)

cusparseStatus_t
cusparseZgtsv2StridedBatch(cusparseHandle_t       handle,
                           int                    m,
                           const cuDoubleComplex* dl,
                           const cuDoubleComplex* d,
                           const cuDoubleComplex* du,
                           cuDoubleComplex*       x,
                           int                    batchCount,
                           int                    batchStride,
                           void*                  pBuffer)
```

This function computes the solution of multiple tridiagonal linear systems for *i*=0, …,`batchCount`:

$$A^{(i)} * \mathrm{y}^{(i)} = \mathrm{x}^{(i)}$$

The coefficient matrix `A` of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (`dl`), main (`d`), and upper (`du`) matrix diagonals; the right-hand sides are stored in the dense matrix `X`. Notice that solution `Y` overwrites right-hand-side matrix `X` on exit. The different matrices are assumed to be of the same size and are stored with a fixed `batchStride` in memory.

The routine does not perform any pivoting and uses a combination of the Cyclic Reduction (CR) and the Parallel Cyclic Reduction (PCR) algorithms to find the solution. It achieves better performance when `m` is a power of 2.

This function requires a buffer size returned by `gtsv2StridedBatch_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`n`	the size of the linear system (must be ≥ 3).
`dl`	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The lower diagonal $dl^{(i)}$ that corresponds to the i^{th} linear system starts at location `dl+batchStride×i` in

	memory. Also, the first element of each lower diagonal must be zero.
d	<type> dense array containing the main diagonal of the tri-diagonal linear system. The main diagonal $d^{(i)}$ that corresponds to the i^{th} linear system starts at location `d+batchStride×i` in memory.
du	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The upper diagonal $du^{(i)}$ that corresponds to the i^{th} linear system starts at location `du+batchStride×i` in memory. Also, the last element of each upper diagonal must be zero.
x	<type> dense array that contains the right-hand-side of the tri-diagonal linear system. The right-hand-side $x^{(i)}$ that corresponds to the i^{th} linear system starts at location `x+batchStride×i`in memory.
batchCount	number of systems to solve.
batchStride	stride (number of elements) that separates the vectors of every system (must be at least `n`).
pBuffer	buffer allocated by the user, the size is return by `gtsv2StridedBatch_bufferSizeExt`.

Output

x	<type> dense array that contains the solution of the tri-diagonal linear system. The solution $x^{(i)}$ that corresponds to the i^{th} linear system starts at location `x+batchStride×i`in memory.

See `cusparseStatus_t` for the description of the return status.

11.4.3. cusparse<t>gtsvInterleavedBatch()

```
cusparseStatus_t
cusparseSgtsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                            int              algo,
                                            int              m,
                                            const float*     dl,
                                            const float*     d,
                                            const float*     du,
                                            const float*     x,
                                            int              batchCount,
                                            size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseDgtsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                            int              algo,
                                            int              m,
```

```
                                             const double*    dl,
                                             const double*    d,
                                             const double*    du,
                                             const double*    x,
                                             int              batchCount,
                                             size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseCgtsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                             int              algo,
                                             int              m,
                                             const cuComplex* dl,
                                             const cuComplex* d,
                                             const cuComplex* du,
                                             const cuComplex* x,
                                             int              batchCount,
                                             size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseZgtsvInterleavedBatch_bufferSizeExt(cusparseHandle_t       handle,
                                             int                    algo,
                                             int                    m,
                                             const cuDoubleComplex* dl,
                                             const cuDoubleComplex* d,
                                             const cuDoubleComplex* du,
                                             const cuDoubleComplex* x,
                                             int
 batchCount,
                                             size_t*
 pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseSgtsvInterleavedBatch(cusparseHandle_t handle,
                              int              algo,
                              int              m,
                              float*           dl,
                              float*           d,
                              float*           du,
                              float*           x,
                              int              batchCount,
                              void*            pBuffer)

cusparseStatus_t
cusparseDgtsvInterleavedBatch(cusparseHandle_t handle,
                              int              algo,
                              int              m,
                              double*          dl,
                              double*          d,
                              double*          du,
                              double*          x,
                              int              batchCount,
                              void*            pBuffer)

cusparseStatus_t
cusparseCgtsvInterleavedBatch(cusparseHandle_t handle,
                              int              algo,
                              int              m,
                              cuComplex*       dl,
```

```
                               cuComplex*       d,
                               cuComplex*       du,
                               cuComplex*       x,
                               int              batchCount,
                               void*            pBuffer)

cusparseStatus_t
cusparseZgtsvInterleavedBatch(cusparseHandle_t handle,
                               int              algo,
                               int              m,
                               cuDoubleComplex* dl,
                               cuDoubleComplex* d,
                               cuDoubleComplex* du,
                               cuDoubleComplex* x,
                               int              batchCount,
                               void*            pBuffer)
```

This function computes the solution of multiple tridiagonal linear systems for *i*=0, …,`batchCount`:

$$A^{(i)} * x^{(i)} = b^{(i)}$$

The coefficient matrix `A` of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (`dl`), main (`d`), and upper (`du`) matrix diagonals; the right-hand sides are stored in the dense matrix `B`. Notice that solution `X` overwrites right-hand-side matrix `B` on exit.

Assuming `A` is of size `m` and base-1, `dl`, `d` and `du` are defined by the following formula:

`dl(i) := A(i, i-1)` for `i=1,2,...,m`

The first element of dl is out-of-bound (`dl(1) := A(1,0)`), so `dl(1) = 0`.

`d(i) = A(i,i)` for `i=1,2,...,m`

`du(i) = A(i,i+1)` for `i=1,2,...,m`

The last element of du is out-of-bound (`du(m) := A(m,m+1)`), so `du(m) = 0`.

The data layout is different from `gtsvStridedBatch` which aggregates all matrices one after another. Instead, `gtsvInterleavedBatch` gathers different matrices of the same element in a continous manner. If `dl` is regarded as a 2-D array of size `m-by-batchCount`, `dl(:,j)` to store `j-th` matrix. `gtsvStridedBatch` uses column-major while `gtsvInterleavedBatch` uses row-major.

The routine provides three different algorithms, selected by parameter `algo`. The first algorithm is `cuThomas` provided by `Barcelona Supercomputing Center`. The second algorithm is LU with partial pivoting and last algorithm is QR. From stability perspective, cuThomas is not numerically stable because it does not have pivoting. LU with partial pivoting and QR are stable. From performance perspective, LU with partial pivoting and QR is about 10% to 20% slower than cuThomas.

This function requires a buffer size returned by `gtsvInterleavedBatch_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

If the user prepares aggregate format, one can use `cublasXgeam` to get interleaved format. However such transformation takes time comparable to solver itself. To reach best performance, the user must prepare interleaved format explicitly.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`algo`	algo = 0: cuThomas (unstable algorithm); algo = 1: LU with pivoting (stable algorithm); algo = 2: QR (stable algorithm)
`m`	the size of the linear system.
`dl`	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.
`d`	<type> dense array containing the main diagonal of the tri-diagonal linear system.
`du`	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.
`x`	<type> dense right-hand-side array of dimensions `(batchCount, n)`.
`pBuffer`	buffer allocated by the user, the size is return by `gtsvInterleavedBatch_bufferSizeExt`.

Output

`x`	<type> dense solution array of dimensions `(batchCount, n)`.

See `cusparseStatus_t` for the description of the return status.

11.5. Batched Pentadiagonal Solve

Different algorithms for batched pentadiagonal solve are discussed in this section.

11.5.1. cusparse<t>gpsvInterleavedBatch()

```
cusparseStatus_t
cusparseSgpsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                            int              algo,
                                            int              m,
```

```
                                                  const float*     ds,
                                                  const float*     dl,
                                                  const float*     d,
                                                  const float*     du,
                                                  const float*     dw,
                                                  const float*     x,
                                                  int              batchCount,
                                                  size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseDgpsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                                  int              algo,
                                                  int              m,
                                                  const double*    ds,
                                                  const double*    dl,
                                                  const double*    d,
                                                  const double*    du,
                                                  const double*    dw,
                                                  const double*    x,
                                                  int              batchCount,
                                                  size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseCgpsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                                  int              algo,
                                                  int              m,
                                                  const cuComplex* ds,
                                                  const cuComplex* dl,
                                                  const cuComplex* d,
                                                  const cuComplex* du,
                                                  const cuComplex* dw,
                                                  const cuComplex* x,
                                                  int              batchCount,
                                                  size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseZgpsvInterleavedBatch_bufferSizeExt(cusparseHandle_t       handle,
                                                  int                    algo,
                                                  int                    m,
                                                  const cuDoubleComplex* ds,
                                                  const cuDoubleComplex* dl,
                                                  const cuDoubleComplex* d,
                                                  const cuDoubleComplex* du,
                                                  const cuDoubleComplex* dw,
                                                  const cuDoubleComplex* x,
                                                  int
 batchCount,
                                                  size_t*
 pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseSgpsvInterleavedBatch(cusparseHandle_t handle,
                                  int              algo,
                                  int              m,
                                  float*           ds,
                                  float*           dl,
                                  float*           d,
```

```
                              float*           du,
                              float*           dw,
                              float*           x,
                              int              batchCount,
                              void*            pBuffer)

cusparseStatus_t
cusparseDgpsvInterleavedBatch(cusparseHandle_t handle,
                              int              algo,
                              int              m,
                              double*          ds,
                              double*          dl,
                              double*          d,
                              double*          du,
                              double*          dw,
                              double*          x,
                              int              batchCount,
                              void*            pBuffer)

cusparseStatus_t
cusparseCgpsvInterleavedBatch(cusparseHandle_t handle,
                              int              algo,
                              int              m,
                              cuComplex*       ds,
                              cuComplex*       dl,
                              cuComplex*       d,
                              cuComplex*       du,
                              cuComplex*       dw,
                              cuComplex*       x,
                              int              batchCount,
                              void*            pBuffer)

cusparseStatus_t
cusparseZgpsvInterleavedBatch(cusparseHandle_t handle,
                              int              algo,
                              int              m,
                              cuDoubleComplex* ds,
                              cuDoubleComplex* dl,
                              cuDoubleComplex* d,
                              cuDoubleComplex* du,
                              cuDoubleComplex* dw,
                              cuDoubleComplex* x,
                              int              batchCount,
                              void*            pBuffer)
```

This function computes the solution of multiple penta-diagonal linear systems for *i*=0, ...,`batchCount`:

$$A^{(i)} * \mathrm{x}^{(i)} = \mathrm{b}^{(i)}$$

The coefficient matrix `A` of each of these penta-diagonal linear system is defined with five vectors corresponding to its lower (`ds, dl`), main (`d`), and upper (`du, dw`) matrix diagonals; the right-hand sides are stored in the dense matrix `B`. Notice that solution `X` overwrites right-hand-side matrix `B` on exit.

Assuming `A` is of size `m` and base-1, `ds`, `dl`, `d`, `du` and `dw` are defined by the following formula:

`ds(i) := A(i, i-2)` for `i=1,2,...,m`

The first two elements of ds is out-of-bound (`ds(1) := A(1,-1)`, `ds(2) := A(2,0)`), so `ds(1) = 0` and `ds(2) = 0`.

`dl(i) := A(i, i-1)` for `i=1,2,...,m`

The first element of dl is out-of-bound (`dl(1) := A(1,0)`), so `dl(1) = 0`.

`d(i) = A(i,i)` for `i=1,2,...,m`

`du(i) = A(i,i+1)` for `i=1,2,...,m`

The last element of du is out-of-bound (`du(m) := A(m,m+1)`), so `du(m) = 0`.

`dw(i) = A(i,i+2)` for `i=1,2,...,m`

The last two elements of dw is out-of-bound (`dw(m-1) := A(m-1,m+1)`, `dw(m) := A(m,m+2)`), so `dw(m-1) = 0` and `dw(m) = 0`.

The data layout is the same as `gtsvStridedBatch`.

The routine is numerically stable because it uses QR to solve the linear system.

This function requires a buffer size returned by `gpsvInterleavedBatch_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

The function supports the following properties if `pBuffer != NULL`

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`algo`	only support algo = 0 (QR)
`m`	the size of the linear system.
`ds`	<type> dense array containing the lower diagonal (distance 2 to the diagonal) of the penta-diagonal linear system. The first two elements must be zero.
`dl`	<type> dense array containing the lower diagonal (distance 1 to the diagonal) of the penta-diagonal linear system. The first element must be zero.
`d`	<type> dense array containing the main diagonal of the penta-diagonal linear system.
`du`	<type> dense array containing the upper diagonal (distance 1 to the diagonal) of the penta-diagonal linear system. The last element must be zero.
`dw`	<type> dense array containing the upper diagonal (distance 2 to the diagonal) of the penta-diagonal linear system. The last two elements must be zero.

`x`	<type> dense right-hand-side array of dimensions `(batchCount, n)`.
`pBuffer`	buffer allocated by the user, the size is return by `gpsvInterleavedBatch_bufferSizeExt`.

Output

`x`	<type> dense solution array of dimensions `(batchCount, n)`.

See `cusparseStatus_t` for the description of the return status.

Please visit cuSPARSE Library Samples - cusparseSgpsvInterleavedBatch for a code example.

Chapter 12. cuSPARSE Reorderings Reference

This chapter describes the reordering routines used to manipulate sparse matrices.

12.1. cusparse<t>csrcolor()

```
cusparseStatus_t
cusparseScsrcolor(cusparseHandle_t         handle,
                  int                      m,
                  int                      nnz,
                  const cusparseMatDescr_t descrA,
                  const float*             csrValA,
                  const int*               csrRowPtrA,
                  const int*               csrColIndA,
                  const float*             fractionToColor,
                  int*                     ncolors,
                  int*                     coloring,
                  int*                     reordering,
                  cusparseColorInfo_t      info)

cusparseStatus_t
cusparseDcsrcolor(cusparseHandle_t         handle,
                  int                      m,
                  int                      nnz,
                  const cusparseMatDescr_t descrA,
                  const double*            csrValA,
                  const int*               csrRowPtrA,
                  const int*               csrColIndA,
                  const double*            fractionToColor,
                  int*                     ncolors,
                  int*                     coloring,
                  int*                     reordering,
                  cusparseColorInfo_t      info)

cusparseStatus_t
cusparseCcsrcolor(cusparseHandle_t         handle,
                  int                      m,
                  int                      nnz,
                  const cusparseMatDescr_t descrA,
                  const cuComplex*         csrValA,
                  const int*               csrRowPtrA,
                  const int*               csrColIndA,
```

```
                    const cuComplex*          fractionToColor,
                    int*                      ncolors,
                    int*                      coloring,
                    int*                      reordering,
                    cusparseColorInfo_t       info)

cusparseStatus_t
cusparseZcsrcolor(cusparseHandle_t          handle,
                    int                       m,
                    int                       nnz,
                    const cusparseMatDescr_t descrA,
                    const cuDoubleComplex*    csrValA,
                    const int*                csrRowPtrA,
                    const int*                csrColIndA,
                    const cuDoubleComplex*    fractionToColor,
                    int*                      ncolors,
                    int*                      coloring,
                    int*                      reordering,
                    cusparseColorInfo_t       info)
```

This function performs the coloring of the adjacency graph associated with the matrix A stored in CSR format. The coloring is an assignment of colors (integer numbers) to nodes, such that neighboring nodes have distinct colors. An approximate coloring algorithm is used in this routine, and is stopped when a certain percentage of nodes has been colored. The rest of the nodes are assigned distinct colors (an increasing sequence of integers numbers, starting from the last integer used previously). The last two auxiliary routines can be used to extract the resulting number of colors, their assignment and the associated reordering. The reordering is such that nodes that have been assigned the same color are reordered to be next to each other.

The matrix A passed to this routine, must be stored as a general matrix and have a symmetric sparsity pattern. If the matrix is nonsymmetric the user should pass A+A^T as a parameter to this routine.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`m`	number of rows of matrix `A`.
`nnz`	number of nonzero elements of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrValA`	<type> array of `nnz` (= `csrRowPtrA(m)` − `csrRowPtrA(0)`) nonzero elements of matrix `A`.

csrRowPtrA	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column indices of the nonzero elements of matrix A.
fractionToColor	fraction of nodes to be colored, which should be in the interval [0.0,1.0], for example 0.8 implies that 80 percent of nodes will be colored.
info	structure with information to be passed to the coloring.

Output

ncolors	The number of distinct colors used (at most the size of the matrix, but likely much smaller).
coloring	The resulting coloring permutation
reordering	The resulting reordering permutation (untouched if NULL)

See cusparseStatus_t for the description of the return status.

Chapter 13. cuSPARSE Format Conversion Reference

This chapter describes the conversion routines between different sparse and dense storage formats.

`coosort`, `csrsort`, `cscsort`, and `csru2csr` are sorting routines without malloc inside, the following table estimates the buffer size.

routine	buffer size	maximum problem size if buffer is limited by 2GB
coosort	> 16*n bytes	125M
csrsort or cscsort	> 20*n bytes	100M
csru2csr	'd' > 28*n bytes ; 'z' > 36*n bytes	71M for 'd' and 55M for 'z'

13.1. cusparse<t>bsr2csr()

```
cusparseStatus_t
cusparseSbsr2csr(cusparseHandle_t         handle,
                 cusparseDirection_t      dir,
                 int                      mb,
                 int                      nb,
                 const cusparseMatDescr_t descrA,
                 const float*             bsrValA,
                 const int*               bsrRowPtrA,
                 const int*               bsrColIndA,
                 int                      blockDim,
                 const cusparseMatDescr_t descrC,
                 float*                   csrValC,
                 int*                     csrRowPtrC,
                 int*                     csrColIndC)

cusparseStatus_t
cusparseDbsr2csr(cusparseHandle_t         handle,
                 cusparseDirection_t      dir,
                 int                      mb,
                 int                      nb,
                 const cusparseMatDescr_t descrA,
                 const double*            bsrValA,
```

```
                    const int*                bsrRowPtrA,
                    const int*                bsrColIndA,
                    int                       blockDim,
                    const cusparseMatDescr_t descrC,
                    double*                   csrValC,
                    int*                      csrRowPtrC,
                    int*                      csrColIndC)

cusparseStatus_t
cusparseCbsr2csr(cusparseHandle_t          handle,
                    cusparseDirection_t       dir,
                    int                       mb,
                    int                       nb,
                    const cusparseMatDescr_t descrA,
                    const cuComplex*          bsrValA,
                    const int*                bsrRowPtrA,
                    const int*                bsrColIndA,
                    int                       blockDim,
                    const cusparseMatDescr_t descrC,
                    cuComplex*                csrValC,
                    int*                      csrRowPtrC,
                    int*                      csrColIndC)

cusparseStatus_t
cusparseZbsr2csr(cusparseHandle_t          handle,
                    cusparseDirection_t       dir,
                    int                       mb,
                    int                       nb,
                    const cusparseMatDescr_t descrA,
                    const cuDoubleComplex*    bsrValA,
                    const int*                bsrRowPtrA,
                    const int*                bsrColIndA,
                    int                       blockDim,
                    const cusparseMatDescr_t descrC,
                    cuDoubleComplex*          csrValC,
                    int*                      csrRowPtrC,
                    int*                      csrColIndC)
```

This function converts a sparse matrix in BSR format that is defined by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`) into a sparse matrix in CSR format that is defined by arrays `csrValC`, `csrRowPtrC`, and `csrColIndC`.

Let `m(=mb*blockDim)` be the number of rows of `A` and `n(=nb*blockDim)` be number of columns of `A`, then `A` and `C` are `m*n` sparse matrices. The BSR format of `A` contains `nnzb(=bsrRowPtrA[mb] - bsrRowPtrA[0])` nonzero blocks, whereas the sparse matrix `A` contains `nnz(=nnzb*blockDim*blockDim)` elements. The user must allocate enough space for arrays `csrRowPtrC`, `csrColIndC`, and `csrValC`. The requirements are as follows:

`csrRowPtrC` of `m+1` elements

`csrValC` of `nnz` elements

`csrColIndC` of `nnz` elements

The general procedure is as follows:

```
// Given BSR format (bsrRowPtrA, bsrcolIndA, bsrValA) and
// blocks of BSR format are stored in column-major order.
cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int m = mb*blockDim;
int nnzb = bsrRowPtrA[mb] - bsrRowPtrA[0]; // number of blocks
```

```
int nnz  = nnzb * blockDim * blockDim; // number of elements
cudaMalloc((void**)&csrRowPtrC, sizeof(int)*(m+1));
cudaMalloc((void**)&csrColIndC, sizeof(int)*nnz);
cudaMalloc((void**)&csrValC, sizeof(float)*nnz);
cusparseSbsr2csr(handle, dir, mb, nb,
        descrA,
        bsrValA, bsrRowPtrA, bsrColIndA,
        blockDim,
        descrC,
        csrValC, csrRowPtrC, csrColIndC);
```

- The routine requires no extra storage
- The routine supports asynchronous execution if `blockDim != 1` or the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if `blockDim != 1` or the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`dir`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`mb`	number of block rows of sparse matrix `A`.
`nb`	number of block columns of sparse matrix `A`.
`descrA`	the descriptor of matrix `A`.
`bsrValA`	<type> array of `nnzb*blockDim*blockDim` nonzero elements of matrix `A`.
`bsrRowPtrA`	integer array of `mb+1` elements that contains the start of every block row and the end of the last block row plus one of matrix `A`.
`bsrColIndA`	integer array of `nnzb` column indices of the nonzero blocks of matrix `A`.
`blockDim`	block dimension of sparse matrix `A`.
`descrC`	the descriptor of matrix `C`.

Output

`csrValC`	<type> array of `nnz(=csrRowPtrC[m]-csrRowPtrC[0])` nonzero elements of matrix `C`.
`csrRowPtrC`	integer array of `m+1` elements that contains the start of every row and the end of the last row plus one of matrix `C`.
`csrColIndC`	integer array of `nnz` column indices of the nonzero elements of matrix `C`.

See `cusparseStatus_t` for the description of the return status.

13.2. cusparse<t>gebsr2gebsc()

```
cusparseStatus_t
cusparseSgebsr2gebsc_bufferSize(cusparseHandle_t handle,
                                int              mb,
                                int              nb,
                                int              nnzb,
                                const float*     bsrVal,
                                const int*       bsrRowPtr,
                                const int*       bsrColInd,
                                int              rowBlockDim,
                                int              colBlockDim,
                                int*             pBufferSize)

cusparseStatus_t
cusparseDgebsr2gebsc_bufferSize(cusparseHandle_t handle,
                                int              mb,
                                int              nb,
                                int              nnzb,
                                const double*    bsrVal,
                                const int*       bsrRowPtr,
                                const int*       bsrColInd,
                                int              rowBlockDim,
                                int              colBlockDim,
                                int*             pBufferSize)

cusparseStatus_t
cusparseCgebsr2gebsc_bufferSize(cusparseHandle_t handle,
                                int              mb,
                                int              nb,
                                int              nnzb,
                                const cuComplex* bsrVal,
                                const int*       bsrRowPtr,
                                const int*       bsrColInd,
                                int              rowBlockDim,
                                int              colBlockDim,
                                int*             pBufferSize)

cusparseStatus_t
cusparseZgebsr2gebsc_bufferSize(cusparseHandle_t      handle,
                                int                   mb,
                                int                   nb,
                                int                   nnzb,
                                const cuDoubleComplex* bsrVal,
                                const int*            bsrRowPtr,
                                const int*            bsrColInd,
                                int                   rowBlockDim,
                                int                   colBlockDim,
                                int*                  pBufferSize)
```

```
cusparseStatus_t
cusparseSgebsr2gebsc(cusparseHandle_t    handle,
                     int                 mb,
                     int                 nb,
                     int                 nnzb,
```

```
                    const float*          bsrVal,
                    const int*            bsrRowPtr,
                    const int*            bsrColInd,
                    int                   rowBlockDim,
                    int                   colBlockDim,
                    float*                bscVal,
                    int*                  bscRowInd,
                    int*                  bscColPtr,
                    cusparseAction_t      copyValues,
                    cusparseIndexBase_t   baseIdx,
                    void*                 pBuffer)

cusparseStatus_t
cusparseDgebsr2gebsc(cusparseHandle_t    handle,
                    int                   mb,
                    int                   nb,
                    int                   nnzb,
                    const double*         bsrVal,
                    const int*            bsrRowPtr,
                    const int*            bsrColInd,
                    int                   rowBlockDim,
                    int                   colBlockDim,
                    double*               bscVal,
                    int*                  bscRowInd,
                    int*                  bscColPtr,
                    cusparseAction_t      copyValues,
                    cusparseIndexBase_t   baseIdx,
                    void*                 pBuffer)

cusparseStatus_t
cusparseCgebsr2gebsc(cusparseHandle_t    handle,
                    int                   mb,
                    int                   nb,
                    int                   nnzb,
                    const cuComplex*      bsrVal,
                    const int*            bsrRowPtr,
                    const int*            bsrColInd,
                    int                   rowBlockDim,
                    int                   colBlockDim,
                    cuComplex*            bscVal,
                    int*                  bscRowInd,
                    int*                  bscColPtr,
                    cusparseAction_t      copyValues,
                    cusparseIndexBase_t   baseIdx,
                    void*                 pBuffer)

cusparseStatus_t
cusparseZgebsr2gebsc(cusparseHandle_t       handle,
                    int                      mb,
                    int                      nb,
                    int                      nnzb,
                    const cuDoubleComplex*   bsrVal,
                    const int*               bsrRowPtr,
                    const int*               bsrColInd,
                    int                      rowBlockDim,
                    int                      colBlockDim,
                    cuDoubleComplex*         bscVal,
                    int*                     bscRowInd,
                    int*                     bscColPtr,
                    cusparseAction_t         copyValues,
                    cusparseIndexBase_t      baseIdx,
```

```
                        void*                           pBuffer)
```

This function can be seen as the same as `csr2csc()` when each block of size `rowBlockDim*colBlockDim` is regarded as a scalar.

This sparsity pattern of the result matrix can also be seen as the transpose of the original sparse matrix, but the memory layout of a block does not change.

The user must call `gebsr2gebsc_bufferSize()` to determine the size of the buffer required by `gebsr2gebsc()`, allocate the buffer, and pass the buffer pointer to `gebsr2gebsc()`.

- The routine requires no extra storage if `pBuffer != NULL`
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`mb`	number of block rows of sparse matrix `A`.
`nb`	number of block columns of sparse matrix `A`.
`nnzb`	number of nonzero blocks of matrix `A`.
`bsrVal`	<type> array of `nnzb*rowBlockDim*colBlockDim` nonzero elements of matrix `A`.
`bsrRowPtr`	integer array of `mb+1` elements that contains the start of every block row and the end of the last block row plus one.
`bsrColInd`	integer array of `nnzb` column indices of the non-zero blocks of matrix `A`.
`rowBlockDim`	number of rows within a block of `A`.
`colBlockDim`	number of columns within a block of `A`.
`copyValues`	`CUSPARSE_ACTION_SYMBOLIC` or `CUSPARSE_ACTION_NUMERIC`.
`baseIdx`	`CUSPARSE_INDEX_BASE_ZERO` or `CUSPARSE_INDEX_BASE_ONE`.
`pBufferSize`	host pointer containing number of bytes of the buffer used in `gebsr2gebsc()`.
`pBuffer`	buffer allocated by the user; the size is return by `gebsr2gebsc_bufferSize()`.

Output

`bscVal`	<type> array of `nnzb*rowBlockDim*colBlockDim` non-zero elements of matrix `A`. It is only filled-in if `copyValues` is set to `CUSPARSE_ACTION_NUMERIC`.

bscRowInd	integer array of nnzb row indices of the non-zero blocks of matrix A.
bscColPtr	integer array of nb+1 elements that contains the start of every block column and the end of the last block column plus one.

See cusparseStatus_t for the description of the return status.

13.3. cusparse<t>gebsr2gebsr()

```
cusparseStatus_t
cusparseSgebsr2gebsr_bufferSize(cusparseHandle_t         handle,
                                cusparseDirection_t      dir,
                                int                      mb,
                                int                      nb,
                                int                      nnzb,
                                const cusparseMatDescr_t descrA,
                                const float*             bsrValA,
                                const int*               bsrRowPtrA,
                                const int*               bsrColIndA,
                                int                      rowBlockDimA,
                                int                      colBlockDimA,
                                int                      rowBlockDimC,
                                int                      colBlockDimC,
                                int*                     pBufferSize)

cusparseStatus_t
cusparseDgebsr2gebsr_bufferSize(cusparseHandle_t         handle,
                                cusparseDirection_t      dir,
                                int                      mb,
                                int                      nb,
                                int                      nnzb,
                                const cusparseMatDescr_t descrA,
                                const double*            bsrValA,
                                const int*               bsrRowPtrA,
                                const int*               bsrColIndA,
                                int                      rowBlockDimA,
                                int                      colBlockDimA,
                                int                      rowBlockDimC,
                                int                      colBlockDimC,
                                int*                     pBufferSize)

cusparseStatus_t
cusparseCgebsr2gebsr_bufferSize(cusparseHandle_t         handle,
                                cusparseDirection_t      dir,
                                int                      mb,
                                int                      nb,
                                int                      nnzb,
                                const cusparseMatDescr_t descrA,
                                const cuComplex*         bsrValA,
                                const int*               bsrRowPtrA,
                                const int*               bsrColIndA,
                                int                      rowBlockDimA,
                                int                      colBlockDimA,
                                int                      rowBlockDimC,
                                int                      colBlockDimC,
```

```
                                int*                     pBufferSize)

cusparseStatus_t
cusparseZgebsr2gebsr_bufferSize(cusparseHandle_t         handle,
                                cusparseDirection_t      dir,
                                int                      mb,
                                int                      nb,
                                int                      nnzb,
                                const cusparseMatDescr_t descrA,
                                const cuDoubleComplex*   bsrValA,
                                const int*               bsrRowPtrA,
                                const int*               bsrColIndA,
                                int                      rowBlockDimA,
                                int                      colBlockDimA,
                                int                      rowBlockDimC,
                                int                      colBlockDimC,
                                int*                     pBufferSize)
```

```
cusparseStatus_t
cusparseXgebsr2gebsrNnz(cusparseHandle_t         handle,
                        cusparseDirection_t      dir,
                        int                      mb,
                        int                      nb,
                        int                      nnzb,
                        const cusparseMatDescr_t descrA,
                        const int*               bsrRowPtrA,
                        const int*               bsrColIndA,
                        int                      rowBlockDimA,
                        int                      colBlockDimA,
                        const cusparseMatDescr_t descrC,
                        int*                     bsrRowPtrC,
                        int                      rowBlockDimC,
                        int                      colBlockDimC,
                        int*                     nnzTotalDevHostPtr,
                        void*                    pBuffer)

cusparseStatus_t
cusparseSgebsr2gebsr(cusparseHandle_t         handle,
                     cusparseDirection_t      dir,
                     int                      mb,
                     int                      nb,
                     int                      nnzb,
                     const cusparseMatDescr_t descrA,
                     const float*             bsrValA,
                     const int*               bsrRowPtrA,
                     const int*               bsrColIndA,
                     int                      rowBlockDimA,
                     int                      colBlockDimA,
                     const cusparseMatDescr_t descrC,
                     float*                   bsrValC,
                     int*                     bsrRowPtrC,
                     int*                     bsrColIndC,
                     int                      rowBlockDimC,
                     int                      colBlockDimC,
                     void*                    pBuffer)

cusparseStatus_t
cusparseDgebsr2gebsr(cusparseHandle_t         handle,
                     cusparseDirection_t      dir,
                     int                      mb,
```

```
                     int                       nb,
                     int                       nnzb,
                     const cusparseMatDescr_t descrA,
                     const double*             bsrValA,
                     const int*                bsrRowPtrA,
                     const int*                bsrColIndA,
                     int                       rowBlockDimA,
                     int                       colBlockDimA,
                     const cusparseMatDescr_t descrC,
                     double*                   bsrValC,
                     int*                      bsrRowPtrC,
                     int*                      bsrColIndC,
                     int                       rowBlockDimC,
                     int                       colBlockDimC,
                     void*                     pBuffer)

cusparseStatus_t
cusparseCgebsr2gebsr(cusparseHandle_t          handle,
                     cusparseDirection_t       dir,
                     int                       mb,
                     int                       nb,
                     int                       nnzb,
                     const cusparseMatDescr_t descrA,
                     const cuComplex*          bsrValA,
                     const int*                bsrRowPtrA,
                     const int*                bsrColIndA,
                     int                       rowBlockDimA,
                     int                       colBlockDimA,
                     const cusparseMatDescr_t descrC,
                     cuComplex*                bsrValC,
                     int*                      bsrRowPtrC,
                     int*                      bsrColIndC,
                     int                       rowBlockDimC,
                     int                       colBlockDimC,
                     void*                     pBuffer)

cusparseStatus_t
cusparseZgebsr2gebsr(cusparseHandle_t          handle,
                     cusparseDirection_t       dir,
                     int                       mb,
                     int                       nb,
                     int                       nnzb,
                     const cusparseMatDescr_t descrA,
                     const cuDoubleComplex*    bsrValA,
                     const int*                bsrRowPtrA,
                     const int*                bsrColIndA,
                     int                       rowBlockDimA,
                     int                       colBlockDimA,
                     const cusparseMatDescr_t descrC,
                     cuDoubleComplex*          bsrValC,
                     int*                      bsrRowPtrC,
                     int*                      bsrColIndC,
                     int                       rowBlockDimC,
                     int                       colBlockDimC,
                     void*                     pBuffer)
```

This function converts a sparse matrix in general BSR format that is defined by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA` into a sparse matrix in another general BSR format that is defined by arrays `bsrValC`, `bsrRowPtrC`, and `bsrColIndC`.

If `rowBlockDimA=1` and `colBlockDimA=1`, `cusparse[S|D|C|Z]gebsr2gebsr()` is the same as `cusparse[S|D|C|Z]csr2gebsr()`.

If `rowBlockDimC=1` and `colBlockDimC=1`, `cusparse[S|D|C|Z]gebsr2gebsr()` is the same as `cusparse[S|D|C|Z]gebsr2csr()`.

`A` is an `m*n` sparse matrix where `m(=mb*rowBlockDim)` is the number of rows of `A`, and `n(=nb*colBlockDim)` is the number of columns of `A`. The general BSR format of `A` contains `nnzb(=bsrRowPtrA[mb] - bsrRowPtrA[0])` nonzero blocks. The matrix `C` is also general BSR format with a different block size, `rowBlockDimC*colBlockDimC`. If `m` is not a multiple of `rowBlockDimC`, or `n` is not a multiple of `colBlockDimC`, zeros are filled in. The number of block rows of `C` is `mc(=(m+rowBlockDimC-1)/rowBlockDimC)`. The number of block rows of `C` is `nc(=(n+colBlockDimC-1)/colBlockDimC)`. The number of nonzero blocks of `C` is `nnzc`.

The implementation adopts a two-step approach to do the conversion. First, the user allocates `bsrRowPtrC` of `mc+1` elements and uses function `cusparseXgebsr2gebsrNnz()` to determine the number of nonzero block columns per block row of matrix `C`. Second, the user gathers `nnzc` (number of non-zero block columns of matrix `C`) from either `(nnzc=*nnzTotalDevHostPtr)` or `(nnzc=bsrRowPtrC[mc]-bsrRowPtrC[0])` and allocates `bsrValC` of `nnzc*rowBlockDimC*colBlockDimC` elements and `bsrColIndC` of `nnzc` integers. Finally the function `cusparse[S|D|C|Z]gebsr2gebsr()` is called to complete the conversion.

The user must call `gebsr2gebsr_bufferSize()` to know the size of the buffer required by `gebsr2gebsr()`, allocate the buffer, and pass the buffer pointer to `gebsr2gebsr()`.

The general procedure is as follows:

```
// Given general BSR format (bsrRowPtrA, bsrColIndA, bsrValA) and
// blocks of BSR format are stored in column-major order.
cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int base, nnzc;
int m = mb*rowBlockDimA;
int n = nb*colBlockDimA;
int mc = (m+rowBlockDimC-1)/rowBlockDimC;
int nc = (n+colBlockDimC-1)/colBlockDimC;
int bufferSize;
void *pBuffer;
cusparseSgebsr2gebsr_bufferSize(handle, dir, mb, nb, nnzb,
    descrA, bsrValA, bsrRowPtrA, bsrColIndA,
    rowBlockDimA, colBlockDimA,
    rowBlockDimC, colBlockDimC,
    &bufferSize);
cudaMalloc((void**)&pBuffer, bufferSize);
cudaMalloc((void**)&bsrRowPtrC, sizeof(int)*(mc+1));
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzc;
cusparseXgebsr2gebsrNnz(handle, dir, mb, nb, nnzb,
    descrA, bsrRowPtrA, bsrColIndA,
    rowBlockDimA, colBlockDimA,
    descrC, bsrRowPtrC,
    rowBlockDimC, colBlockDimC,
    nnzTotalDevHostPtr,
    pBuffer);
if (NULL != nnzTotalDevHostPtr){
    nnzc = *nnzTotalDevHostPtr;
}else{
    cudaMemcpy(&nnzc, bsrRowPtrC+mc, sizeof(int), cudaMemcpyDeviceToHost);
    cudaMemcpy(&base, bsrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
    nnzc -= base;
}
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzc);
```

```
cudaMalloc((void**)&bsrValC, sizeof(float)*(rowBlockDimC*colBlockDimC)*nnzc);
cusparseSgebsr2gebsr(handle, dir, mb, nb, nnzb,
    descrA, bsrValA, bsrRowPtrA, bsrColIndA,
    rowBlockDimA, colBlockDimA,
    descrC, bsrValC, bsrRowPtrC, bsrColIndC,
    rowBlockDimC, colBlockDimC,
    pBuffer);
```

- The routines require no extra storage if `pBuffer != NULL`
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routines do **not** support CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`dir`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`mb`	number of block rows of sparse matrix `A`.
`nb`	number of block columns of sparse matrix `A`.
`nnzb`	number of nonzero blocks of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`bsrValA`	<type> array of `nnzb*rowBlockDimA*colBlockDimA` non-zero elements of matrix `A`.
`bsrRowPtrA`	integer array of `mb+1` elements that contains the start of every block row and the end of the last block row plus one of matrix `A`.
`bsrColIndA`	integer array of `nnzb` column indices of the non-zero blocks of matrix `A`.
`rowBlockDimA`	number of rows within a block of `A`.
`colBlockDimA`	number of columns within a block of `A`.
`descrC`	the descriptor of matrix `C`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`rowBlockDimC`	number of rows within a block of `C`.
`colBlockDimC`	number of columns within a block of `C`.
`pBufferSize`	host pointer containing number of bytes of the buffer used in `gebsr2gebsr()`.
`pBuffer`	buffer allocated by the user; the size is return by `gebsr2gebsr_bufferSize()`.

Output

bsrValC	<type> array of nnzc*rowBlockDimC*colBlockDimC non-zero elements of matrix C.
bsrRowPtrC	integer array of mc+1 elements that contains the start of every block row and the end of the last block row plus one of matrix C.
bsrColIndC	integer array of nnzc block column indices of the nonzero blocks of matrix C.
nnzTotalDevHostPtr	total number of nonzero blocks of C. *nnzTotalDevHostPtr is the same as bsrRowPtrC[mc]-bsrRowPtrC[0].

See cusparseStatus_t for the description of the return status.

13.4. cusparse<t>gebsr2csr()

```
cusparseStatus_t
cusparseSgebsr2csr(cusparseHandle_t         handle,
                   cusparseDirection_t      dir,
                   int                      mb,
                   int                      nb,
                   const cusparseMatDescr_t descrA,
                   const float*             bsrValA,
                   const int*               bsrRowPtrA,
                   const int*               bsrColIndA,
                   int                      rowBlockDim,
                   int                      colBlockDim,
                   const cusparseMatDescr_t descrC,
                   float*                   csrValC,
                   int*                     csrRowPtrC,
                   int*                     csrColIndC)

cusparseStatus_t
cusparseDgebsr2csr(cusparseHandle_t         handle,
                   cusparseDirection_t      dir,
                   int                      mb,
                   int                      nb,
                   const cusparseMatDescr_t descrA,
                   const double*            bsrValA,
                   const int*               bsrRowPtrA,
                   const int*               bsrColIndA,
                   int                      rowBlockDim,
                   int                      colBlockDim,
                   const cusparseMatDescr_t descrC,
                   double*                  csrValC,
                   int*                     csrRowPtrC,
                   int*                     csrColIndC)

cusparseStatus_t
cusparseCgebsr2csr(cusparseHandle_t         handle,
                   cusparseDirection_t      dir,
                   int                      mb,
```

```
                     int                      nb,
                     const cusparseMatDescr_t descrA,
                     const cuComplex*         bsrValA,
                     const int*               bsrRowPtrA,
                     const int*               bsrColIndA,
                     int                      rowBlockDim,
                     int                      colBlockDim,
                     const cusparseMatDescr_t descrC,
                     cuComplex*               csrValC,
                     int*                     csrRowPtrC,
                     int*                     csrColIndC)

cusparseStatus_t
cusparseZgebsr2csr(cusparseHandle_t         handle,
                     cusparseDirection_t      dir,
                     int                      mb,
                     int                      nb,
                     const cusparseMatDescr_t descrA,
                     const cuDoubleComplex*   bsrValA,
                     const int*               bsrRowPtrA,
                     const int*               bsrColIndA,
                     int                      rowBlockDim,
                     int                      colBlockDim,
                     const cusparseMatDescr_t descrC,
                     cuDoubleComplex*         csrValC,
                     int*                     csrRowPtrC,
                     int*                     csrColIndC)
```

This function converts a sparse matrix in general BSR format that is defined by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA` into a sparse matrix in CSR format that is defined by arrays `csrValC`, `csrRowPtrC`, and `csrColIndC`.

Let `m(=mb*rowBlockDim)` be number of rows of `A` and `n(=nb*colBlockDim)` be number of columns of `A`, then `A` and `C` are `m*n` sparse matrices. The general BSR format of `A` contains `nnzb(=bsrRowPtrA[mb] - bsrRowPtrA[0])` non-zero blocks, whereas sparse matrix `A` contains `nnz(=nnzb*rowBlockDim*colBlockDim)` elements. The user must allocate enough space for arrays `csrRowPtrC`, `csrColIndC`, and `csrValC`. The requirements are as follows:

`csrRowPtrC` of `m+1` elements

`csrValC` of `nnz` elements

`csrColIndC` of `nnz` elements

The general procedure is as follows:

```
// Given general BSR format (bsrRowPtrA, bsrColIndA, bsrValA) and
// blocks of BSR format are stored in column-major order.
cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int m = mb*rowBlockDim;
int n = nb*colBlockDim;
int nnzb = bsrRowPtrA[mb] - bsrRowPtrA[0]; // number of blocks
int nnz  = nnzb * rowBlockDim * colBlockDim; // number of elements
cudaMalloc((void**)&csrRowPtrC, sizeof(int)*(m+1));
cudaMalloc((void**)&csrColIndC, sizeof(int)*nnz);
cudaMalloc((void**)&csrValC, sizeof(float)*nnz);
cusparseSgebsr2csr(handle, dir, mb, nb,
        descrA,
        bsrValA, bsrRowPtrA, bsrColIndA,
        rowBlockDim, colBlockDim,
        descrC,
        csrValC, csrRowPtrC, csrColIndC);
```

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`dir`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`mb`	number of block rows of sparse matrix `A`.
`nb`	number of block columns of sparse matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`bsrValA`	<type> array of `nnzb*rowBlockDim*colBlockDim` non-zero elements of matrix `A`.
`bsrRowPtrA`	integer array of `mb+1` elements that contains the start of every block row and the end of the last block row plus one of matrix `A`.
`bsrColIndA`	integer array of `nnzb` column indices of the non-zero blocks of matrix `A`.
`rowBlockDim`	number of rows within a block of `A`.
`colBlockDim`	number of columns within a block of `A`.
`descrC`	the descriptor of matrix `C`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.

Output

`csrValC`	<type> array of `nnz` non-zero elements of matrix `C`.
`csrRowPtrC`	integer array of `m+1` elements that contains the start of every row and the end of the last row plus one of matrix `C`.
`csrColIndC`	integer array of `nnz` column indices of the non-zero elements of matrix `C`.

See `cusparseStatus_t` for the description of the return status.

13.5. cusparse<t>csr2gebsr()

```
cusparseStatus_t
cusparseScsr2gebsr_bufferSize(cusparseHandle_t         handle,
                              cusparseDirection_t      dir,
                              int                      m,
                              int                      n,
                              const cusparseMatDescr_t descrA,
                              const float*             csrValA,
                              const int*               csrRowPtrA,
                              const int*               csrColIndA,
                              int                      rowBlockDim,
                              int                      colBlockDim,
                              int*                     pBufferSize)

cusparseStatus_t
cusparseDcsr2gebsr_bufferSize(cusparseHandle_t         handle,
                              cusparseDirection_t      dir,
                              int                      m,
                              int                      n,
                              const cusparseMatDescr_t descrA,
                              const double*            csrValA,
                              const int*               csrRowPtrA,
                              const int*               csrColIndA,
                              int                      rowBlockDim,
                              int                      colBlockDim,
                              int*                     pBufferSize)

cusparseStatus_t
cusparseCcsr2gebsr_bufferSize(cusparseHandle_t         handle,
                              cusparseDirection_t      dir,
                              int                      m,
                              int                      n,
                              const cusparseMatDescr_t descrA,
                              const cuComplex*         csrValA,
                              const int*               csrRowPtrA,
                              const int*               csrColIndA,
                              int                      rowBlockDim,
                              int                      colBlockDim,
                              int*                     pBufferSize)

cusparseStatus_t
cusparseZcsr2gebsr_bufferSize(cusparseHandle_t         handle,
                              cusparseDirection_t      dir,
                              int                      m,
                              int                      n,
                              const cusparseMatDescr_t descrA,
                              const cuDoubleComplex*   csrValA,
                              const int*               csrRowPtrA,
                              const int*               csrColIndA,
                              int                      rowBlockDim,
                              int                      colBlockDim,
                              int*                     pBufferSize)
```

```
cusparseStatus_t
```

```
cusparseXcsr2gebsrNnz(cusparseHandle_t         handle,
                      cusparseDirection_t      dir,
                      int                      m,
                      int                      n,
                      const cusparseMatDescr_t descrA,
                      const int*               csrRowPtrA,
                      const int*               csrColIndA,
                      const cusparseMatDescr_t descrC,
                      int*                     bsrRowPtrC,
                      int                      rowBlockDim,
                      int                      colBlockDim,
                      int*                     nnzTotalDevHostPtr,
                      void*                    pBuffer)

cusparseStatus_t
cusparseScsr2gebsr(cusparseHandle_t         handle,
                   cusparseDirection_t      dir,
                   int                      m,
                   int                      n,
                   const cusparseMatDescr_t descrA,
                   const float*             csrValA,
                   const int*               csrRowPtrA,
                   const int*               csrColIndA,
                   const cusparseMatDescr_t descrC,
                   float*                   bsrValC,
                   int*                     bsrRowPtrC,
                   int*                     bsrColIndC,
                   int                      rowBlockDim,
                   int                      colBlockDim,
                   void*                    pBuffer)

cusparseStatus_t
cusparseDcsr2gebsr(cusparseHandle_t         handle,
                   cusparseDirection_t      dir,
                   int                      m,
                   int                      n,
                   const cusparseMatDescr_t descrA,
                   const double*            csrValA,
                   const int*               csrRowPtrA,
                   const int*               csrColIndA,
                   const cusparseMatDescr_t descrC,
                   double*                  bsrValC,
                   int*                     bsrRowPtrC,
                   int*                     bsrColIndC,
                   int                      rowBlockDim,
                   int                      colBlockDim,
                   void*                    pBuffer)

cusparseStatus_t
cusparseCcsr2gebsr(cusparseHandle_t         handle,
                   cusparseDirection_t      dir,
                   int                      m,
                   int                      n,
                   const cusparseMatDescr_t descrA,
                   const cuComplex*         csrValA,
                   const int*               csrRowPtrA,
                   const int*               csrColIndA,
                   const cusparseMatDescr_t descrC,
                   cuComplex*               bsrValC,
                   int*                     bsrRowPtrC,
                   int*                     bsrColIndC,
```

```
                     int                          rowBlockDim,
                     int                          colBlockDim,
                     void*                        pBuffer)

cusparseStatus_t
cusparseZcsr2gebsr(cusparseHandle_t           handle,
                     cusparseDirection_t         dir,
                     int                          m,
                     int                          n,
                     const cusparseMatDescr_t descrA,
                     const cuDoubleComplex*     csrValA,
                     const int*                   csrRowPtrA,
                     const int*                   csrColIndA,
                     const cusparseMatDescr_t descrC,
                     cuDoubleComplex*             bsrValC,
                     int*                         bsrRowPtrC,
                     int*                         bsrColIndC,
                     int                          rowBlockDim,
                     int                          colBlockDim,
                     void*                        pBuffer)
```

This function converts a sparse matrix `A` in CSR format (that is defined by arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`) into a sparse matrix `C` in general BSR format (that is defined by the three arrays `bsrValC`, `bsrRowPtrC`, and `bsrColIndC`).

The matrix A is a `m*n` sparse matrix and matrix `C` is a `(mb*rowBlockDim)*(nb*colBlockDim)` sparse matrix, where `mb(=(m+rowBlockDim-1)/rowBlockDim)` is the number of block rows of `C`, and `nb(=(n+colBlockDim-1)/colBlockDim)` is the number of block columns of `C`.

The block of `C` is of size `rowBlockDim*colBlockDim`. If `m` is not multiple of `rowBlockDim` or `n` is not multiple of `colBlockDim`, zeros are filled in.

The implementation adopts a two-step approach to do the conversion. First, the user allocates `bsrRowPtrC` of `mb+1` elements and uses function `cusparseXcsr2gebsrNnz()` to determine the number of nonzero block columns per block row. Second, the user gathers `nnzb` (number of nonzero block columns of matrix `C`) from either `(nnzb=*nnzTotalDevHostPtr)` or `(nnzb=bsrRowPtrC[mb]-bsrRowPtrC[0])` and allocates `bsrValC` of `nnzb*rowBlockDim*colBlockDim` elements and `bsrColIndC` of `nnzb` integers. Finally function `cusparse[S|D|C|Z]csr2gebsr()` is called to complete the conversion.

The user must obtain the size of the buffer required by `csr2gebsr()` by calling `csr2gebsr_bufferSize()`, allocate the buffer, and pass the buffer pointer to `csr2gebsr()`.

The general procedure is as follows:

```
// Given CSR format (csrRowPtrA, csrColIndA, csrValA) and
// blocks of BSR format are stored in column-major order.
cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int base, nnzb;
int mb = (m + rowBlockDim-1)/rowBlockDim;
int nb = (n + colBlockDim-1)/colBlockDim;
int bufferSize;
void *pBuffer;
cusparseScsr2gebsr_bufferSize(handle, dir, m, n,
    descrA, csrValA, csrRowPtrA, csrColIndA,
    rowBlockDim, colBlockDim,
    &bufferSize);
cudaMalloc((void**)&pBuffer, bufferSize);
cudaMalloc((void**)&bsrRowPtrC, sizeof(int) *(mb+1));
// nnzTotalDevHostPtr points to host memory
```

```
int *nnzTotalDevHostPtr = &nnzb;
cusparseXcsr2gebsrNnz(handle, dir, m, n,
    descrA, csrRowPtrA, csrColIndA,
    descrC, bsrRowPtrC, rowBlockDim, colBlockDim,
    nnzTotalDevHostPtr,
    pBuffer);
if (NULL != nnzTotalDevHostPtr){
    nnzb = *nnzTotalDevHostPtr;
}else{
    cudaMemcpy(&nnzb, bsrRowPtrC+mb, sizeof(int), cudaMemcpyDeviceToHost);
    cudaMemcpy(&base, bsrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
    nnzb -= base;
}
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzb);
cudaMalloc((void**)&bsrValC, sizeof(float)*(rowBlockDim*colBlockDim)*nnzb);
cusparseScsr2gebsr(handle, dir, m, n,
        descrA,
        csrValA, csrRowPtrA, csrColIndA,
        descrC,
        bsrValC, bsrRowPtrC, bsrColIndC,
        rowBlockDim, colBlockDim,
        pBuffer);
```

The routine `cusparseXcsr2gebsrNnz()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

The routine `cusparse<t>csr2gebsr()` has the following properties:

- The routine requires no extra storage if `pBuffer != NULL`
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

`handle`	handle to the cuSPARSE library context.
`dir`	storage format of blocks, either `CUSPARSE_DIRECTION_ROW` or `CUSPARSE_DIRECTION_COLUMN`.
`m`	number of rows of sparse matrix `A`.
`n`	number of columns of sparse matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrValA`	<type> array of `nnz` nonzero elements of matrix `A`.

csrRowPtrA	integer array of m+1 elements that contains the start of every row and the end of the last row plus one of matrix A.
csrColIndA	integer array of nnz column indices of the nonzero elements of matrix A.
descrC	the descriptor of matrix C. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
rowBlockDim	number of rows within a block of C.
colBlockDim	number of columns within a block of C.
pBuffer	buffer allocated by the user, the size is return by csr2gebsr_bufferSize().

Output

bsrValC	<type> array of nnzb*rowBlockDim*colBlockDim nonzero elements of matrix C.
bsrRowPtrC	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix C.
bsrColIndC	integer array of nnzb column indices of the nonzero blocks of matrix C.
nnzTotalDevHostPtr	total number of nonzero blocks of matrix C. Pointer nnzTotalDevHostPtr can point to a device memory or host memory.

See cusparseStatus_t for the description of the return status.

13.6. cusparse<t>coo2csr()

```
cusparseStatus_t
cusparseXcoo2csr(cusparseHandle_t   handle,
                 const int*         cooRowInd,
                 int                nnz,
                 int                m,
                 int*               csrRowPtr,
                 cusparseIndexBase_t idxBase)
```

This function converts the array containing the uncompressed row indices (corresponding to COO format) into an array of compressed row pointers (corresponding to CSR format).

It can also be used to convert the array containing the uncompressed column indices (corresponding to COO format) into an array of column pointers (corresponding to CSC format).

- The routine requires no extra storage

- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

handle	handle to the cuSPARSE library context.
cooRowInd	integer array of nnz uncompressed row indices.
nnz	number of non-zeros of the sparse matrix (that is also the length of array cooRowInd).
m	number of rows of matrix A.
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.

Output

csrRowPtr	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.

See cusparseStatus_t for the description of the return status.

13.7. cusparse<t>csr2bsr()

```
cusparseStatus_t
cusparseXcsr2bsrNnz(cusparseHandle_t         handle,
                    cusparseDirection_t      dir,
                    int                      m,
                    int                      n,
                    const cusparseMatDescr_t descrA,
                    const int*               csrRowPtrA,
                    const int*               csrColIndA,
                    int                      blockDim,
                    const cusparseMatDescr_t descrC,
                    int*                     bsrRowPtrC,
                    int*                     nnzTotalDevHostPtr)

cusparseStatus_t
cusparseScsr2bsr(cusparseHandle_t         handle,
                 cusparseDirection_t      dir,
                 int                      m,
                 int                      n,
                 const cusparseMatDescr_t descrA,
                 const float*             csrValA,
                 const int*               csrRowPtrA,
                 const int*               csrColIndA,
                 int                      blockDim,
                 const cusparseMatDescr_t descrC,
                 float*                   bsrValC,
                 int*                     bsrRowPtrC,
                 int*                     bsrColIndC)

cusparseStatus_t
cusparseDcsr2bsr(cusparseHandle_t         handle,
```

```
                   cusparseDirection_t      dir,
                   int                      m,
                   int                      n,
                   const cusparseMatDescr_t descrA,
                   const double*            csrValA,
                   const int*               csrRowPtrA,
                   const int*               csrColIndA,
                   int                      blockDim,
                   const cusparseMatDescr_t descrC,
                   double*                  bsrValC,
                   int*                     bsrRowPtrC,
                   int*                     bsrColIndC)

cusparseStatus_t
cusparseCcsr2bsr(cusparseHandle_t         handle,
                   cusparseDirection_t      dir,
                   int                      m,
                   int                      n,
                   const cusparseMatDescr_t descrA,
                   const cuComplex*         csrValA,
                   const int*               csrRowPtrA,
                   const int*               csrColIndA,
                   int                      blockDim,
                   const cusparseMatDescr_t descrC,
                   cuComplex*               bsrValC,
                   int*                     bsrRowPtrC,
                   int*                     bsrColIndC)

cusparseStatus_t
cusparseZcsr2bsr(cusparseHandle_t         handle,
                   cusparseDirection_t      dir,
                   int                      m,
                   int                      n,
                   const cusparseMatDescr_t descrA,
                   const cuDoubleComplex*   csrValA,
                   const int*               csrRowPtrA,
                   const int*               csrColIndA,
                   int                      blockDim,
                   const cusparseMatDescr_t descrC,
                   cuDoubleComplex*         bsrValC,
                   int*                     bsrRowPtrC,
                   int*                     bsrColIndC)
```

This function converts a sparse matrix in CSR format that is defined by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA` into a sparse matrix in BSR format that is defined by arrays `bsrValC`, `bsrRowPtrC`, and `bsrColIndC`.

`A` is an `m*n` sparse matrix. The BSR format of `A` has `mb` block rows, `nb` block columns, and `nnzb` nonzero blocks, where `mb=((m+blockDim-1)/blockDim)` and `nb=(n+blockDim-1)/blockDim`.

If `m` or `n` is not multiple of `blockDim`, zeros are filled in.

The conversion in cuSPARSE entails a two-step approach. First, the user allocates `bsrRowPtrC` of `mb+1` elements and uses function `cusparseXcsr2bsrNnz()` to determine the number of nonzero block columns per block row. Second, the user gathers `nnzb` (number of non-zero block columns of matrix `C`) from either `(nnzb=*nnzTotalDevHostPtr)` or `(nnzb=bsrRowPtrC[mb]-bsrRowPtrC[0])` and allocates bsrValC of nnzb*blockDim*blockDim elements and bsrColIndC of nnzb elements. Finally function cusparse[S|D|C|Z]csr2bsr90 is called to complete the conversion.

The general procedure is as follows:

```
// Given CSR format (csrRowPtrA, csrcolIndA, csrValA) and
// blocks of BSR format are stored in column-major order.
cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int base, nnzb;
int mb = (m + blockDim-1)/blockDim;
cudaMalloc((void**)&bsrRowPtrC, sizeof(int) *(mb+1));
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzb;
cusparseXcsr2bsrNnz(handle, dir, m, n,
        descrA, csrRowPtrA, csrColIndA,
        blockDim,
        descrC, bsrRowPtrC,
        nnzTotalDevHostPtr);
if (NULL != nnzTotalDevHostPtr){
    nnzb = *nnzTotalDevHostPtr;
}else{
    cudaMemcpy(&nnzb, bsrRowPtrC+mb, sizeof(int), cudaMemcpyDeviceToHost);
    cudaMemcpy(&base, bsrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
    nnzb -= base;
}
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzb);
cudaMalloc((void**)&bsrValC, sizeof(float)*(blockDim*blockDim)*nnzb);
cusparseScsr2bsr(handle, dir, m, n,
        descrA,
        csrValA, csrRowPtrA, csrColIndA,
        blockDim,
        descrC,
        bsrValC, bsrRowPtrC, bsrColIndC);
```

The routine cusparse<t>csr2bsr() has the following properties:

- This function requires temporary extra storage that is allocated internally if blockDim > 16
- The routine support asynchronous execution if blockDim != 1 and the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if blockDim != 1 or the Stream Ordered Memory Allocator is available

The routine cusparseXcsr2bsrNnz() has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine support asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.

m	number of rows of sparse matrix A.
n	number of columns of sparse matrix A.
descrA	the descriptor of matrix A.
csrValA	<type> array of nnz (=csrRowPtrA[m]-csrRowPtr[0]) non-zero elements of matrix A.
csrRowPtrA	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz column indices of the non-zero elements of matrix A.
blockDim	block dimension of sparse matrix A. The range of blockDim is between 1 and min(m,n).
descrC	the descriptor of matrix C.

Output

bsrValC	<type> array of nnzb*blockDim*blockDim nonzero elements of matrix C.
bsrRowPtrC	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix C.
bsrColIndC	integer array of nnzb column indices of the non-zero blocks of matrix C.
nnzTotalDevHostPtr	total number of nonzero elements in device or host memory. It is equal to (bsrRowPtrC[mb]-bsrRowPtrC[0]).

See cusparseStatus_t for the description of the return status.

13.8. cusparse<t>csr2coo()

```
cusparseStatus_t
cusparseXcsr2coo(cusparseHandle_t    handle,
                 const int*          csrRowPtr,
                 int                 nnz,
                 int                 m,
                 int*                cooRowInd,
                 cusparseIndexBase_t idxBase)
```

This function converts the array containing the compressed row pointers (corresponding to CSR format) into an array of uncompressed row indices (corresponding to COO format).

It can also be used to convert the array containing the compressed column indices (corresponding to CSC format) into an array of uncompressed column indices (corresponding to COO format).

- The routine requires no extra storage

- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

handle	handle to the cuSPARSE library context.
csrRowPtr	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
nnz	number of nonzeros of the sparse matrix (that is also the length of array cooRowInd).
m	number of rows of matrix A.
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.

Output

cooRowInd	integer array of nnz uncompressed row indices.

See cusparseStatus_t for the description of the return status.

13.9. cusparseCsr2cscEx2()

```
cusparseStatus_t
cusparseCsr2cscEx2_bufferSize(cusparseHandle_t     handle,
                              int                  m,
                              int                  n,
                              int                  nnz,
                              const void*          csrVal,
                              const int*           csrRowPtr,
                              const int*           csrColInd,
                              void*                cscVal,
                              int*                 cscColPtr,
                              int*                 cscRowInd,
                              cudaDataType         valType,
                              cusparseAction_t     copyValues,
                              cusparseIndexBase_t  idxBase,
                              cusparseCsr2CscAlg_t alg,
                              size_t*              bufferSize)
```

```
cusparseStatus_t
cusparseCsr2cscEx2(cusparseHandle_t     handle,
                   int                  m,
                   int                  n,
                   int                  nnz,
                   const void*          csrVal,
                   const int*           csrRowPtr,
                   const int*           csrColInd,
                   void*                cscVal,
                   int*                 cscColPtr,
                   int*                 cscRowInd,
```

```
                         cudaDataType          valType,
                         cusparseAction_t      copyValues,
                         cusparseIndexBase_t   idxBase,
                         cusparseCsr2CscAlg_t alg,
                         void*                 buffer)
```

This function converts a sparse matrix in CSR format (that is defined by the three arrays `csrVal`, `csrRowPtr`, and `csrColInd`) into a sparse matrix in CSC format (that is defined by arrays `cscVal`, `cscRowInd`, and `cscColPtr`). The resulting matrix can also be seen as the transpose of the original sparse matrix. Notice that this routine can also be used to convert a matrix in CSC format into a matrix in CSR format.

The routine requires extra storage proportional to the number of nonzero values `nnz`. It provides in output always the same matrix.

It is executed asynchronously with respect to the host, and it may return control to the application on the host before the result is ready.

The function `cusparseCsr2cscEx2_bufferSize()` returns the size of the workspace needed by `cusparseCsr2cscEx2()`. User needs to allocate a buffer of this size and give that buffer to `cusparseCsr2cscEx2()` as an argument.

If `nnz == 0`, then `csrColInd`, `csrVal`, `cscVal`, and `cscRowInd` could have `NULL` value. In this case, `cscColPtr` is set to `idxBase` for all values.

If `m == 0` or `n == 0`, the pointers are not checked and the routine returns `CUSPARSE_STATUS_SUCCESS`.

Input

`handle`	handle to the cuSPARSE library context
`m`	number of rows of the CSR input matrix; number of columns of the CSC ouput matrix
`n`	number of columns of the CSR input matrix; number of rows of the CSC ouput matrix
`nnz`	number of nonzero elements of the CSR and CSC matrices
`csrVal`	value array of size `nnz` of the CSR matrix; of same type as `valType`
`csrRowPtr`	integer array of size `m + 1` that containes the CSR row offsets
`csrColInd`	integer array of size `nnz` that containes the CSR column indices
`cscVal`	value array of size `nnz` of the CSC matrix; of same type as `valType`
`cscColPtr`	integer array of size `n + 1` that containes the CSC column offsets
`cscRowInd`	integer array of size `nnz` that containes the CSC row indices
valType	value type for both CSR and CSC matrices
`copyValues`	`CUSPARSE_ACTION_SYMBOLIC` or `CUSPARSE_ACTION_NUMERIC`

idxBase	Index base CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.
alg	algorithm implementation. see cusparseCsr2CscAlg_t for possible values.
bufferSize	number of bytes of workspace needed by cusparseCsr2cscEx2()
buffer	pointer to workspace buffer

cusparseCsr2cscEx2() supports the following data types:

X/Y
CUDA_R_8I
CUDA_R_16F
CUDA_R_16BF
CUDA_R_32F
CUDA_R_64F
CUDA_C_16F
CUDA_C_16BF
CUDA_C_32F
CUDA_C_64F

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine does **not** support CUDA graph capture

See cusparseStatus_t for the description of the return status.

13.10. cusparse<t>csr2csr_compress()

```
cusparseStatus_t
cusparseScsr2csr_compress(cusparseHandle_t         handle,
                          int                      m,
                          int                      n,
                          const cusparseMatDescr_t descrA,
                          const float*             csrValA,
                          const int*               csrColIndA,
                          const int*               csrRowPtrA,
                          int                      nnzA,
                          const int*               nnzPerRow,
                          float*                   csrValC,
                          int*                     csrColIndC,
                          int*                     csrRowPtrC,
                          float                    tol)

cusparseStatus_t
cusparseDcsr2csr_compress(cusparseHandle_t         handle,
```

```
                                int                         m,
                                int                         n,
                                const cusparseMatDescr_t descrA,
                                const double*               csrValA,
                                const int*                  csrColIndA,
                                const int*                  csrRowPtrA,
                                int                         nnzA,
                                const int*                  nnzPerRow,
                                double*                     csrValC,
                                int*                        csrColIndC,
                                int*                        csrRowPtrC,
                                double                      tol)

cusparseStatus_t
cusparseCcsr2csr_compress(cusparseHandle_t          handle,
                                int                         m,
                                int                         n,
                                const cusparseMatDescr_t descrA,
                                const cuComplex*            csrValA,
                                const int*                  csrColIndA,
                                const int*                  csrRowPtrA,
                                int                         nnzA,
                                const int*                  nnzPerRow,
                                cuComplex*                  csrValC,
                                int*                        csrColIndC,
                                int*                        csrRowPtrC,
                                cuComplex                   tol)

cusparseStatus_t
cusparseZcsr2csr_compress(cusparseHandle_t          handle,
                                int                         m,
                                int                         n,
                                const cusparseMatDescr_t descrA,
                                const cuDoubleComplex*      csrValA,
                                const int*                  csrColIndA,
                                const int*                  csrRowPtrA,
                                int                         nnzA,
                                const int*                  nnzPerRow,
                                cuDoubleComplex*            csrValC,
                                int*                        csrColIndC,
                                int*                        csrRowPtrC,
                                cuDoubleComplex             tol)
```

This function compresses the sparse matrix in CSR format into compressed CSR format. Given a sparse matrix A and a non-negative value threshold, the function returns a sparse matrix C, defined by

$$C(i,j) = A(i,j) \quad \text{if } |A(i,j)| > |\text{threshold}|$$

The implementation adopts a two-step approach to do the conversion. First, the user allocates csrRowPtrC of m+1 elements and uses function cusparse<t>nnz_compress() to determine nnzPerRow(the number of nonzeros columns per row) and nnzC(the total number of nonzeros). Second, the user allocates csrValC of nnzC elements and csrColIndC of nnzC integers. Finally function cusparse<t>csr2csr_compress() is called to complete the conversion.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available

- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<type> array of nnz ($=$ csrRowPtrA(m) − csrRowPtrA(0)) elements of matrix A.
csrColIndA	integer array of nnz ($=$ csrRowPtrA(m) − csrRowPtrA(0)) column indices of the elements of matrix A.
csrRowPtrA	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
nnzA	number of nonzero elements in matrix A.
nnzPerRow	this array contains the number of elements kept in the compressed matrix, by row.
tol	on input, this contains the non-negative tolerance value used for compression. Any values in matrix A less than or equal to this value will be dropped during compression.

Output

csrValC	on output, this array contains the typed values of elements kept in the compressed matrix. Size = nnzC.
csrColIndC	on output, this integer array contains the column indices of elements kept in the compressed matrix. Size = nnzC.
csrRowPtrC	on output, this integer array contains the row pointers for elements kept in the compressed matrix. Size = m+1

See cusparseStatus_t for the description of the return status

The following is a sample code to show how to use this API.

```
#include <stdio.h>
#include <sys/time.h>
#include <cusparse.h>

#define ERR_NE(X,Y) do { if ((X) != (Y)) { \
```

```
                             fprintf(stderr,"Error in %s at %s:%d
\n",__func__,__FILE__,__LINE__); \
                             exit(-1);}} while(0)
#define CUDA_CALL(X) ERR_NE((X),cudaSuccess)
#define CUSPARSE_CALL(X) ERR_NE((X),CUSPARSE_STATUS_SUCCESS)
int main(){
    int m = 6, n = 5;
    cusparseHandle_t  handle;
    CUSPARSE_CALL( cusparseCreate(&handle) );
    cusparseMatDescr_t descrX;
    CUSPARSE_CALL(cusparseCreateMatDescr(&descrX));
    // Initialize sparse matrix
    float *X;
    CUDA_CALL(cudaMallocManaged( &X, sizeof(float) * m * n ));
    memset( X, 0, sizeof(float) * m * n );
    X[0 + 0*m] = 1.0;  X[0 + 1*m] = 3.0;
    X[1 + 1*m] = -4.0; X[1 + 2*m] = 5.0;
    X[2 + 0*m] = 2.0;  X[2 + 3*m] = 7.0;  X[2 + 4*m] = 8.0;
    X[3 + 2*m] = 6.0;  X[3 + 4*m] = 9.0;
    X[4 + 3*m] = 3.5;  X[4 + 4*m] = 5.5;
    X[5 + 0*m] = 6.5;  X[5 + 2*m] = -9.9;
    // Initialize total_nnz, and  nnzPerRowX for cusparseSdense2csr()
    int total_nnz = 13;
    int *nnzPerRowX;
    CUDA_CALL( cudaMallocManaged( &nnzPerRowX, sizeof(int) * m ));
    nnzPerRowX[0] = 2;  nnzPerRowX[1] = 2;  nnzPerRowX[2] = 3;
    nnzPerRowX[3] = 2;  nnzPerRowX[4] = 2;  nnzPerRowX[5] = 2;

    float *csrValX;
    int *csrRowPtrX;
    int *csrColIndX;
    CUDA_CALL( cudaMallocManaged( &csrValX, sizeof(float) * total_nnz) );
    CUDA_CALL( cudaMallocManaged( &csrRowPtrX, sizeof(int) * (m+1))) ;
    CUDA_CALL( cudaMallocManaged( &csrColIndX, sizeof(int) * total_nnz)) ;
```

Before calling this API, call two APIs to prepare the input.

```
/** Call cusparseSdense2csr to generate CSR format as the inputs for
    cusparseScsr2csr_compress   **/
    CUSPARSE_CALL( cusparseSdense2csr( handle, m, n, descrX, X,
                                       m, nnzPerRowX, csrValX,
                                       csrRowPtrX, csrColIndX )) ;
    float tol = 3.5;
    int *nnzPerRowY;
    int *testNNZTotal;
    CUDA_CALL  (cudaMallocManaged( &nnzPerRowY, sizeof(int) * m ));
    CUDA_CALL  (cudaMallocManaged( &testNNZTotal, sizeof(int)));
    memset( nnzPerRowY, 0, sizeof(int) * m );
    // cusparseSnnz_compress generates nnzPerRowY and testNNZTotal
    CUSPARSE_CALL( cusparseSnnz_compress(handle, m, descrX, csrValX,
                                         csrRowPtrX, nnzPerRowY,
                                         testNNZTotal, tol));

    float *csrValY;
    int *csrRowPtrY;
    int *csrColIndY;
    CUDA_CALL( cudaMallocManaged( &csrValY, sizeof(float) * (*testNNZTotal)));
    CUDA_CALL( cudaMallocManaged( &csrRowPtrY, sizeof(int) * (m+1)));
    CUDA_CALL( cudaMallocManaged( &csrColIndY, sizeof(int) * (*testNNZTotal)));

    CUSPARSE_CALL( cusparseScsr2csr_compress( handle, m, n, descrX, csrValX,
                                              csrColIndX, csrRowPtrX,
                                              total_nnz,  nnzPerRowY,
                                              csrValY, csrColIndY,
                                              csrRowPtrY, tol));
```

```
    /* Expect results
    nnzPerRowY:  0 2 2 2 1 2
    csrValY:     -4 5 7 8 6 9 5.5 6.5 -9.9
    csrColIndY:  1 2 3 4 2 4 4 0 2
    csrRowPtrY:  0 0 2 4 6 7 9
    */
    cudaFree(X);
    cusparseDestroy(handle);
    cudaFree(nnzPerRowX);
    cudaFree(csrValX);
    cudaFree(csrRowPtrX);
    cudaFree(csrColIndX);
    cudaFree(csrValY);
    cudaFree(nnzPerRowY);
    cudaFree(testNNZTotal);
    cudaFree(csrRowPtrY);
    cudaFree(csrColIndY);
    return 0;
}
```

13.11. cusparse<t>nnz()

```
cusparseStatus_t
cusparseSnnz(cusparseHandle_t         handle,
             cusparseDirection_t      dirA,
             int                      m,
             int                      n,
             const cusparseMatDescr_t descrA,
             const float*             A,
             int                      lda,
             int*                     nnzPerRowColumn,
             int*                     nnzTotalDevHostPtr)

cusparseStatus_t
cusparseDnnz(cusparseHandle_t         handle,
             cusparseDirection_t      dirA,
             int                      m,
             int                      n,
             const cusparseMatDescr_t descrA,
             const double*            A,
             int                      lda,
             int*                     nnzPerRowColumn,
             int*                     nnzTotalDevHostPtr)

cusparseStatus_t
cusparseCnnz(cusparseHandle_t         handle,
             cusparseDirection_t      dirA,
             int                      m,
             int                      n,
             const cusparseMatDescr_t descrA,
             const cuComplex*         A,
             int                      lda,
             int*                     nnzPerRowColumn,
             int*                     nnzTotalDevHostPtr)

cusparseStatus_t
cusparseZnnz(cusparseHandle_t         handle,
             cusparseDirection_t      dirA,
```

```
              int                        m,
              int                        n,
              const cusparseMatDescr_t   descrA,
              const cuDoubleComplex*     A,
              int                        lda,
              int*                       nnzPerRowColumn,
              int*                       nnzTotalDevHostPtr)
```

This function computes the number of nonzero elements per row or column and the total number of nonzero elements in a dense matrix.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`dirA`	direction that specifies whether to count nonzero elements by `CUSPARSE_DIRECTION_ROW` or by `CUSPARSE_DIRECTION_COLUMN`.
`m`	number of rows of matrix `A`.
`n`	number of columns of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`A`	array of dimensions `(lda, n)`.
`lda`	leading dimension of dense array `A`.

Output

`nnzPerRowColumn`	array of size `m` or `n` containing the number of nonzero elements per row or column, respectively.
`nnzTotalDevHostPtr`	total number of nonzero elements in device or host memory.

See `cusparseStatus_t` for the description of the return status.

13.12. cusparseCreateIdentityPermutation()

```
cusparseStatus_t
cusparseCreateIdentityPermutation(cusparseHandle_t handle,
                                  int              n,
                                  int*             p);
```

This function creates an identity map. The output parameter `p` represents such map by `p = 0:1:(n-1)`.

This function is typically used with `coosort`, `csrsort`, `cscsort`.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
n	host	size of the map.

Output

parameter	device or host	description
p	device	integer array of dimensions `n`.

See `cusparseStatus_t` for the description of the return status.

13.13. cusparseXcoosort()

```
cusparseStatus_t
cusparseXcoosort_bufferSizeExt(cusparseHandle_t handle,
                               int              m,
                               int              n,
                               int              nnz,
                               const int*       cooRows,
                               const int*       cooCols,
                               size_t*          pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseXcoosortByRow(cusparseHandle_t handle,
                      int              m,
                      int              n,
                      int              nnz,
                      int*             cooRows,
                      int*             cooCols,
                      int*             P,
                      void*            pBuffer)

cusparseStatus_t
cusparseXcoosortByColumn(cusparseHandle_t handle,
                         int              m,
                         int              n,
                         int              nnz,
                         int*             cooRows,
                         int*             cooCols,
                         int*             P,
```

```
                                    void*                   pBuffer);
```

This function sorts COO format. The sorting is in-place. Also the user can sort by row or sort by column.

`A` is an `m×n` sparse matrix that is defined in COO storage format by the three arrays `cooVals`, `cooRows`, and `cooCols`.

There is no assumption for the base index of the matrix. `coosort` uses stable sort on signed integer, so the value of `cooRows` or `cooCols` can be negative.

This function `coosort()` requires buffer size returned by `coosort_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

The parameter `P` is both input and output. If the user wants to compute sorted `cooVal`, `P` must be set as 0:1:(nnz-1) before `coosort()`, and after `coosort()`, new sorted value array satisfies `cooVal_sorted = cooVal(P)`.

Remark: the dimension `m` and `n` are not used. If the user does not know the value of `m` or `n`, just passes a value positive. This usually happens if the user only reads a COO array first and needs to decide the dimension `m` or `n` later.

- The routine requires no extra storage if `pBuffer != NULL`
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix `A`.
n	host	number of columns of matrix `A`.
nnz	host	number of nonzero elements of matrix `A`.
cooRows	device	integer array of `nnz` unsorted row indices of `A`.
cooCols	device	integer array of `nnz` unsorted column indices of `A`.
P	device	integer array of `nnz` unsorted map indices. To construct `cooVal`, the user has to set `P=0:1:(nnz-1)`.
pBuffer	device	buffer allocated by the user; the size is returned by `coosort_bufferSizeExt()`.

Output

parameter	device or host	description
cooRows	device	integer array of `nnz` sorted row indices of `A`.
cooCols	device	integer array of `nnz` sorted column indices of `A`.
P	device	integer array of `nnz` sorted map indices.

pBufferSizeInBytes	host	number of bytes of the buffer.

See cusparseStatus_t for the description of the return status

Please visit cuSPARSE Library Samples - cusparseXcoosortByRow for a code example.

13.14. cusparseXcsrsort()

```
cusparseStatus_t
cusparseXcsrsort_bufferSizeExt(cusparseHandle_t handle,
                               int              m,
                               int              n,
                               int              nnz,
                               const int*       csrRowPtr,
                               const int*       csrColInd,
                               size_t*          pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseXcsrsort(cusparseHandle_t         handle,
                 int                      m,
                 int                      n,
                 int                      nnz,
                 const cusparseMatDescr_t descrA,
                 const int*               csrRowPtr,
                 int*                     csrColInd,
                 int*                     P,
                 void*                    pBuffer)
```

This function sorts CSR format. The stable sorting is in-place.

The matrix type is regarded as `CUSPARSE_MATRIX_TYPE_GENERAL` implicitly. In other words, any symmetric property is ignored.

This function `csrsort()` requires buffer size returned by `csrsort_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

The parameter `P` is both input and output. If the user wants to compute sorted `csrVal`, `P` must be set as 0:1:(nnz-1) before `csrsort()`, and after `csrsort()`, new sorted value array satisfies `csrVal_sorted = csrVal(P)`.

The general procedure is as follows:

```
// A is a 3x3 sparse matrix, base-0
//     | 1 2 3 |
// A = | 4 5 6 |
//     | 7 8 9 |
const int m = 3;
const int n = 3;
const int nnz = 9;
csrRowPtr[m+1] = { 0, 3, 6, 9}; // on device
csrColInd[nnz] = { 2, 1, 0, 0, 2,1, 1, 2, 0}; // on device
csrVal[nnz] = { 3, 2, 1, 4, 6, 5, 8, 9, 7}; // on device
```

```
size_t pBufferSizeInBytes = 0;
void *pBuffer = NULL;
int *P = NULL;

// step 1: allocate buffer
cusparseXcsrsort_bufferSizeExt(handle, m, n, nnz, csrRowPtr, csrColInd,
 &pBufferSizeInBytes);
cudaMalloc( &pBuffer, sizeof(char)* pBufferSizeInBytes);

// step 2: setup permutation vector P to identity
cudaMalloc( (void**)&P, sizeof(int)*nnz);
cusparseCreateIdentityPermutation(handle, nnz, P);

// step 3: sort CSR format
cusparseXcsrsort(handle, m, n, nnz, descrA, csrRowPtr, csrColInd, P, pBuffer);

// step 4: gather sorted csrVal
cusparseDgthr(handle, nnz, csrVal, csrVal_sorted, P, CUSPARSE_INDEX_BASE_ZERO);
```

- The routine requires no extra storage if `pBuffer != NULL`
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix `A`.
n	host	number of columns of matrix `A`.
nnz	host	number of nonzero elements of matrix `A`.
csrRowsPtr	device	integer array of `m+1` elements that contains the start of every row and the end of the last row plus one.
csrColInd	device	integer array of `nnz` unsorted column indices of `A`.
P	device	integer array of `nnz` unsorted map indices. To construct `csrVal`, the user has to set `P=0:1:(nnz-1)`.
pBuffer	device	buffer allocated by the user; the size is returned by `csrsort_bufferSizeExt()`.

Output

parameter	device or host	description
csrColInd	device	integer array of `nnz` sorted column indices of `A`.
P	device	integer array of `nnz` sorted map indices.
pBufferSizeInBytes	host	number of bytes of the buffer.

See cusparseStatus_t for the description of the return status.

13.15. cusparseXcscsort()

```
cusparseStatus_t
cusparseXcscsort_bufferSizeExt(cusparseHandle_t handle,
                               int              m,
                               int              n,
                               int              nnz,
                               const int*       cscColPtr,
                               const int*       cscRowInd,
                               size_t*          pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseXcscsort(cusparseHandle_t         handle,
                 int                      m,
                 int                      n,
                 int                      nnz,
                 const cusparseMatDescr_t descrA,
                 const int*               cscColPtr,
                 int*                     cscRowInd,
                 int*                     P,
                 void*                    pBuffer)
```

This function sorts CSC format. The stable sorting is in-place.

The matrix type is regarded as `CUSPARSE_MATRIX_TYPE_GENERAL` implicitly. In other words, any symmetric property is ignored.

This function `cscsort()` requires buffer size returned by `cscsort_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

The parameter `P` is both input and output. If the user wants to compute sorted `cscVal`, `P` must be set as 0:1:(nnz-1) before `cscsort()`, and after `cscsort()`, new sorted value array satisfies `cscVal_sorted = cscVal(P)`.

The general procedure is as follows:

```
// A is a 3x3 sparse matrix, base-0
//     | 1 2  |
// A = | 4 0  |
//     | 0 8  |
const int m = 3;
const int n = 2;
const int nnz = 4;
cscColPtr[n+1] = { 0, 2, 4}; // on device
cscRowInd[nnz] = { 1, 0, 2, 0}; // on device
cscVal[nnz]    = { 4.0, 1.0, 8.0, 2.0 }; // on device
size_t pBufferSizeInBytes = 0;
void *pBuffer = NULL;
int *P = NULL;

// step 1: allocate buffer
cusparseXcscsort_bufferSizeExt(handle, m, n, nnz, cscColPtr, cscRowInd,
 &pBufferSizeInBytes);
cudaMalloc( &pBuffer, sizeof(char)* pBufferSizeInBytes);
```

```
// step 2: setup permutation vector P to identity
cudaMalloc( (void**)&P, sizeof(int)*nnz);
cusparseCreateIdentityPermutation(handle, nnz, P);

// step 3: sort CSC format
cusparseXcscsort(handle, m, n, nnz, descrA, cscColPtr, cscRowInd, P, pBuffer);

// step 4: gather sorted cscVal
cusparseDgthr(handle, nnz, cscVal, cscVal_sorted, P, CUSPARSE_INDEX_BASE_ZERO);
```

- The routine requires no extra storage if `pBuffer != NULL`
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix `A`.
n	host	number of columns of matrix `A`.
nnz	host	number of nonzero elements of matrix `A`.
cscColPtr	device	integer array of `n+1` elements that contains the start of every column and the end of the last column plus one.
cscRowInd	device	integer array of `nnz` unsorted row indices of `A`.
P	device	integer array of `nnz` unsorted map indices. To construct `cscVal`, the user has to set `P=0:1:(nnz-1)`.
pBuffer	device	buffer allocated by the user; the size is returned by `cscsort_bufferSizeExt()`.

Output

parameter	device or host	description
cscRowInd	device	integer array of `nnz` sorted row indices of `A`.
P	device	integer array of `nnz` sorted map indices.
pBufferSizeInBytes	host	number of bytes of the buffer.

See `cusparseStatus_t` for the description of the return status.

13.16. cusparseXcsru2csr()

```
cusparseStatus_t
cusparseCreateCsru2csrInfo(csru2csrInfo_t *info);

cusparseStatus_t
```

```
cusparseDestroyCsru2csrInfo(csru2csrInfo_t info);

cusparseStatus_t
cusparseScsru2csr_bufferSizeExt(cusparseHandle_t handle,
                                int              m,
                                int              n,
                                int              nnz,
                                float*           csrVal,
                                const int*       csrRowPtr,
                                int*             csrColInd,
                                csru2csrInfo_t   info,
                                size_t*          pBufferSizeInBytes)

cusparseStatus_t
cusparseDcsru2csr_bufferSizeExt(cusparseHandle_t handle,
                                int              m,
                                int              n,
                                int              nnz,
                                double*          csrVal,
                                const int*       csrRowPtr,
                                int*             csrColInd,
                                csru2csrInfo_t   info,
                                size_t*          pBufferSizeInBytes)

cusparseStatus_t
cusparseCcsru2csr_bufferSizeExt(cusparseHandle_t handle,
                                int              m,
                                int              n,
                                int              nnz,
                                cuComplex*       csrVal,
                                const int*       csrRowPtr,
                                int*             csrColInd,
                                csru2csrInfo_t   info,
                                size_t*          pBufferSizeInBytes)

cusparseStatus_t
cusparseZcsru2csr_bufferSizeExt(cusparseHandle_t handle,
                                int              m,
                                int              n,
                                int              nnz,
                                cuDoubleComplex* csrVal,
                                const int*       csrRowPtr,
                                int*             csrColInd,
                                csru2csrInfo_t   info,
                                size_t*          pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseScsru2csr(cusparseHandle_t         handle,
                  int                      m,
                  int                      n,
                  int                      nnz,
                  const cusparseMatDescr_t descrA,
                  float*                   csrVal,
                  const int*               csrRowPtr,
                  int*                     csrColInd,
                  csru2csrInfo_t           info,
                  void*                    pBuffer)

cusparseStatus_t
cusparseDcsru2csr(cusparseHandle_t         handle,
```

```
                  int                       m,
                  int                       n,
                  int                       nnz,
                  const cusparseMatDescr_t  descrA,
                  double*                   csrVal,
                  const int*                csrRowPtr,
                  int*                      csrColInd,
                  csru2csrInfo_t            info,
                  void*                     pBuffer)

cusparseStatus_t
cusparseCcsru2csr(cusparseHandle_t          handle,
                  int                       m,
                  int                       n,
                  int                       nnz,
                  const cusparseMatDescr_t  descrA,
                  cuComplex*                csrVal,
                  const int*                csrRowPtr,
                  int*                      csrColInd,
                  csru2csrInfo_t            info,
                  void*                     pBuffer)

cusparseStatus_t
cusparseZcsru2csr(cusparseHandle_t          handle,
                  int                       m,
                  int                       n,
                  int                       nnz,
                  const cusparseMatDescr_t  descrA,
                  cuDoubleComplex*          csrVal,
                  const int*                csrRowPtr,
                  int*                      csrColInd,
                  csru2csrInfo_t            info,
                  void*                     pBuffer)
```

```
cusparseStatus_t
cusparseScsr2csru(cusparseHandle_t          handle,
                  int                       m,
                  int                       n,
                  int                       nnz,
                  const cusparseMatDescr_t  descrA,
                  float*                    csrVal,
                  const int*                csrRowPtr,
                  int*                      csrColInd,
                  csru2csrInfo_t            info,
                  void*                     pBuffer)

cusparseStatus_t
cusparseDcsr2csru(cusparseHandle_t          handle,
                  int                       m,
                  int                       n,
                  int                       nnz,
                  const cusparseMatDescr_t  descrA,
                  double*                   csrVal,
                  const int*                csrRowPtr,
                  int*                      csrColInd,
                  csru2csrInfo_t            info,
                  void*                     pBuffer)

cusparseStatus_t
cusparseCcsr2csru(cusparseHandle_t          handle,
```

```
                    int                         m,
                    int                         n,
                    int                         nnz,
                    const cusparseMatDescr_t descrA,
                    cuComplex*                  csrVal,
                    const int*                  csrRowPtr,
                    int*                        csrColInd,
                    csru2csrInfo_t              info,
                    void*                       pBuffer)

cusparseStatus_t
cusparseZcsr2csru(cusparseHandle_t          handle,
                    int                         m,
                    int                         n,
                    int                         nnz,
                    const cusparseMatDescr_t descrA,
                    cuDoubleComplex*            csrVal,
                    const int*                  csrRowPtr,
                    int*                        csrColInd,
                    csru2csrInfo_t              info,
                    void*                       pBuffer)
```

This function transfers unsorted CSR format to CSR format, and vice versa. The operation is in-place.

This function is a wrapper of `csrsort` and `gthr`. The usecase is the following scenario.

If the user has a matrix `A` of CSR format which is unsorted, and implements his own code (which can be CPU or GPU kernel) based on this special order (for example, diagonal first, then lower triangle, then upper triangle), and wants to convert it to CSR format when calling CUSPARSE library, and then convert it back when doing something else on his/her kernel. For example, suppose the user wants to solve a linear system `Ax=b` by the following iterative scheme

$$x^{(k+1)} = x^{(k)} + L^{(-1)} * (b - Ax^{(k)})$$

The code heavily uses SpMv and triangular solve. Assume that the user has an in-house design of SpMV (Sparse Matrix-Vector multiplication) based on special order of `A`. However the user wants to use CUSAPRSE library for triangular solver. Then the following code can work.

do

step 1: compute residual vector $r = b - A\ x^{(k)}$ by

step 2: B := sort(A), and L is lower triangular part of B

(only sort A once and keep the permutation vector)

step 3: solve $z = L^{(-1)} * (\ b$

step 4: add correction $x^{(k+1)} = x^{(k)} + z$

step 5: A := unsort(B)

(use permutation vector to get back the unsorted CSR)

until convergence

The requirements of step 2 and step 5 are

1. In-place operation.

2. The permutation vector `P` is hidden in an opaque structure.

3. No cudaMalloc inside the conversion routine. Instead, the user has to provide the buffer explicitly.

4. The conversion between unsorted CSR and sorted CSR may needs several times, but the function only generates the permutation vector P once.

5. The function is based on csrsort, gather and scatter operations.

The operation is called csru2csr, which means unsorted CSR to sorted CSR. Also we provide the inverse operation, called csr2csru.

In order to keep the permutation vector invisible, we need an opaque structure called csru2csrInfo. Then two functions (cusparseCreateCsru2csrInfo, cusparseDestroyCsru2csrInfo) are used to initialize and to destroy the opaque structure.

cusparse[S|D|C|Z]csru2csr_bufferSizeExt returns the size of the buffer. The permutation vector P is also allcated inside csru2csrInfo. The lifetime of the permutation vector is the same as the lifetime of csru2csrInfo.

cusparse[S|D|C|Z]csru2csr performs forward transformation from unsorted CSR to sorted CSR. First call uses csrsort to generate the permutation vector P, and subsequent call uses P to do transformation.

cusparse[S|D|C|Z]csr2csru performs backward transformation from sorted CSR to unsorted CSR. P is used to get unsorted form back.

The routine cusparse<t>csru2csr() has the following properties:

- The routine requires no extra storage if pBuffer != NULL
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

The routine cusparse<t>csr2csru() has the following properties if pBuffer != NULL:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

The following tables describe parameters of csr2csru_bufferSizeExt and csr2csru.

Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix A.
n	host	number of columns of matrix A.
nnz	host	number of nonzero elements of matrix A.
descrA	host	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, Also, the supported

		index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrVal	device	<type> array of nnz unsorted nonzero elements of matrix A.
csrRowsPtr	device	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColInd	device	integer array of nnz unsorted column indices of A.
info	host	opaque structure initialized using cusparseCreateCsru2csrInfo().
pBuffer	device	buffer allocated by the user; the size is returned by csru2csr_bufferSizeExt().

Output

parameter	device or host	description
csrVal	device	<type> array of nnz sorted nonzero elements of matrix A.
csrColInd	device	integer array of nnz sorted column indices of A.
pBufferSizeInBytes	host	number of bytes of the buffer.

See cusparseStatus_t for the description of the return status.

13.17. cusparseXpruneDense2csr()

```
cusparseStatus_t
cusparseHpruneDense2csr_bufferSizeExt(cusparseHandle_t         handle,
                                       int                      m,
                                       int                      n,
                                       const __half*            A,
                                       int                      lda,
                                       const __half*            threshold,
                                       const cusparseMatDescr_t descrC,
                                       const __half*            csrValC,
                                       const int*               csrRowPtrC,
                                       const int*               csrColIndC,
                                       size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseSpruneDense2csr_bufferSizeExt(cusparseHandle_t         handle,
                                       int                      m,
                                       int                      n,
                                       const float*             A,
                                       int                      lda,
                                       const float*             threshold,
                                       const cusparseMatDescr_t descrC,
                                       const float*             csrValC,
                                       const int*               csrRowPtrC,
                                       const int*               csrColIndC,
                                       size_t*
 pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseDpruneDense2csr_bufferSizeExt(cusparseHandle_t         handle,
                                       int                      m,
                                       int                      n,
                                       const double*            A,
                                       int                      lda,
                                       const double*            threshold,
                                       const cusparseMatDescr_t descrC,
                                       const double*            csrValC,
                                       const int*               csrRowPtrC,
                                       const int*               csrColIndC,
                                       size_t*
 pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseHpruneDense2csrNnz(cusparseHandle_t         handle,
                           int                      m,
                           int                      n,
                           const __half*            A,
                           int                      lda,
                           const __half*            threshold,
                           const cusparseMatDescr_t descrC,
                           int*                     csrRowPtrC,
                           int*                     nnzTotalDevHostPtr,
                           void*                    pBuffer)

cusparseStatus_t
cusparseSpruneDense2csrNnz(cusparseHandle_t         handle,
                           int                      m,
                           int                      n,
                           const float*             A,
                           int                      lda,
                           const float*             threshold,
                           const cusparseMatDescr_t descrC,
                           int*                     csrRowPtrC,
                           int*                     nnzTotalDevHostPtr,
                           void*                    pBuffer)

cusparseStatus_t
cusparseDpruneDense2csrNnz(cusparseHandle_t         handle,
                           int                      m,
                           int                      n,
                           const double*            A,
                           int                      lda,
                           const double*            threshold,
                           const cusparseMatDescr_t descrC,
                           int*                     csrRowPtrC,
                           int*                     nnzTotalDevHostPtr,
                           void*                    pBuffer)
```

```
cusparseStatus_t
cusparseHpruneDense2csr(cusparseHandle_t         handle,
                        int                      m,
                        int                      n,
                        const __half*            A,
                        int                      lda,
                        const __half*            threshold,
                        const cusparseMatDescr_t descrC,
```

```
                         __half*                   csrValC,
                         const int*                csrRowPtrC,
                         int*                      csrColIndC,
                         void*                     pBuffer)

cusparseStatus_t
cusparseSpruneDense2csr(cusparseHandle_t          handle,
                         int                       m,
                         int                       n,
                         const float*              A,
                         int                       lda,
                         const float*              threshold,
                         const cusparseMatDescr_t descrC,
                         float*                    csrValC,
                         const int*                csrRowPtrC,
                         int*                      csrColIndC,
                         void*                     pBuffer)

cusparseStatus_t
cusparseDpruneDense2csr(cusparseHandle_t          handle,
                         int                       m,
                         int                       n,
                         const double*             A,
                         int                       lda,
                         const double*             threshold,
                         const cusparseMatDescr_t descrC,
                         double*                   csrValC,
                         const int*                csrRowPtrC,
                         int*                      csrColIndC,
                         void*                     pBuffer)
```

This function prunes a dense matrix to a sparse matrix with CSR format.

Given a dense matrix `A` and a non-negative value `threshold`, the function returns a sparse matrix `C`, defined by

$$C(i,j) = A(i,j) \quad \text{if } |A(i,j)| > \text{threshold}$$

The implementation adopts a two-step approach to do the conversion. First, the user allocates `csrRowPtrC` of `m+1` elements and uses function `pruneDense2csrNnz()` to determine the number of nonzeros columns per row. Second, the user gathers `nnzC` (number of nonzeros of matrix `C`) from either `(nnzC=*nnzTotalDevHostPtr)` or `(nnzC=csrRowPtrC[m]-csrRowPtrC[0])` and allocates `csrValC` of `nnzC` elements and `csrColIndC` of `nnzC` integers. Finally function `pruneDense2csr()` is called to complete the conversion.

The user must obtain the size of the buffer required by `pruneDense2csr()` by calling `pruneDense2csr_bufferSizeExt()`, allocate the buffer, and pass the buffer pointer to `pruneDense2csr()`.

Appendix section provides a simple example of `pruneDense2csr()`.

The routine `cusparse<t>pruneDense2csrNnz()` has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available

- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

The routine `cusparse<t>DpruneDense2csr()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix A.
n	host	number of columns of matrix A.
A	device	array of dimension (lda, n).
lda	device	leading dimension of A. It must be at least max(1, m).
threshold	host or device	a value to drop the entries of A. threshold can point to a device memory or host memory.
descrC	host	the descriptor of matrix C. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
pBuffer	device	buffer allocated by the user; the size is returned by pruneDense2csr_bufferSizeExt().

Output

parameter	device or host	description
nnzTotalDevHostPtr	device or host	total number of nonzero of matrix C. nnzTotalDevHostPtr can point to a device memory or host memory.
csrValC	device	<type> array of nnzC nonzero elements of matrix C.
csrRowsPtrC	device	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndC	device	integer array of nnzC column indices of C.
pBufferSizeInBytes	host	number of bytes of the buffer.

See cusparseStatus_t for the description of the return status.

13.18. cusparseXpruneCsr2csr()

```
cusparseStatus_t
cusparseHpruneCsr2csr_bufferSizeExt(cusparseHandle_t         handle,
                                    int                      m,
                                    int                      n,
```

```
                                    int                     nnzA,
                                    const cusparseMatDescr_t descrA,
                                    const __half*           csrValA,
                                    const int*              csrRowPtrA,
                                    const int*              csrColIndA,
                                    const __half*           threshold,
                                    const cusparseMatDescr_t descrC,
                                    const __half*           csrValC,
                                    const int*              csrRowPtrC,
                                    const int*              csrColIndC,
                                    size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseSpruneCsr2csr_bufferSizeExt(cusparseHandle_t         handle,
                                    int                     m,
                                    int                     n,
                                    int                     nnzA,
                                    const cusparseMatDescr_t descrA,
                                    const float*            csrValA,
                                    const int*              csrRowPtrA,
                                    const int*              csrColIndA,
                                    const float*            threshold,
                                    const cusparseMatDescr_t descrC,
                                    const float*            csrValC,
                                    const int*              csrRowPtrC,
                                    const int*              csrColIndC,
                                    size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseDpruneCsr2csr_bufferSizeExt(cusparseHandle_t         handle,
                                    int                     m,
                                    int                     n,
                                    int                     nnzA,
                                    const cusparseMatDescr_t descrA,
                                    const double*           csrValA,
                                    const int*              csrRowPtrA,
                                    const int*              csrColIndA,
                                    const double*           threshold,
                                    const cusparseMatDescr_t descrC,
                                    const double*           csrValC,
                                    const int*              csrRowPtrC,
                                    const int*              csrColIndC,
                                    size_t*
 pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseHpruneCsr2csrNnz(cusparseHandle_t         handle,
                         int                     m,
                         int                     n,
                         int                     nnzA,
                         const cusparseMatDescr_t descrA,
                         const __half*           csrValA,
                         const int*              csrRowPtrA,
                         const int*              csrColIndA,
                         const __half*           threshold,
                         const cusparseMatDescr_t descrC,
                         int*                    csrRowPtrC,
                         int*                    nnzTotalDevHostPtr,
```

```
                          void*                    pBuffer)

cusparseStatus_t
cusparseSpruneCsr2csrNnz(cusparseHandle_t         handle,
                          int                      m,
                          int                      n,
                          int                      nnzA,
                          const cusparseMatDescr_t descrA,
                          const float*             csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          const float*             threshold,
                          const cusparseMatDescr_t descrC,
                          int*                     csrRowPtrC,
                          int*                     nnzTotalDevHostPtr,
                          void*                    pBuffer)

cusparseStatus_t
cusparseDpruneCsr2csrNnz(cusparseHandle_t         handle,
                          int                      m,
                          int                      n,
                          int                      nnzA,
                          const cusparseMatDescr_t descrA,
                          const double*            csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          const double*            threshold,
                          const cusparseMatDescr_t descrC,
                          int*                     csrRowPtrC,
                          int*                     nnzTotalDevHostPtr,
                          void*                    pBuffer)
```

```
cusparseStatus_t
cusparseHpruneCsr2csr(cusparseHandle_t         handle,
                       int                      m,
                       int                      n,
                       int                      nnzA,
                       const cusparseMatDescr_t descrA,
                       const __half*            csrValA,
                       const int*               csrRowPtrA,
                       const int*               csrColIndA,
                       const __half*            threshold,
                       const cusparseMatDescr_t descrC,
                       __half*                  csrValC,
                       const int*               csrRowPtrC,
                       int*                     csrColIndC,
                       void*                    pBuffer)

cusparseStatus_t
cusparseSpruneCsr2csr(cusparseHandle_t         handle,
                       int                      m,
                       int                      n,
                       int                      nnzA,
                       const cusparseMatDescr_t descrA,
                       const float*             csrValA,
                       const int*               csrRowPtrA,
                       const int*               csrColIndA,
                       const float*             threshold,
                       const cusparseMatDescr_t descrC,
                       float*                   csrValC,
```

```
                            const int*              csrRowPtrC,
                            int*                    csrColIndC,
                            void*                   pBuffer)

cusparseStatus_t
cusparseDpruneCsr2csr(cusparseHandle_t          handle,
                            int                     m,
                            int                     n,
                            int                     nnzA,
                            const cusparseMatDescr_t descrA,
                            const double*           csrValA,
                            const int*              csrRowPtrA,
                            const int*              csrColIndA,
                            const double*           threshold,
                            const cusparseMatDescr_t descrC,
                            double*                 csrValC,
                            const int*              csrRowPtrC,
                            int*                    csrColIndC,
                            void*                   pBuffer)
```

This function prunes a sparse matrix to a sparse matrix with CSR format.

Given a sparse matrix `A` and a non-negative value `threshold`, the function returns a sparse matrix `C`, defined by

$$C(i,j) = A(i,j) \quad \text{if } |A(i,j)| > \text{threshold}$$

The implementation adopts a two-step approach to do the conversion. First, the user allocates `csrRowPtrC` of `m+1` elements and uses function `pruneCsr2csrNnz()` to determine the number of nonzeros columns per row. Second, the user gathers `nnzC` (number of nonzeros of matrix `C`) from either `(nnzC=*nnzTotalDevHostPtr)` or `(nnzC=csrRowPtrC[m]-csrRowPtrC[0])` and allocates `csrValC` of `nnzC` elements and `csrColIndC` of `nnzC` integers. Finally function `pruneCsr2csr()` is called to complete the conversion.

The user must obtain the size of the buffer required by `pruneCsr2csr()` by calling `pruneCsr2csr_bufferSizeExt()`, allocate the buffer, and pass the buffer pointer to `pruneCsr2csr()`.

Appendix section provides a simple example of `pruneCsr2csr()`.

The routine `cusparse<t>pruneCsr2csrNnz()` has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

The routine `cusparse<t>pruneCsr2csr()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix A.
n	host	number of columns of matrix A.
nnzA	host	number of nonzeros of matrix A.
descrA	host	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	device	<type> array of nnzA nonzero elements of matrix A.
csrRowsPtrA	device	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	device	integer array of nnzA column indices of A.
threshold	host or device	a value to drop the entries of A. threshold can point to a device memory or host memory.
descrC	host	the descriptor of matrix C. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
pBuffer	device	buffer allocated by the user; the size is returned by pruneCsr2csr_bufferSizeExt().

Output

parameter	device or host	description
nnzTotalDevHostPtr	device or host	total number of nonzero of matrix C. nnzTotalDevHostPtr can point to a device memory or host memory.
csrValC	device	<type> array of nnzC nonzero elements of matrix C.
csrRowsPtrC	device	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndC	device	integer array of nnzC column indices of C.
pBufferSizeInBytes	host	number of bytes of the buffer.

See cusparseStatus_t for the description of the return status.

13.19. cusparseXpruneDense2csrPercentage()

```
cusparseStatus_t
cusparseHpruneDense2csrByPercentage_bufferSizeExt(cusparseHandle_t
 handle,
                                                   int
 m,
```

```
                                                    int
 n,
                                                    const __half*
 A,
                                                    int
 lda,
                                                    float
 percentage,
                                                    const cusparseMatDescr_t
 descrC,
                                                    const __half*
 csrValC,
                                                    const int*
 csrRowPtrC,
                                                    const int*
 csrColIndC,
                                                    pruneInfo_t
 info,
                                                    size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseSpruneDense2csrByPercentage_bufferSizeExt(cusparseHandle_t
 handle,
                                                    int
 m,
                                                    int
 n,
                                                    const float*
 A,
                                                    int
 lda,
                                                    float
 percentage,
                                                    const cusparseMatDescr_t
 descrC,
                                                    const float*
 csrValC,
                                                    const int*
 csrRowPtrC,
                                                    const int*
 csrColIndC,
                                                    pruneInfo_t
 info,
                                                    size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseDpruneDense2csrByPercentage_bufferSizeExt(cusparseHandle_t
 handle,
                                                    int
 m,
                                                    int
 n,
                                                    const double*
 A,
                                                    int
 lda,
                                                    float
 percentage,
```

```
                                                  const cusparseMatDescr_t
 descrC,
                                                  const double*
 csrValC,
                                                  const int*
 csrRowPtrC,
                                                  const int*
 csrColIndC,
                                                  pruneInfo_t
 info,
                                                  size_t*
 pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseHpruneDense2csrNnzByPercentage(cusparseHandle_t         handle,
                                       int                      m,
                                       int                      n,
                                       const __half*            A,
                                       int                      lda,
                                       float                    percentage,
                                       const cusparseMatDescr_t descrC,
                                       int*                     csrRowPtrC,
                                       int*
 nnzTotalDevHostPtr,
                                       pruneInfo_t              info,
                                       void*                    pBuffer)

cusparseStatus_t
cusparseSpruneDense2csrNnzByPercentage(cusparseHandle_t         handle,
                                       int                      m,
                                       int                      n,
                                       const float*             A,
                                       int                      lda,
                                       float                    percentage,
                                       const cusparseMatDescr_t descrC,
                                       int*                     csrRowPtrC,
                                       int*
 nnzTotalDevHostPtr,
                                       pruneInfo_t              info,
                                       void*                    pBuffer)

cusparseStatus_t
cusparseDpruneDense2csrNnzByPercentage(cusparseHandle_t         handle,
                                       int                      m,
                                       int                      n,
                                       const double*            A,
                                       int                      lda,
                                       float                    percentage,
                                       const cusparseMatDescr_t descrC,
                                       int*                     csrRowPtrC,
                                       int*
 nnzTotalDevHostPtr,
                                       pruneInfo_t              info,
                                       void*                    pBuffer)
```

```
cusparseStatus_t
cusparseHpruneDense2csrByPercentage(cusparseHandle_t         handle,
                                    int                      m,
                                    int                      n,
```

```
                                        const __half*            A,
                                        int                      lda,
                                        float                    percentage,
                                        const cusparseMatDescr_t descrC,
                                        __half*                  csrValC,
                                        const int*               csrRowPtrC,
                                        int*                     csrColIndC,
                                        pruneInfo_t              info,
                                        void*                    pBuffer)

cusparseStatus_t
cusparseSpruneDense2csrByPercentage(cusparseHandle_t         handle,
                                    int                      m,
                                    int                      n,
                                    const float*             A,
                                    int                      lda,
                                    float                    percentage,
                                    const cusparseMatDescr_t descrC,
                                    float*                   csrValC,
                                    const int*               csrRowPtrC,
                                    int*                     csrColIndC,
                                    pruneInfo_t              info,
                                    void*                    pBuffer)

cusparseStatus_t
cusparseDpruneDense2csrByPercentage(cusparseHandle_t         handle,
                                    int                      m,
                                    int                      n,
                                    const double*            A,
                                    int                      lda,
                                    float                    percentage,
                                    const cusparseMatDescr_t descrC,
                                    double*                  csrValC,
                                    const int*               csrRowPtrC,
                                    int*                     csrColIndC,
                                    pruneInfo_t              info,
                                    void*                    pBuffer)
```

This function prunes a dense matrix to a sparse matrix by percentage.

Given a dense matrix `A` and a non-negative value `percentage`, the function computes sparse matrix `C` by the following three steps:

Step 1: sort absolute value of `A` in ascending order.

$$\text{key} := \text{sort}(\ |A|\ )$$

Step 2: choose threshold by the parameter `percentage`

$$\begin{aligned}
&\text{pos} = \text{ceil}(m*n*(\text{percentage}/100)) - 1\\
&\text{pos} = \min(\text{pos},\ m*n-1)\\
&\text{pos} = \max(\text{pos},\ 0)\\
&\text{threshold} = \text{key}[\text{pos}]
\end{aligned}$$

Step 3: call `pruneDense2csr()` by with the parameter `threshold`.

The implementation adopts a two-step approach to do the conversion. First, the user allocates `csrRowPtrC` of `m+1` elements and uses function `pruneDense2csrNnzByPercentage()` to determine the number of nonzeros columns per row. Second, the user gathers `nnzC` (number of nonzeros of matrix `C`) from either `(nnzC=*nnzTotalDevHostPtr)` or (nnzC=csrRowPtrC[m]-csrRowPtrC[0]) and allocates csrValC of nnzC elements and csrColIndC of nnzC integers. Finally function pruneDense2csrByPercentage() is called to complete the conversion.

The user must obtain the size of the buffer required by pruneDense2csrByPercentage() by calling pruneDense2csrByPercentage_bufferSizeExt(), allocate the buffer, and pass the buffer pointer to pruneDense2csrByPercentage().

Remark 1: the value of percentage must be not greater than 100. Otherwise, CUSPARSE_STATUS_INVALID_VALUE is returned.

Remark 2: the zeros of A are not ignored. All entries are sorted, including zeros. This is different from pruneCsr2csrByPercentage()

Appendix section provides a simple example of pruneDense2csrNnzByPercentage().

The routine cusparse<t>pruneDense2csrNnzByPercentage() has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

The routine cusparse<t>pruneDense2csrByPercentage() has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix A.
n	host	number of columns of matrix A.
A	device	array of dimension (lda, n).
lda	device	leading dimension of A. It must be at least max(1, m).
percentage	host	percentage <=100 and percentage >= 0
descrC	host	the descriptor of matrix C. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
pBuffer	device	buffer allocated by the user; the size is returned by pruneDense2csrByPercentage_bufferSizeExt().

Output

parameter	device or host	description

nnzTotalDevHostPtr	device or host	total number of nonzero of matrix C. nnzTotalDevHostPtr can point to a device memory or host memory.
csrValC	device	<type> array of nnzC nonzero elements of matrix C.
csrRowsPtrC	device	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndC	device	integer array of nnzC column indices of C.
pBufferSizeInBytes	host	number of bytes of the buffer.

See cusparseStatus_t for the description of the return status.

13.20. cusparseXpruneCsr2csrByPercentage()

```
cusparseStatus_t
cusparseHpruneCsr2csrByPercentage_bufferSizeExt(cusparseHandle_t
 handle,
                                                int                      m,
                                                int                      n,
                                                int
 nnzA,
                                                const cusparseMatDescr_t
 descrA,
                                                const __half*
 csrValA,
                                                const int*
 csrRowPtrA,
                                                const int*
 csrColIndA,
                                                float
 percentage,
                                                const cusparseMatDescr_t
 descrC,
                                                const __half*
 csrValC,
                                                const int*
 csrRowPtrC,
                                                const int*
 csrColIndC,
                                                pruneInfo_t
 info,
                                                size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseSpruneCsr2csrByPercentage_bufferSizeExt(cusparseHandle_t
 handle,
                                                int                      m,
                                                int                      n,
                                                int
 nnzA,
                                                const cusparseMatDescr_t
 descrA,
                                                const float*
 csrValA,
```

```
                                                   const int*
 csrRowPtrA,
                                                   const int*
 csrColIndA,
                                                   float
 percentage,
                                                   const cusparseMatDescr_t
 descrC,
                                                   const float*
 csrValC,
                                                   const int*
 csrRowPtrC,
                                                   const int*
 csrColIndC,
                                                   pruneInfo_t
 info,
                                                   size_t*
 pBufferSizeInBytes)

cusparseStatus_t
cusparseDpruneCsr2csrByPercentage_bufferSizeExt(cusparseHandle_t
 handle,
                                                   int                     m,
                                                   int                     n,
                                                   int
 nnzA,
                                                   const cusparseMatDescr_t
 descrA,
                                                   const double*
 csrValA,
                                                   const int*
 csrRowPtrA,
                                                   const int*
 csrColIndA,
                                                   float
 percentage,
                                                   const cusparseMatDescr_t
 descrC,
                                                   const double*
 csrValC,
                                                   const int*
 csrRowPtrC,
                                                   const int*
 csrColIndC,
                                                   pruneInfo_t
 info,
                                                   size_t*
 pBufferSizeInBytes)
```

```
cusparseStatus_t
cusparseHpruneCsr2csrNnzByPercentage(cusparseHandle_t        handle,
                                     int                      m,
                                     int                      n,
                                     int                      nnzA,
                                     const cusparseMatDescr_t descrA,
                                     const __half*            csrValA,
                                     const int*               csrRowPtrA,
                                     const int*               csrColIndA,
                                     float                    percentage,
                                     const cusparseMatDescr_t descrC,
```

```
                                    int*                     csrRowPtrC,
                                    int*
 nnzTotalDevHostPtr,
                                    pruneInfo_t              info,
                                    void*                    pBuffer)

cusparseStatus_t
cusparseSpruneCsr2csrNnzByPercentage(cusparseHandle_t        handle,
                                    int                      m,
                                    int                      n,
                                    int                      nnzA,
                                    const cusparseMatDescr_t descrA,
                                    const float*             csrValA,
                                    const int*               csrRowPtrA,
                                    const int*               csrColIndA,
                                    float                    percentage,
                                    const cusparseMatDescr_t descrC,
                                    int*                     csrRowPtrC,
                                    int*
 nnzTotalDevHostPtr,
                                    pruneInfo_t              info,
                                    void*                    pBuffer)

cusparseStatus_t
cusparseDpruneCsr2csrNnzByPercentage(cusparseHandle_t        handle,
                                    int                      m,
                                    int                      n,
                                    int                      nnzA,
                                    const cusparseMatDescr_t descrA,
                                    const double*            csrValA,
                                    const int*               csrRowPtrA,
                                    const int*               csrColIndA,
                                    float                    percentage,
                                    const cusparseMatDescr_t descrC,
                                    int*                     csrRowPtrC,
                                    int*
 nnzTotalDevHostPtr,
                                    pruneInfo_t              info,
                                    void*                    pBuffer)
```

```
cusparseStatus_t
cusparseHpruneCsr2csrByPercentage(cusparseHandle_t         handle,
                                  int                      m,
                                  int                      n,
                                  int                      nnzA,
                                  const cusparseMatDescr_t descrA,
                                  const __half*            csrValA,
                                  const int*               csrRowPtrA,
                                  const int*               csrColIndA,
                                  float                    percentage,
                                  const cusparseMatDescr_t descrC,
                                  __half*                  csrValC,
                                  const int*               csrRowPtrC,
                                  int*                     csrColIndC,
                                  pruneInfo_t              info,
                                  void*                    pBuffer)

cusparseStatus_t
cusparseSpruneCsr2csrByPercentage(cusparseHandle_t         handle,
                                  int                      m,
```

```
                                   int                        n,
                                   int                        nnzA,
                                   const cusparseMatDescr_t descrA,
                                   const float*               csrValA,
                                   const int*                 csrRowPtrA,
                                   const int*                 csrColIndA,
                                   float                      percentage,
                                   const cusparseMatDescr_t descrC,
                                   float*                     csrValC,
                                   const int*                 csrRowPtrC,
                                   int*                       csrColIndC,
                                   pruneInfo_t                info,
                                   void*                      pBuffer)

cusparseStatus_t
cusparseDpruneCsr2csrByPercentage(cusparseHandle_t         handle,
                                   int                        m,
                                   int                        n,
                                   int                        nnzA,
                                   const cusparseMatDescr_t descrA,
                                   const double*              csrValA,
                                   const int*                 csrRowPtrA,
                                   const int*                 csrColIndA,
                                   float                      percentage,
                                   const cusparseMatDescr_t descrC,
                                   double*                    csrValC,
                                   const int*                 csrRowPtrC,
                                   int*                       csrColIndC,
                                   pruneInfo_t                info,
                                   void*                      pBuffer)
```

This function prunes a sparse matrix to a sparse matrix by percentage.

Given a sparse matrix `A` and a non-negative value `percentage`, the function computes sparse matrix `C` by the following three steps:

Step 1: sort absolute value of `A` in ascending order.

$$\text{key} := \text{sort}(\ |\text{csrValA}|\ )$$

Step 2: choose threshold by the parameter `percentage`

$$\begin{aligned}
&\text{pos} = \text{ceil}(\text{nnzA}*(\text{percentage}/100)) - 1 \\
&\text{pos} = \min(\text{pos},\ \text{nnzA}-1) \\
&\text{pos} = \max(\text{pos},\ 0) \\
&\text{threshold} = \text{key}[\text{pos}]
\end{aligned}$$

Step 3: call `pruneCsr2csr()` by with the parameter `threshold`.

The implementation adopts a two-step approach to do the conversion. First, the user allocates `csrRowPtrC` of `m+1` elements and uses function `pruneCsr2csrNnzByPercentage()` to determine the number of nonzeros columns per row. Second, the user gathers `nnzC` (number of nonzeros of matrix `C`) from either `(nnzC=*nnzTotalDevHostPtr)` or `(nnzC=csrRowPtrC[m]-csrRowPtrC[0])` and allocates `csrValC` of `nnzC` elements and `csrColIndC` of `nnzC` integers. Finally function `pruneCsr2csrByPercentage()` is called to complete the conversion.

The user must obtain the size of the buffer required by `pruneCsr2csrByPercentage()` by calling `pruneCsr2csrByPercentage_bufferSizeExt()`, allocate the buffer, and pass the buffer pointer to `pruneCsr2csrByPercentage()`.

Remark 1: the value of `percentage` must be not greater than 100. Otherwise, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Appendix section provides a simple example of `pruneCsr2csrByPercentage()`.

The routine `cusparse<t>pruneCsr2csrNnzByPercentage()` has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

The routine `cusparse<t>pruneCsr2csrByPercentage()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix `A`.
n	host	number of columns of matrix `A`.
nnzA	host	number of nonzeros of matrix `A`.
descrA	host	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`, Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
csrValA	device	<type> array of `nnzA` nonzero elements of matrix `A`.
csrRowsPtrA	device	integer array of `m+1` elements that contains the start of every row and the end of the last row plus one.
csrColIndA	device	integer array of `nnzA` column indices of `A`.
percentage	host	percentage <=100 and percentage >= 0
descrC	host	the descriptor of matrix `C`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`, Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
pBuffer	device	buffer allocated by the user; the size is returned by `pruneCsr2csrByPercentage_bufferSizeExt()`.

Output

parameter	device or host	description
nnzTotalDevHostPtr	device or host	total number of nonzero of matrix C. nnzTotalDevHostPtr can point to a device memory or host memory.
csrValC	device	<type> array of nnzC nonzero elements of matrix C.
csrRowsPtrC	device	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndC	device	integer array of nnzC column indices of C.
pBufferSizeInBytes	host	number of bytes of the buffer.

See cusparseStatus_t for the description of the return status.

13.21. cusparse<t>nnz_compress()

```
cusparseStatus_t
cusparseSnnz_compress(cusparseHandle_t         handle,
                      int                      m,
                      const cusparseMatDescr_t descr,
                      const float*             csrValA,
                      const int*               csrRowPtrA,
                      int*                     nnzPerRow,
                      int*                     nnzC,
                      float                    tol)

cusparseStatus_t
cusparseDnnz_compress(cusparseHandle_t         handle,
                      int                      m,
                      const cusparseMatDescr_t descr,
                      const double*            csrValA,
                      const int*               csrRowPtrA,
                      int*                     nnzPerRow,
                      int*                     nnzC,
                      double                   tol)

cusparseStatus_t
cusparseCnnz_compress(cusparseHandle_t         handle,
                      int                      m,
                      const cusparseMatDescr_t descr,
                      const cuComplex*         csrValA,
                      const int*               csrRowPtrA,
                      int*                     nnzPerRow,
                      int*                     nnzC,
                      cuComplex                tol)

cusparseStatus_t
cusparseZnnz_compress(cusparseHandle_t         handle,
                      int                      m,
                      const cusparseMatDescr_t descr,
                      const cuDoubleComplex*   csrValA,
                      const int*               csrRowPtrA,
                      int*                     nnzPerRow,
                      int*                     nnzC,
                      cuDoubleComplex          tol)
```

This function is the step one to convert from csr format to compressed csr format.

Given a sparse matrix A and a non-negative value threshold, the function returns nnzPerRow(the number of nonzeros columns per row) and nnzC(the total number of nonzeros) of a sparse matrix C, defined by

$$C(i,j) = A(i,j) \quad \text{if } |A(i,j)| > \text{threshold}$$

A key assumption for the cuComplex and cuDoubleComplex case is that this tolerance is given as the real part. For example tol = 1e-8 + 0*i and we extract cureal, that is the x component of this struct.

- This function requires temporary extra storage that is allocated internally
- The routine supports asynchronous execution if the Stream Ordered Memory Allocator is available
- The routine supports CUDA graph capture if the Stream Ordered Memory Allocator is available

Input

`handle`	handle to the cuSPARSE library context.
`m`	number of rows of matrix `A`.
`descrA`	the descriptor of matrix `A`. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.
`csrValA`	csr noncompressed values array
`csrRowPtrA`	the corresponding input noncompressed row pointer.
`tol`	non-negative tolerance to determine if a number less than or equal to it.

Output

`nnzPerRow`	this array contains the number of elements whose absolute values are greater than tol per row.
`nnzC`	host/device pointer of the total number of elements whose absolute values are greater than tol.

See `cusparseStatus_t` for the description of the return status.

Chapter 14. cuSPARSE Generic API Reference

The cuSPARSE Generic APIs allow computing the most common sparse linear algebra operations, such as sparse matrix-vector (SpMV) and sparse matrix-matrix multiplication (SpMM), in a flexible way. The new APIs have the following capabilities and features:

- Set matrix data layouts, number of batches, and storage formats (for example, CSR, COO, and so on)
- Set input/output/compute data types. This also allows mixed data-type computation
- Set types of sparse matrix indices
- Choose the algorithm for the computation
- Provide external device memory for internal operations
- Provide extensive consistency checks across input matrices and vectors for a given routine. This includes the validation of matrix sizes, data types, layout, allowed operations, etc.
- Provide constant descriptors for vector and matrix inputs to support const-safe interface and guarantee that the APIs do not modify their inputs.

14.1. Generic Types Reference

The cuSPARSE generic type references are described in this section.

14.1.1. cudaDataType_t

The section describes the types shared by multiple CUDA Libraries and defined in the header file `library_types.h`. The `cudaDataType` type is an enumerator to specify the data precision. It is used when the data reference does not carry the type itself (e.g. `void*`). For example, it is used in the routine `cusparseSpMM()`.

Value	Meaning	Data Type	Header
`CUDA_R_16F`	The data type is 16-bit IEEE-754 floating-point	`__half`	`cuda_fp16.h`

Value	Meaning	Data Type	Header
CUDA_C_16F	The data type is 16-bit complex IEEE-754 floating-point	__half2	cuda_fp16.h
CUDA_R_16BF	The data type is 16-bit bfloat floating-point	__nv_bfloat16	cuda_bf16.h
CUDA_C_16BF	The data type is 16-bit complex bfloat floating-point	__nv_bfloat162	cuda_bf16.h
CUDA_R_32F	The data type is 32-bit IEEE-754 floating-point	float	
CUDA_C_32F	The data type is 32-bit complex IEEE-754 floating-point	cuComplex	cuComplex.h
CUDA_R_64F	The data type is 64-bit IEEE-754 floating-point	double	
CUDA_C_64F	The data type is 64-bit complex IEEE-754 floating-point	cuDoubleComplex	cuComplex.h
CUDA_R_8I	The data type is 8-bit integer	int8_t	stdint.h
CUDA_R_32I	The data type is 32-bit integer	int32_t	stdint.h

IMPORTANT: The Generic API routines allow all data types reported in the respective section of the documentation only on GPU architectures with *native* support for them. If a specific GPU model does not provide *native* support for a given data type, the routine returns `CUSPARSE_STATUS_ARCH_MISMATCH` error.

Unsupported data types and Compute Capability (CC):

- `__half` on GPUs with `CC < 53` (e.g. Kepler)
- `__nv_bfloat16` on GPUs with `CC < 80` (e.g. Kepler, Maxwell, Pascal, Volta, Turing)

see https://developer.nvidia.com/cuda-gpus

14.1.2. cusparseFormat_t

This type indicates the format of the sparse matrix.

Value	Meaning
CUSPARSE_FORMAT_COO	The matrix is stored in Coordinate (COO) format organized in *Structure of Arrays (SoA)* layout
CUSPARSE_FORMAT_COO_AOS	The matrix is stored in Coordinate (COO) format organized in *Array of Structures (SoA)* layout
CUSPARSE_FORMAT_CSR	The matrix is stored in Compressed Sparse Row (CSR) format
CUSPARSE_FORMAT_CSC	The matrix is stored in Compressed Sparse Column (CSC) format
CUSPARSE_FORMAT_BLOCKED_ELL	The matrix is stored in Blocked-Ellpack (Blocked-ELL) format

14.1.3. cusparseOrder_t

This type indicates the memory layout of a dense matrix.

Value	Meaning
CUSPARSE_ORDER_ROW	The matrix is stored in row-major
CUSPARSE_ORDER_COL	The matrix is stored in column-major

14.1.4. cusparseIndexType_t

This type indicates the index type for representing the sparse matrix indices.

Value	Meaning
CUSPARSE_INDEX_16U	16-bit unsigned integer [1, 65535]
CUSPARSE_INDEX_32I	32-bit signed integer [1, 2^31 - 1]
CUSPARSE_INDEX_64I	64-bit signed integer [1, 2^63 - 1]

14.2. Sparse Vector APIs

The cuSPARSE helper functions for sparse vector descriptor are described in this section.

14.2.1. cusparseCreateSpVec()

```
cusparseStatus_t
cusparseCreateSpVec(cusparseSpVecDescr_t* spVecDescr,
                    int64_t               size,
                    int64_t               nnz,
                    void*                 indices,
                    void*                 values,
                    cusparseIndexType_t   idxType,
                    cusparseIndexBase_t   idxBase,
                    cudaDataType          valueType)
```

```
cusparseStatus_t
cusparseCreateConstSpVec(cusparseSpVecDescr_t* spVecDescr, //const
 descriptor
                    int64_t               size,
                    int64_t               nnz,
                    void*                 indices,
                    void*                 values,
                    cusparseIndexType_t   idxType,
                    cusparseIndexBase_t   idxBase,
                    cudaDataType          valueType)
```

This function initializes the sparse matrix descriptor `spVecDescr`.

Param.	Memory	In/out	Meaning
spVecDescr	HOST	OUT	Sparse vector descriptor
size	HOST	IN	Size of the sparse vector
nnz	HOST	IN	Number of non-zero entries of the sparse vector
indices	DEVICE	IN	Indices of the sparse vector. Array of size nnz

Param.	Memory	In/out	Meaning
values	DEVICE	IN	Values of the sparse vector. Array of size nnz
idxType	HOST	IN	Enumerator specifying the data type of indices
idxBase	HOST	IN	Enumerator specifying the the base index of indices
valueType	HOST	IN	Enumerator specifying the datatype of values

Note:

- It is recommended to use constness (i.e., by using cusparseCreateConst...) for sparse and dense vector and matrix pointers if the descriptor will not be used as an output parameter of a routine (e.g., conversion functions).
- The new generic API functions pass input vector and matrix pointers as constant descriptors (i.e., //const descriptors).

See cusparseStatus_t for the description of the return status.

14.2.2. cusparseDestroySpVec()

```
cusparseStatus_t
cusparseDestroySpVec(cusparseSpVecDescr_t spVecDescr)
```

This function releases the host memory allocated for the sparse vector descriptor spVecDescr.

Param.	Memory	In/out	Meaning
spVecDescr	HOST	IN	Sparse vector descriptor

See cusparseStatus_t for the description of the return status.

14.2.3. cusparseSpVecGet()

```
cusparseStatus_t
cusparseSpVecGet(cusparseSpVecDescr_t spVecDescr,
                 int64_t*             size,
                 int64_t*             nnz,
                 void**               indices,
                 void**               values,
                 cusparseIndexType_t* idxType,
                 cusparseIndexBase_t* idxBase,
                 cudaDataType*        valueType)
```

```
cusparseStatus_t
cusparseConstSpVecGet(cusparseSpVecDescr_t spVecDescr, //const descriptor
                 int64_t*             size,
                 int64_t*             nnz,
                 void**               indices,
                 void**               values,
                 cusparseIndexType_t* idxType,
                 cusparseIndexBase_t* idxBase,
```

```
                    cudaDataType*          valueType)
```

This function returns the fields of the sparse vector descriptor `spVecDescr`.

Param.	Memory	In/out	Meaning
`spVecDescr`	HOST	IN	Sparse vector descriptor
`size`	HOST	OUT	Size of the sparse vector
`nnz`	HOST	OUT	Number of non-zero entries of the sparse vector
`indices`	DEVICE	OUT	Indices of the sparse vector. Array of size `nnz`
`values`	DEVICE	OUT	Values of the sparse vector. Array of size `nnz`
`idxType`	HOST	OUT	Enumerator specifying the data type of `indices`
`idxBase`	HOST	OUT	Enumerator specifying the the base index of `indices`
`valueType`	HOST	OUT	Enumerator specifying the datatype of `values`

See `cusparseStatus_t` for the description of the return status

14.2.4. cusparseSpVecGetIndexBase()

```
cusparseStatus_t
cusparseSpVecGetIndexBase(cusparseSpVecDescr_t spVecDescr, //const
 descriptor
                          cusparseIndexBase_t* idxBase)
```

This function returns the `idxBase` field of the sparse vector descriptor `spVecDescr`.

Param.	Memory	In/out	Meaning
`spVecDescr`	HOST	IN	Sparse vector descriptor
`idxBase`	HOST	OUT	Enumerator specifying the the base index of `indices`

See `cusparseStatus_t` for the description of the return status

14.2.5. cusparseSpVecGetValues()

```
cusparseStatus_t
cusparseSpVecGetValues(cusparseSpVecDescr_t spVecDescr,
                       void**               values)
```

```
cusparseStatus_t
cusparseConstSpVecGetValues(cusparseSpVecDescr_t spVecDescr, //const
 descriptor
                       void**                    values)
```

This function returns the `values` field of the sparse vector descriptor `spVecDescr`.

Param.	Memory	In/out	Meaning
`spVecDescr`	HOST	IN	Sparse vector descriptor
`values`	DEVICE	OUT	Values of the sparse vector. Array of size `nnz`

See cusparseStatus_t for the description of the return status

14.2.6. cusparseSpVecSetValues()

```
cusparseStatus_t
cusparseSpVecSetValues(cusparseSpVecDescr_t spVecDescr,
                       void*                values)
```

This function set the `values` field of the sparse vector descriptor `spVecDescr`.

Param.	Memory	In/out	Meaning
`spVecDescr`	HOST	IN	Sparse vector descriptor
`values`	DEVICE	IN	Values of the sparse vector. Array of size `nnz`

See cusparseStatus_t for the description of the return status

14.3. Sparse Matrix APIs

The cuSPARSE helper functions for sparse matrix descriptor are described in this section.

14.3.1. cusparseCreateCoo()

```
cusparseStatus_t
cusparseCreateCoo(cusparseSpMatDescr_t* spMatDescr,
                  int64_t               rows,
                  int64_t               cols,
                  int64_t               nnz,
                  void*                 cooRowInd,
                  void*                 cooColInd,
                  void*                 cooValues,
                  cusparseIndexType_t   cooIdxType,
                  cusparseIndexBase_t   idxBase,
                  cudaDataType          valueType)
```

```
cusparseStatus_t
cusparseCreateConstCoo(cusparseSpMatDescr_t* spMatDescr, //const descriptor
                  int64_t               rows,
                  int64_t               cols,
                  int64_t               nnz,
                  void*                 cooRowInd,
                  void*                 cooColInd,
                  void*                 cooValues,
                  cusparseIndexType_t   cooIdxType,
                  cusparseIndexBase_t   idxBase,
                  cudaDataType          valueType)
```

This function initializes the sparse matrix descriptor `spMatDescr` in the COO format (Structure of Arrays layout).

Param.	Memory	In/out	Meaning
spMatDescr	HOST	OUT	Sparse matrix descriptor
rows	HOST	IN	Number of rows of the sparse matrix
cols	HOST	IN	Number of columns of the sparse matrix
nnz	HOST	IN	Number of non-zero entries of the sparse matrix
cooRowInd	DEVICE	IN	Row indices of the sparse matrix. Array of size nnz
cooColInd	DEVICE	IN	Column indices of the sparse matrix. Array of size nnz
cooValues	DEVICE	IN	Values of the sparse matrix. Array of size nnz
cooIdxType	HOST	IN	Data type of cooRowInd and cooColInd
idxBase	HOST	IN	Base index of cooRowInd and cooColInd
valueType	HOST	IN	Datatype of cooValues

NOTE: it is safe to cast away constness (const_cast) for input pointers if the descriptor will not be used as an output parameter of a routine (e.g. conversion functions).

See cusparseStatus_t for the description of the return status

14.3.2. cusparseCreateCsr()

```
cusparseStatus_t
cusparseCreateCsr(cusparseSpMatDescr_t* spMatDescr,
                  int64_t               rows,
                  int64_t               cols,
                  int64_t               nnz,
                  void*                 csrRowOffsets,
                  void*                 csrColInd,
                  void*                 csrValues,
                  cusparseIndexType_t   csrRowOffsetsType,
                  cusparseIndexType_t   csrColIndType,
                  cusparseIndexBase_t   idxBase,
                  cudaDataType          valueType)
```

```
cusparseStatus_t
cusparseCreateConstCsr(cusparseSpMatDescr_t* spMatDescr, //const descriptor
                  int64_t               rows,
                  int64_t               cols,
                  int64_t               nnz,
                  void*                 csrRowOffsets,
                  void*                 csrColInd,
                  void*                 csrValues,
                  cusparseIndexType_t   csrRowOffsetsType,
                  cusparseIndexType_t   csrColIndType,
                  cusparseIndexBase_t   idxBase,
                  cudaDataType          valueType)
```

This function initializes the sparse matrix descriptor spMatDescr in the CSR format.

Param.	Memory	In/out	Meaning
spMatDescr	HOST	OUT	Sparse matrix descriptor
rows	HOST	IN	Number of rows of the sparse matrix
cols	HOST	IN	Number of columns of the sparse matrix
nnz	HOST	IN	Number of non-zero entries of the sparse matrix
csrRowOffsets	DEVICE	IN	Row offsets of the sparse matrix. Array of size rows + 1
csrColInd	DEVICE	IN	Column indices of the sparse matrix. Array of size nnz
csrValues	DEVICE	IN	Values of the sparse matrix. Array of size nnz
csrRowOffsetsType	HOST	IN	Data type of csrRowOffsets
csrColIndType	HOST	IN	Data type of csrColInd
idxBase	HOST	IN	Base index of csrRowOffsets and csrColInd
valueType	HOST	IN	Datatype of csrValues

NOTE: it is safe to cast away constness (const_cast) for input pointers if the descriptor will not be used as an output parameter of a routine (e.g. conversion functions).

See cusparseStatus_t for the description of the return status

14.3.3. cusparseCreateCsc()

```
cusparseStatus_t
cusparseCreateCsc(cusparseSpMatDescr_t* spMatDescr,
                  int64_t               rows,
                  int64_t               cols,
                  int64_t               nnz,
                  void*                 cscColOffsets,
                  void*                 cscRowInd,
                  void*                 cscValues,
                  cusparseIndexType_t   cscColOffsetsType,
                  cusparseIndexType_t   cscRowIndType,
                  cusparseIndexBase_t   idxBase,
                  cudaDataType          valueType)
```

```
cusparseStatus_t
cusparseCreateConstCsc(cusparseSpMatDescr_t* spMatDescr, //const descriptor
                  int64_t               rows,
                  int64_t               cols,
                  int64_t               nnz,
                  void*                 cscColOffsets,
                  void*                 cscRowInd,
                  void*                 cscValues,
                  cusparseIndexType_t   cscColOffsetsType,
                  cusparseIndexType_t   cscRowIndType,
                  cusparseIndexBase_t   idxBase,
                  cudaDataType          valueType)
```

This function initializes the sparse matrix descriptor spMatDescr in the CSC format.

Param.	Memory	In/out	Meaning
spMatDescr	HOST	OUT	Sparse matrix descriptor
rows	HOST	IN	Number of rows of the sparse matrix
cols	HOST	IN	Number of columns of the sparse matrix
nnz	HOST	IN	Number of non-zero entries of the sparse matrix
cscColOffsets	DEVICE	IN	Column offsets of the sparse matrix. Array of size `cols + 1`
cscRowInd	DEVICE	IN	Row indices of the sparse matrix. Array of size `nnz`
cscValues	DEVICE	IN	Values of the sparse matrix. Array of size `nnz`
cscColOffsetsType	HOST	IN	Data type of `cscColOffsets`
cscRowIndType	HOST	IN	Data type of `cscRowInd`
idxBase	HOST	IN	Base index of `cscColOffsets` and `cscRowInd`
valueType	HOST	IN	Datatype of `cscValues`

NOTE: it is safe to cast away constness (`const_cast`) for input pointers if the descriptor will not be used as an output parameter of a routine (e.g. conversion functions).

See `cusparseStatus_t` for the description of the return status

14.3.4. cusparseCreateBlockedEll()

```
cusparseStatus_t
cusparseCreateBlockedEll(cusparseSpMatDescr_t* spMatDescr,
                         int64_t               rows,
                         int64_t               cols,
                         int64_t               ellBlockSize,
                         int64_t               ellCols,
                         void*                 ellColInd,
                         void*                 ellValue,
                         cusparseIndexType_t   ellIdxType,
                         cusparseIndexBase_t   idxBase,
                         cudaDataType          valueType)
```

```
cusparseStatus_t
cusparseCreateConstBlockedEll(cusparseSpMatDescr_t* spMatDescr, //const
 descriptor
                         int64_t               rows,
                         int64_t               cols,
                         int64_t               ellBlockSize,
                         int64_t               ellCols,
                         void*                 ellColInd,
                         void*                 ellValue,
                         cusparseIndexType_t   ellIdxType,
                         cusparseIndexBase_t   idxBase,
                         cudaDataType          valueType)
```

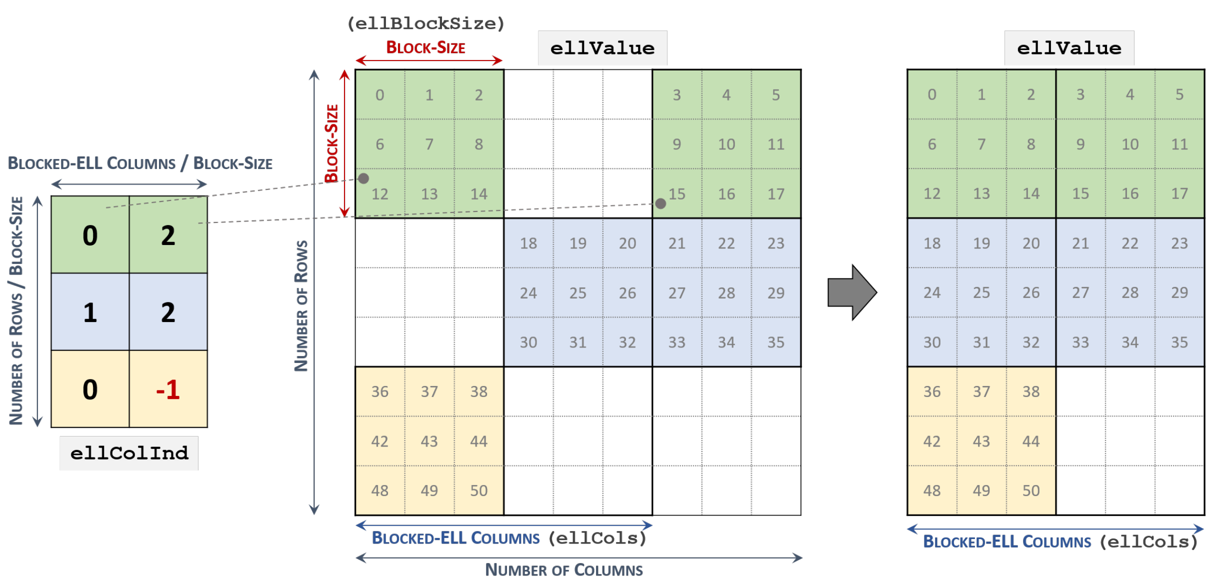
This function initializes the sparse matrix descriptor `spMatDescr` for the Blocked-Ellpack (ELL) format.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	OUT	Sparse matrix descriptor
`rows`	HOST	IN	Number of rows of the sparse matrix
`cols`	HOST	IN	Number of columns of the sparse matrix
`ellBlockSize`	HOST	IN	Size of the ELL-Block
`ellCols`	HOST	IN	Actual number of columns of the Blocked-Ellpack format (`ellValue` columns)
`ellColInd`	DEVICE	IN	Blocked-ELL Column indices. Array of size `[ellCols / ellBlockSize][rows / ellBlockSize]`
`ellValue`	DEVICE	IN	Values of the sparse matrix. Array of size `rows * ellCols`
`ellIdxType`	HOST	IN	Data type of `ellColInd`
`idxBase`	HOST	IN	Base index of `ellColInd`
`valueType`	HOST	IN	Datatype of `ellValue`

Blocked-ELL Column indices (`ellColInd`) are in the range `[0, cols / ellBlockSize -1]`. The array can contain `-1` values for indicating empty blocks.

Note: It is safe to cast away constness (`const_cast`) for input pointers if the descriptor will not be used as an output parameter of a routine (e.g. conversion functions).

Figure 1. Blocked-ELL representation



See [`cusparseStatus_t`]() for the description of the return status.

14.3.5. cusparseDestroySpMat()

```
cusparseStatus_t
cusparseDestroySpMat(cusparseSpMatDescr_t spMatDescr /*const descriptor*/)
```

This function releases the host memory allocated for the sparse matrix descriptor `spMatDescr`.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor

See `cusparseStatus_t` for the description of the return status

14.3.6. cusparseCooGet()

```
cusparseStatus_t
cusparseCooGet(cusparseSpMatDescr_t spMatDescr,
               int64_t*             rows,
               int64_t*             cols,
               int64_t*             nnz,
               void**               cooRowInd,
               void**               cooColInd,
               void**               cooValues,
               cusparseIndexType_t* idxType,
               cusparseIndexBase_t* idxBase,
               cudaDataType*        valueType)
```

```
cusparseStatus_t
cusparseConstCooGet(cusparseSpMatDescr_t spMatDescr, //const descriptor
               int64_t*             rows,
               int64_t*             cols,
               int64_t*             nnz,
               void**               cooRowInd,
               void**               cooColInd,
               void**               cooValues,
               cusparseIndexType_t* idxType,
               cusparseIndexBase_t* idxBase,
               cudaDataType*        valueType)
```

This function returns the fields of the sparse matrix descriptor `spMatDescr` stored in COO format (Array of Structures layout).

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor
`rows`	HOST	OUT	Number of rows of the sparse matrix
`cols`	HOST	OUT	Number of columns of the sparse matrix
`nnz`	HOST	OUT	Number of non-zero entries of the sparse matrix

Param.	Memory	In/out	Meaning
cooRowInd	DEVICE	OUT	Row indices of the sparse matrix. Array of size nnz
cooColInd	DEVICE	OUT	Column indices of the sparse matrix. Array of size nnz
cooValues	DEVICE	OUT	Values of the sparse matrix. Array of size nnz
cooIdxType	HOST	OUT	Data type of cooRowInd and cooColInd
idxBase	HOST	OUT	Base index of cooRowInd and cooColInd
valueType	HOST	OUT	Datatype of cooValues

See cusparseStatus_t for the description of the return status.

14.3.7. cusparseCsrGet()

```
cusparseStatus_t
cusparseCsrGet(cusparseSpMatDescr_t spMatDescr,
               int64_t*             rows,
               int64_t*             cols,
               int64_t*             nnz,
               void**               csrRowOffsets,
               void**               csrColInd,
               void**               csrValues,
               cusparseIndexType_t* csrRowOffsetsType,
               cusparseIndexType_t* csrColIndType,
               cusparseIndexBase_t* idxBase,
               cudaDataType*        valueType)
```

```
cusparseStatus_t
cusparseConstCsrGet(cusparseSpMatDescr_t spMatDescr, //const descriptor
               int64_t*             rows,
               int64_t*             cols,
               int64_t*             nnz,
               void**               csrRowOffsets,
               void**               csrColInd,
               void**               csrValues,
               cusparseIndexType_t* csrRowOffsetsType,
               cusparseIndexType_t* csrColIndType,
               cusparseIndexBase_t* idxBase,
               cudaDataType*        valueType)
```

This function returns the fields of the sparse matrix descriptor spMatDescr stored in CSR format.

Param.	Memory	In/out	Meaning
spMatDescr	HOST	IN	Sparse matrix descriptor
rows	HOST	OUT	Number of rows of the sparse matrix
cols	HOST	OUT	Number of columns of the sparse matrix
nnz	HOST	OUT	Number of non-zero entries of the sparse matrix
csrRowOffsets	DEVICE	OUT	Row offsets of the sparse matrix. Array of size rows + 1

Param.	Memory	In/out	Meaning
csrColInd	DEVICE	OUT	Column indices of the sparse matrix. Array of size nnz
csrValues	DEVICE	OUT	Values of the sparse matrix. Array of size nnz
csrRowOffsetsType	HOST	OUT	Data type of csrRowOffsets
csrColIndType	HOST	OUT	Data type of csrColInd
idxBase	HOST	OUT	Base index of csrRowOffsets and csrColInd
valueType	HOST	OUT	Datatype of csrValues

See cusparseStatus_t for the description of the return status

14.3.8. cusparseCscGet()

```
cusparseStatus_t
cusparseCscGet(cusparseSpMatDescr_t spMatDescr,
               int64_t*             rows,
               int64_t*             cols,
               int64_t*             nnz,
               void**               cscRowOffsets,
               void**               cscColInd,
               void**               cscValues,
               cusparseIndexType_t* cscRowOffsetsType,
               cusparseIndexType_t* cscColIndType,
               cusparseIndexBase_t* idxBase,
               cudaDataType*        valueType)
```

```
cusparseStatus_t
cusparseConstCscGet(cusparseSpMatDescr_t spMatDescr, //const descriptor
               int64_t*             rows,
               int64_t*             cols,
               int64_t*             nnz,
               void**               cscRowOffsets,
               void**               cscColInd,
               void**               cscValues,
               cusparseIndexType_t* cscRowOffsetsType,
               cusparseIndexType_t* cscColIndType,
               cusparseIndexBase_t* idxBase,
               cudaDataType*        valueType)
```

This function returns the fields of the sparse matrix descriptor spMatDescr stored in CSC format.

Param.	Memory	In/out	Meaning
spMatDescr	HOST	IN	Sparse matrix descriptor
rows	HOST	OUT	Number of rows of the sparse matrix
cols	HOST	OUT	Number of columns of the sparse matrix
nnz	HOST	OUT	Number of non-zero entries of the sparse matrix
cscRowOffsets	DEVICE	OUT	Row offsets of the sparse matrix. Array of size rows + 1

Param.	Memory	In/out	Meaning
cscColInd	DEVICE	OUT	Column indices of the sparse matrix. Array of size nnz
cscValues	DEVICE	OUT	Values of the sparse matrix. Array of size nnz
cscRowOffsetsType	HOST	OUT	Data type of cscRowOffsets
cscColIndType	HOST	OUT	Data type of cscColInd
idxBase	HOST	OUT	Base index of cscRowOffsets and cscColInd
valueType	HOST	OUT	Datatype of cscValues

See cusparseStatus_t for the description of the return status

14.3.9. cusparseCsrSetPointers()

```
cusparseStatus_t
cusparseCsrSetPointers(cusparseSpMatDescr_t spMatDescr,
                       void*                csrRowOffsets,
                       void*                csrColInd,
                       void*                csrValues)
```

This function sets the pointers of the sparse matrix descriptor spMatDescr.

Param.	Memory	In/out	Meaning
spMatDescr	HOST	IN	Sparse matrix descriptor
csrRowOffsets	DEVICE	IN	Row offsets of the sparse matrix. Array of size rows + 1
csrColInd	DEVICE	IN	Column indices of the sparse matrix. Array of size nnz
csrValues	DEVICE	IN	Values of the sparse matrix. Array of size nnz

See cusparseStatus_t for the description of the return status.

14.3.10. cusparseCscSetPointers()

```
cusparseStatus_t
cusparseCscSetPointers(cusparseSpMatDescr_t spMatDescr,
                       void*                cscColOffsets,
                       void*                cscRowInd,
                       void*                cscValues)
```

This function sets the pointers of the sparse matrix descriptor spMatDescr.

Param.	Memory	In/out	Meaning
spMatDescr	HOST	IN	Sparse matrix descriptor
cscColOffsets	DEVICE	IN	Col offsets of the sparse matrix. Array of size cols + 1
cscRowInd	DEVICE	IN	Row indices of the sparse matrix. Array of size nnz
cscValues	DEVICE	IN	Values of the sparse matrix. Array of size nnz

See cusparseStatus_t for the description of the return status.

14.3.11. cusparseCooSetPointers()

```
cusparseStatus_t
cusparseCooSetPointers(cusparseSpMatDescr_t spMatDescr,
                       void*                cooRows,
                       void*                cooColumns,
                       void*                cooValues)
```

This function sets the pointers of the sparse matrix descriptor `spMatDescr`.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor
`cooRows`	DEVICE	IN	Row indices of the sparse matrix. Array of size `nnz`
`cooColumns`	DEVICE	IN	Column indices of the sparse matrix. Array of size `nnz`
`cooValues`	DEVICE	IN	Values of the sparse matrix. Array of size `nnz`

See cusparseStatus_t for the description of the return status.

14.3.12. cusparseBlockedEllGet()

```
cusparseStatus_t
cusparseBlockedEllGet(cusparseSpMatDescr_t spMatDescr,
                      int64_t*             rows,
                      int64_t*             cols,
                      int64_t*             ellBlockSize,
                      int64_t*             ellCols,
                      void**               ellColInd,
                      void**               ellValue,
                      cusparseIndexType_t* ellIdxType,
                      cusparseIndexBase_t* idxBase,
                      cudaDataType*        valueType)
```

```
cusparseStatus_t
cusparseConstBlockedEllGet(cusparseSpMatDescr_t spMatDescr, //const
 descriptor
                      int64_t*             rows,
                      int64_t*             cols,
                      int64_t*             ellBlockSize,
                      int64_t*             ellCols,
                      void**               ellColInd,
                      void**               ellValue,
                      cusparseIndexType_t* ellIdxType,
                      cusparseIndexBase_t* idxBase,
                      cudaDataType*        valueType)
```

This function returns the fields of the sparse matrix descriptor `spMatDescr` stored in Blocked-Ellpack (ELL) format.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor
`rows`	HOST	OUT	Number of rows of the sparse matrix
`cols`	HOST	OUT	Number of columns of the sparse matrix
`ellBlockSize`	HOST	OUT	Size of the ELL-Block
`ellCols`	HOST	OUT	Actual number of columns of the Blocked-Ellpack format
`ellColInd`	DEVICE	OUT	Column indices for the ELL-Block. Array of size `[cols / ellBlockSize][rows / ellBlockSize]`
`ellValue`	DEVICE	OUT	Values of the sparse matrix. Array of size `rows * ellCols`
`ellIdxType`	HOST	OUT	Data type of `ellColInd`
`idxBase`	HOST	OUT	Base index of `ellColInd`
`valueType`	HOST	OUT	Datatype of `ellValue`

See `cusparseStatus_t` for the description of the return status.

14.3.13. cusparseSpMatGetSize()

```
cusparseStatus_t
cusparseSpMatGetSize(cusparseSpMatDescr_t spMatDescr, //const descriptor
                     int64_t*             rows,
                     int64_t*             cols,
                     int64_t*             nnz)
```

This function returns the sizes of the sparse matrix `spMatDescr`.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor
`rows`	HOST	OUT	Number of rows of the sparse matrix
`cols`	HOST	OUT	Number of columns of the sparse matrix
`nnz`	HOST	OUT	Number of non-zero entries of the sparse matrix

See `cusparseStatus_t` for the description of the return status.

14.3.14. cusparseSpMatGetFormat()

```
cusparseStatus_t
cusparseSpMatGetFormat(cusparseSpMatDescr_t spMatDescr, //const descriptor
```

```
                                cusparseFormat_t*     format)
```

This function returns the `format` field of the sparse matrix descriptor `spMatDescr`.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor
`format`	HOST	OUT	Storage format of the sparse matrix

See [cusparseStatus_t](#) for the description of the return status

14.3.15. cusparseSpMatGetIndexBase()

```
cusparseStatus_t
cusparseSpMatGetIndexBase(cusparseSpMatDescr_t spMatDescr, //const
 descriptor
                          cusparseIndexBase_t* idxBase)
```

This function returns the `idxBase` field of the sparse matrix descriptor `spMatDescr`.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor
`idxBase`	HOST	OUT	Base index of the sparse matrix

See [cusparseStatus_t](#) for the description of the return status

14.3.16. cusparseSpMatGetValues()

```
cusparseStatus_t
cusparseSpMatGetValues(cusparseSpMatDescr_t spMatDescr,
                       void**               values)
```

```
cusparseStatus_t
cusparseConstSpMatGetValues(cusparseSpMatDescr_t spMatDescr, //const
 descriptor
                       void**               values)
```

This function returns the `values` field of the sparse matrix descriptor `spMatDescr`.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor
`values`	DEVICE	OUT	Values of the sparse matrix. Array of size `nnz`

See [cusparseStatus_t](#) for the description of the return status

14.3.17. cusparseSpMatSetValues()

```
cusparseStatus_t
cusparseSpMatSetValues(cusparseSpMatDescr_t spMatDescr,
                       void*                values)
```

This function sets the values field of the sparse matrix descriptor spMatDescr.

Param.	Memory	In/out	Meaning
spMatDescr	HOST	IN	Sparse matrix descriptor
values	DEVICE	IN	Values of the sparse matrix. Array of size nnz

See cusparseStatus_t for the description of the return status.

14.3.18. cusparseSpMatGetStridedBatch()

```
cusparseStatus_t
cusparseSpMatGetStridedBatch(cusparseSpMatDescr_t spMatDescr, //const
 descriptor
                             int*                 batchCount)
```

This function returns the batchCount field of the sparse matrixdescriptor spMatDescr.

Param.	Memory	In/out	Meaning
spMatDescr	HOST	IN	Sparse matrix descriptor
batchCount	HOST	OUT	Number of batches of the sparse matrix

See cusparseStatus_t for the description of the return status

14.3.19. cusparseCooSetStridedBatch()

```
cusparseStatus_t
cusparseCooSetStridedBatch(cusparseSpMatDescr_t spMatDescr,
                           int                  batchCount,
                           int64_t              batchStride)
```

This function sets the batchCount and the batchStride fields of the sparse matrix descriptor spMatDescr.

Param.	Memory	In/out	Meaning
spMatDescr	HOST	IN	Sparse matrix descriptor
batchCount	HOST	IN	Number of batches of the sparse matrix
batchStride	HOST	IN	address offset between consecutive batches

See cusparseStatus_t for the description of the return status.

14.3.20. cusparseCsrSetStridedBatch()

```
cusparseStatus_t
cusparseCsrSetStridedBatch(cusparseSpMatDescr_t spMatDescr,
                           int                  batchCount,
                           int64_t              offsetsBatchStride,
                           int64_t              columnsValuesBatchStride)
```

This function sets the `batchCount` and the `batchStride` fields of the sparse matrix descriptor `spMatDescr`.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor
`batchCount`	HOST	IN	Number of batches of the sparse matrix
`offsetsBatchStride`	HOST	IN	Address offset between consecutive batches for the row offset array
`columnsValuesBatchStride`	HOST	IN	Address offset between consecutive batches for the column and value arrays

See cusparseStatus_t for the description of the return status.

14.3.21. cusparseSpMatGetAttribute()

```
cusparseStatus_t
cusparseSpMatGetAttribute(cusparseSpMatDescr_t     spMatDescr, //const
 descriptor
                          cusparseSpMatAttribute_t attribute,
                          void*                    data,
                          size_t                   dataSize)
```

The function gets the attributes of the sparse matrix descriptor `spMatDescr`.

Param.	Memory	In/out	Meaning
`spMatDescr`	HOST	IN	Sparse matrix descriptor
`attribute`	HOST	IN	Attribute enumerator
`data`	HOST	OUT	Attribute value
`dataSize`	HOST	IN	Size of the attribute in bytes for safety

Attribute	Meaning	Possible Values
`CUSPARSE_SPMAT_FILL_MODE`	Indicates if the lower or upper part of a matrix is stored in sparse storage	`CUSPARSE_FILL_MODE_LOWER` `CUSPARSE_FILL_MODE_UPPER`

Attribute	Meaning	Possible Values
CUSPARSE_SPMAT_DIAG_TYPE	Indicates if the matrix diagonal entries are unity	CUSPARSE_DIAG_TYPE_NON_UNIT CUSPARSE_DIAG_TYPE_UNIT

See cusparseStatus_t for the description of the return status.

14.3.22. cusparseSpMatSetAttribute()

```
cusparseStatus_t
cusparseSpMatSetAttribute(cusparseSpMatDescr_t     spMatDescr,
                          cusparseSpMatAttribute_t attribute,
                          const void*              data,
                          size_t                   dataSize)
```

The function sets the attributes of the sparse matrix descriptor `spMatDescr`

Param.	Memory	In/out	Meaning
spMatDescr	HOST	OUT	Sparse matrix descriptor
attribute	HOST	IN	Attribute enumerator
data	HOST	IN	Attribute value
dataSize	HOST	IN	Size of the attribute in bytes for safety

Attribute	Meaning	Possible Values
CUSPARSE_SPMAT_FILL_MODE	Indicates if the lower or upper part of a matrix is stored in sparse storage	CUSPARSE_FILL_MODE_LOWER CUSPARSE_FILL_MODE_UPPER
CUSPARSE_SPMAT_DIAG_TYPE	Indicates if the matrix diagonal entries are unity	CUSPARSE_DIAG_TYPE_NON_UNIT CUSPARSE_DIAG_TYPE_UNIT

See cusparseStatus_t for the description of the return status.

14.4. Dense Vector APIs

The cuSPARSE helper functions for dense vector descriptor are described in this section.

14.4.1. cusparseCreateDnVec()

```
cusparseStatus_t
cusparseCreateDnVec(cusparseDnVecDescr_t* dnVecDescr,
```

```
                    int64_t                 size,
                    void*                   values,
                    cudaDataType            valueType)
```

```
cusparseStatus_t
cusparseCreateConstDnVec(cusparseDnVecDescr_t* dnVecDescr, //const
 descriptor
                    int64_t                 size,
                    void*                   values,
                    cudaDataType            valueType)
```

This function initializes the dense vector descriptor `dnVecDescr`.

Param.	Memory	In/out	Meaning
`dnVecDescr`	HOST	OUT	Dense vector descriptor
`size`	HOST	IN	Size of the dense vector
`values`	DEVICE	IN	Values of the dense vector. Array of size `size`
`valueType`	HOST	IN	Enumerator specifying the datatype of `values`

NOTE: it is safe to cast away constness (`const_cast`) for input pointers if the descriptor will not be used as an output parameter of a routine (e.g. conversion functions).

See `cusparseStatus_t` for the description of the return status

14.4.2. cusparseDestroyDnVec()

```
cusparseStatus_t
cusparseDestroyDnVec(cusparseDnVecDescr_t dnVecDescr /*const descriptor*/)
```

This function releases the host memory allocated for the dense vector descriptor `dnVecDescr`.

Param.	Memory	In/out	Meaning
`dnVecDescr`	HOST	IN	Dense vector descriptor

See `cusparseStatus_t` for the description of the return status

14.4.3. cusparseDnVecGet()

```
cusparseStatus_t
cusparseDnVecGet(cusparseDnVecDescr_t dnVecDescr,
                 int64_t*             size,
                 void**               values,
                 cudaDataType*        valueType)
```

```
cusparseStatus_t
cusparseConstDnVecGet(cusparseDnVecDescr_t dnVecDescr, //const descriptor
                 int64_t*             size,
                 void**               values,
                 cudaDataType*        valueType)
```

This function returns the fields of the dense vector descriptor `dnVecDescr`.

Param.	Memory	In/out	Meaning
`dnVecDescr`	HOST	IN	Dense vector descriptor
`size`	HOST	OUT	Size of the dense vector
`values`	DEVICE	OUT	Values of the dense vector. Array of size `nnz`
`valueType`	HOST	OUT	Enumerator specifying the datatype of `values`

See `cusparseStatus_t` for the description of the return status

14.4.4. cusparseDnVecGetValues()

```
cusparseStatus_t
cusparseDnVecGetValues(cusparseDnVecDescr_t dnVecDescr,
                       void**               values)
```

```
cusparseStatus_t
cusparseConstDnVecGetValues(cusparseDnVecDescr_t dnVecDescr, //const
 descriptor
                       void**               values)
```

This function returns the `values` field of the dense vector descriptor `dnVecDescr`.

Param.	Memory	In/out	Meaning
`dnVecDescr`	HOST	IN	Dense vector descriptor
`values`	DEVICE	OUT	Values of the dense vector

See `cusparseStatus_t` for the description of the return status

14.4.5. cusparseDnVecSetValues()

```
cusparseStatus_t
cusparseDnVecSetValues(cusparseDnVecDescr_t dnVecDescr,
                       void*                values)
```

This function set the `values` field of the dense vector descriptor `dnVecDescr`.

Param.	Memory	In/out	Meaning
`dnVecDescr`	HOST	IN	Dense vector descriptor
`values`	DEVICE	IN	Values of the dense vector. Array of size `size`

The possible error values returned by this function and their meanings are listed below :

See `cusparseStatus_t` for the description of the return status

14.5. Dense Matrix APIs

The cuSPARSE helper functions for dense matrix descriptor are described in this section.

14.5.1. cusparseCreateDnMat()

```
cusparseStatus_t
cusparseCreateDnMat(cusparseDnMatDescr_t* dnMatDescr,
                    int64_t               rows,
                    int64_t               cols,
                    int64_t               ld,
                    void*                 values,
                    cudaDataType          valueType,
                    cusparseOrder_t       order)
```

```
cusparseStatus_t
cusparseCreateConstDnMat(cusparseDnMatDescr_t* dnMatDescr, //const
 descriptor
                    int64_t               rows,
                    int64_t               cols,
                    int64_t               ld,
                    void*                 values,
                    cudaDataType          valueType,
                    cusparseOrder_t       order)
```

The function initializes the dense matrix descriptor `dnMatDescr`.

Param.	Memory	In/out	Meaning
`dnMatDescr`	HOST	OUT	Dense matrix descriptor
`rows`	HOST	IN	Number of rows of the dense matrix
`cols`	HOST	IN	Number of columns of the dense matrix
`ld`	HOST	IN	Leading dimension of the dense matrix
`values`	DEVICE	IN	Values of the dense matrix. Array of size `size`
`valueType`	HOST	IN	Enumerator specifying the datatype of `values`
`order`	HOST	IN	Enumerator specifying the memory layout of the dense matrix

NOTE: it is safe to cast away constness (`const_cast`) for input pointers if the descriptor will not be used as an output parameter of a routine (e.g. conversion functions).

See [`cusparseStatus_t`] for the description of the return status

14.5.2. cusparseDestroyDnMat()

```
cusparseStatus_t
cusparseDestroyDnMat(cusparseDnMatDescr_t dnMatDescr /*const descriptor*/)
```

This function releases the host memory allocated for the dense matrix descriptor `dnMatDescr`.

Param.	Memory	In/out	Meaning
`dnMatDescr`	HOST	IN	Dense matrix descriptor

See `cusparseStatus_t` for the description of the return status

14.5.3. cusparseDnMatGet()

```
cusparseStatus_t
cusparseDnMatGet(cusparseDnMatDescr_t dnMatDescr,
                 int64_t*             rows,
                 int64_t*             cols,
                 int64_t*             ld,
                 void**               values,
                 cudaDataType*        type,
                 cusparseOrder_t*     order)
```

```
cusparseStatus_t
cusparseConstDnMatGet(cusparseDnMatDescr_t dnMatDescr, //const descriptor
                 int64_t*             rows,
                 int64_t*             cols,
                 int64_t*             ld,
                 void**               values,
                 cudaDataType*        type,
                 cusparseOrder_t*     order)
```

This function returns the fields of the dense matrix descriptor `dnMatDescr`.

Param.	Memory	In/out	Meaning
`dnMatDescr`	HOST	IN	Dense matrix descriptor
`rows`	HOST	OUT	Number of rows of the dense matrix
`cols`	HOST	OUT	Number of columns of the dense matrix
`ld`	HOST	OUT	Leading dimension of the dense matrix
`values`	DEVICE	OUT	Values of the dense matrix. Array of size `ld * cols`
`valueType`	HOST	OUT	Enumerator specifying the datatype of `values`
`order`	HOST	OUT	Enumerator specifying the memory layout of the dense matrix

See `cusparseStatus_t` for the description of the return status.

14.5.4. cusparseDnMatGetValues()

```
cusparseStatus_t
cusparseDnMatGetValues(cusparseDnMatDescr_t dnMatDescr,
                       void**               values)
```

```
cusparseStatus_t
cusparseConstDnMatGetValues(cusparseDnMatDescr_t dnMatDescr, //const
 descriptor
                       void**               values)
```

This function returns the `values` field of the dense matrix descriptor `dnMatDescr`.

Param.	Memory	In/out	Meaning
`dnMatDescr`	HOST	IN	Dense matrix descriptor
`values`	DEVICE	OUT	Values of the dense matrix. Array of size `ld * cols`

See cusparseStatus_t for the description of the return status

14.5.5. cusparseDnSetValues()

```
cusparseStatus_t
cusparseDnMatSetValues(cusparseDnMatDescr_t dnMatDescr,
                       void*                values)
```

This function sets the `values` field of the dense matrix descriptor `dnMatDescr`.

Param.	Memory	In/out	Meaning
`dnMatDescr`	HOST	IN	Dense matrix descriptor
`values`	DEVICE	IN	Values of the dense matrix. Array of size `ld * cols`

See cusparseStatus_t for the description of the return status.

14.5.6. cusparseDnMatGetStridedBatch()

```
cusparseStatus_t
cusparseDnMatGetStridedBatch(cusparseDnMatDescr_t dnMatDescr, //const
 descriptor
                             int*                 batchCount,
                             int64_t*             batchStride)
```

The function returns the number of batches and the batch stride of the dense matrix descriptor `dnMatDescr`.

Param.	Memory	In/out	Meaning
`dnMatDescr`	HOST	IN	Dense matrix descriptor
`batchCount`	HOST	OUT	Number of batches of the dense matrix
`batchStride`	HOST	OUT	Address offset between a matrix and the next one in the batch

See cusparseStatus_t for the description of the return status

14.5.7. cusparseDnMatSetStridedBatch()

```
cusparseStatus_t
cusparseDnMatSetStridedBatch(cusparseDnMatDescr_t dnMatDescr,
                             int                  batchCount,
                             int64_t              batchStride)
```

The function sets the number of batches and the batch stride of the dense matrix descriptor `dnMatDescr`.

Param.	Memory	In/out	Meaning
`dnMatDescr`	HOST	IN	Dense matrix descriptor
`batchCount`	HOST	IN	Number of batches of the dense matrix
`batchStride`	HOST	IN	Address offset between a matrix and the next one in the batch. `batchStride ≥ ld * cols` if the matrix uses column-major layout, `batchStride ≥ ld * rows` otherwise

See `cusparseStatus_t` for the description of the return status

14.6. Generic API Functions

14.6.1. cusparseSparseToDense()

```
cusparseStatus_t
cusparseSparseToDense_bufferSize(cusparseHandle_t           handle,
                                 cusparseSpMatDescr_t       matA, //const
 descriptor
                                 cusparseDnMatDescr_t       matB,
                                 cusparseSparseToDenseAlg_t alg,
                                 size_t*                    bufferSize)
```

```
cusparseStatus_t
cusparseSparseToDense(cusparseHandle_t           handle,
                      cusparseSpMatDescr_t       matA, //const descriptor
                      cusparseDnMatDescr_t       matB,
                      cusparseSparseToDenseAlg_t alg,
                      void*                      buffer)
```

The function converts the sparse matrix `matA` in CSR, CSC, or COO format into its dense representation `matB`. Blocked-ELL is not currently supported.

The function `cusparseSparseToDense_bufferSize()` returns the size of the workspace needed by `cusparseSparseToDense()`

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`matA`	HOST	IN	Sparse matrix `A`
`matB`	HOST	OUT	Dense matrix `B`
`alg`	HOST	IN	Algorithm for the computation

Param.	Memory	In/out	Meaning
`bufferSize`	HOST	OUT	Number of bytes of workspace needed by `cusparseSparseToDense()`
`buffer`	DEVICE	IN	Pointer to workspace buffer

`cusparseSparseToDense()` supports the following index type for representing the sparse matrix `matA`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseSparseToDense()` supports the following data types:

A/B
`CUDA_R_16F`
`CUDA_R_16BF`
`CUDA_R_32F`
`CUDA_R_64F`
`CUDA_C_16F`
`CUDA_C_16BF`
`CUDA_C_32F`
`CUDA_C_64F`

`cusparseSparse2Dense()` supports the following algorithm:

Algorithm	Notes
`CUSPARSE_SPARSETODENSE_ALG_DEFAULT`	Default algorithm

`cusparseSparseToDense()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run

`cusparseSparseToDense()` supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See `cusparseStatus_t` for the description of the return status

Please visit cuSPARSE Library Samples - cusparseSparseToDense for a code example.

14.6.2. cusparseDenseToSparse()

```
cusparseStatus_t
cusparseDenseToSparse_bufferSize(cusparseHandle_t           handle,
                                 cusparseDnMatDescr_t       matA, //const
 descriptor
                                 cusparseSpMatDescr_t       matB,
                                 cusparseDenseToSparseAlg_t alg,
                                 size_t*                    bufferSize)
```

```
cusparseStatus_t
cusparseDenseToSparse_analysis(cusparseHandle_t           handle,
                               cusparseDnMatDescr_t       matA, //const
 descriptor
                               cusparseSpMatDescr_t       matB,
                               cusparseDenseToSparseAlg_t alg,
                               void*                      buffer)
```

```
cusparseStatus_t
cusparseDenseToSparse_convert(cusparseHandle_t           handle,
                              cusparseDnMatDescr_t       matA, //const
 descriptor
                              cusparseSpMatDescr_t       matB,
                              cusparseDenseToSparseAlg_t alg,
                              void*                      buffer)
```

The function converts the dense matrix `matA` into a sparse matrix `matB` in CSR, CSC, COO, or Blocked-ELL format.

The function `cusparseDenseToSparse_bufferSize()` returns the size of the workspace needed by `cusparseDenseToSparse_analysis()`.

The function `cusparseDenseToSparse_analysis()` updates the number of non-zero elements in the sparse matrix descriptor `matB`. The user is responsible to allocate the memory required by the sparse matrix:

- Row/Column indices and value arrays for CSC and CSR respectively
- Row, column, value arrays for COO
- Column (`ellColInd`), value (`ellValue`) arrays for Blocked-ELL

Finally, we call `cusparseDenseToSparse_convert()` for filling the arrays allocated in the previous step.

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`matA`	HOST	IN	Dense matrix `A`
`matB`	HOST	OUT	Sparse matrix `B`
`alg`	HOST	IN	Algorithm for the computation

Param.	Memory	In/out	Meaning
`bufferSize`	HOST	OUT	Number of bytes of workspace needed by `cusparseDenseToSparse_analysis()`
`buffer`	DEVICE	IN	Pointer to workspace buffer

`cusparseDenseToSparse()` supports the following index type for representing the sparse vector `matB`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseDenseToSparse()` supports the following data types:

A/B
`CUDA_R_16F`
`CUDA_R_16BF`
`CUDA_R_32F`
`CUDA_R_64F`
`CUDA_C_16F`
`CUDA_C_16BF`
`CUDA_C_32F`
`CUDA_C_64F`

`cusparseDense2Sparse()` supports the following algorithm:

Algorithm	Notes
`CUSPARSE_DENSETOSPARSE_ALG_DEFAULT`	Default algorithm

`cusparseDenseToSparse()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run

`cusparseDenseToSparse()` supports the following optimizations:

- The routine supports does **not** support CUDA graph capture for CSR, CSC, COO formats
- Hardware Memory Compression

See `cusparseStatus_t` for the description of the return status.

Please visit cuSPARSE Library Samples - cusparseDenseToSparse (CSR) and cuSPARSE Library Samples - cusparseDenseToSparse (Blocked-ELL) for code examples.

14.6.3. cusparseAxpby()

```
cusparseStatus_t
cusparseAxpby(cusparseHandle_t     handle,
              const void*          alpha,
              cusparseSpVecDescr_t vecX, //const descriptor
              const void*          beta,
              cusparseDnVecDescr_t vecY)
```

The function computes the sum of a sparse vector `vecX` and a dense vector `vecY`

$$\mathbf{Y} = \alpha\mathbf{X} + \beta\mathbf{Y}$$

In other words,

```
for i=0 to n-1
    Y[i] = beta * Y[i]
for i=0 to nnz-1
    Y[X_indices[i]] += alpha * X_values[i]
```

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`alpha`	HOST or DEVICE	IN	α scalar used for multiplication of compute type
`vecX`	HOST	IN	Sparse vector `X`
`beta`	HOST or DEVICE	IN	β scalar used for multiplication of compute type
`vecY`	HOST	IN/OUT	Dense vector `Y`

`cusparseAxpby` supports the following index type for representing the sparse vector `vecX`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseAxpby` supports the following data types:

Uniform-precision computation:

`X`/`Y`/`compute`
`CUDA_R_32F`
`CUDA_R_64F`
`CUDA_C_32F`
`CUDA_C_64F`

Mixed-precision computation:

`X`/`Y`	`compute`
`CUDA_R_16F`	`CUDA_R_32F`

X/Y	compute
CUDA_R_16BF	
CUDA_C_16F	CUDA_C_32F
CUDA_C_16BF	

cusparseAxpby() has the following constraints:

- The arrays representing the sparse vector vecX must be aligned to 16 bytes

cusparseAxpby() has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the the sparse vector vecX indices are distinct

cusparseAxpby() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See cusparseStatus_t for the description of the return status

Please visit cuSPARSE Library Samples - cusparseAxpby for a code example.

14.6.4. cusparseGather()

```
cusparseStatus_t
cusparseGather(cusparseHandle_t     handle,
               cusparseDnVecDescr_t vecY, //const descriptor
               cusparseSpVecDescr_t vecX)
```

The function gathers the elements of the dense vector vecY into the sparse vector vecX

In other words,

```
for i=0 to nnz-1
    X_values[i] = Y[X_indices[i]]
```

Param.	Memory	In/out	Meaning
handle	HOST	IN	Handle to the cuSPARSE library context
vecX	HOST	OUT	Sparse vector X
vecY	HOST	IN	Dense vector Y

cusparseGather supports the following index type for representing the sparse vector vecX:

- 32-bit indices (CUSPARSE_INDEX_32I)

- 64-bit indices (CUSPARSE_INDEX_64I)

cusparseGather supports the following data types:

X/Y
CUDA_R_8I
CUDA_R_16F
CUDA_R_16BF
CUDA_R_32F
CUDA_R_64F
CUDA_C_16F
CUDA_C_16BF
CUDA_C_32F
CUDA_C_64F

cusparseGather() has the following constraints:

- The arrays representing the sparse vector vecX must be aligned to 16 bytes

cusparseGather() has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the the sparse vector vecX indices are distinct

cusparseGather() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See cusparseStatus_t for the description of the return status

Please visit cuSPARSE Library Samples - cusparseGather for a code example.

14.6.5. cusparseScatter()

```
cusparseStatus_t
cusparseScatter(cusparseHandle_t     handle,
                cusparseSpVecDescr_t vecX, //const descriptor
                cusparseDnVecDescr_t vecY)
```

The function scatters the elements of the sparse vector vecX into the dense vector vecY

In other words,

```
for i=0 to nnz-1
    Y[X_indices[i]] = X_values[i]
```

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`vecX`	HOST	IN	Sparse vector `X`
`vecY`	HOST	OUT	Dense vector `Y`

`cusparseScatter` supports the following index type for representing the sparse vector `vecX`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseScatter` supports the following data types:

`X`/`Y`
`CUDA_R_8I`
`CUDA_R_16F`
`CUDA_R_16BF`
`CUDA_R_32F`
`CUDA_R_64F`
`CUDA_C_16F`
`CUDA_C_16BF`
`CUDA_C_32F`
`CUDA_C_64F`

`cusparseScatter()` has the following constraints:

- The arrays representing the sparse vector `vecX` must be aligned to 16 bytes

`cusparseScatter()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the the sparse vector `vecX` indices are distinct

`cusparseScatter()` supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See `cusparseStatus_t` for the description of the return status

Please visit cuSPARSE Library Samples - cusparseScatter for a code example.

14.6.6. cusparseRot()

```
cusparseStatus_t
cusparseRot(cusparseHandle_t     handle,
            const void*          c_coeff,
            const void*          s_coeff,
            cusparseSpVecDescr_t vecX,
            cusparseDnVecDescr_t vecY)
```

The function computes the Givens rotation matrix

$$G = \begin{bmatrix} c & s \\ -s & c \end{bmatrix}$$

to a sparse `vecX` and a dense vector `vecY`

In other words,

```
for i=0 to nnz-1
    Y[X_indices[i]] = c * Y[X_indices[i]] - s * X_values[i]
    X_values[i]     = c * X_values[i]     + s * Y[X_indices[i]]
```

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`c_coeff`	HOST or DEVICE	IN	cosine element of the rotation matrix
`vecX`	HOST	IN/OUT	Sparse vector `X`
`s_coeff`	HOST or DEVICE	IN	sine element of the rotation matrix
`vecY`	HOST	IN/OUT	Dense vector `Y`

`cusparseRot` supports the following index type for representing the sparse vector `vecX`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseRot` supports the following data types:

Uniform-precision computation:

`X`/`Y`/`compute`
`CUDA_R_32F`
`CUDA_R_64F`
`CUDA_C_32F`
`CUDA_C_64F`

Mixed-precision computation:

X/Y	compute
CUDA_R_16F	CUDA_R_32F
CUDA_R_16BF	
CUDA_C_16F	CUDA_C_32F
CUDA_C_16BF	

cusparseRot() has the following constraints:

- The arrays representing the sparse vector vecX must be aligned to 16 bytes

cusparseRot() has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the the sparse vector vecX indices are distinct

cusparseRot() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See cusparseStatus_t for the description of the return status

Please visit cuSPARSE Library Samples - cusparseRot for a code example.

14.6.7. cusparseSpVV()

```
cusparseStatus_t
cusparseSpVV_bufferSize(cusparseHandle_t     handle,
                        cusparseOperation_t  opX,
                        cusparseSpVecDescr_t vecX, //const descriptor
                        cusparseDnVecDescr_t vecY, //const descriptor
                        void*                result,
                        cudaDataType         computeType,
                        size_t*              bufferSize)
```

```
cusparseStatus_t
cusparseSpVV(cusparseHandle_t     handle,
             cusparseOperation_t  opX,
             cusparseSpVecDescr_t vecX, //const descriptor
             cusparseDnVecDescr_t vecY, //const descriptor
             void*                result,
             cudaDataType         computeType,
             void*                externalBuffer)
```

The function computes the inner dot product of a sparse vector vecX and a dense vector vecY

$$result = \mathbf{X}' \cdot \mathbf{Y}$$

In other words,

```
result = 0;
for i=0 to nnz-1
    result += X_values[i] * Y[X_indices[i]]
```

$$\mathrm{op}(X) = \begin{cases} X & \text{if op(X) == CUSPARSE\_OPERATION\_ NON\_TRANSPOSE} \\ \overline{X} & \text{if op(X) == CUSPARSE\_OPERATION\_ CONJUGATE\_TRANSPOSE} \end{cases}$$

The function `cusparseSpVV_bufferSize()` returns the size of the workspace needed by `cusparseSpVV()`

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`opX`	HOST	IN	Operation `op(X)` that is non-transpose or conjugate transpose
`vecX`	HOST	IN	Sparse vector `X`
`vecY`	HOST	IN	Dense vector `Y`
`result`	HOST or DEVICE	OUT	The resulting dot product
`computeType`	HOST	IN	Datatype in which the computation is executed
`bufferSize`	HOST	OUT	Number of bytes of workspace needed by `cusparseSpVV`
`externalBuffer`	DEVICE	IN	Pointer to a workspace buffer of at least `bufferSize` bytes

`cusparseSpVV` supports the following index type for representing the sparse vector `vecX`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

The data types combinations currently supported for `cusparseSpVV` are listed below:

Uniform-precision computation:

`X/Y/computeType`
`CUDA_R_32F`
`CUDA_R_64F`
`CUDA_C_32F`
`CUDA_C_64F`

Mixed-precision computation:

`X/Y`	`computeType/result`
`CUDA_R_8I`	`CUDA_R_32I`
`CUDA_R_8I`	`CUDA_R_32F`
`CUDA_R_16F`	
`CUDA_R_16BF`	

X/Y	computeType/result
CUDA_C_16F	CUDA_C_32F
CUDA_C_16BF	

cusparseSpVV() has the following constraints:

- The arrays representing the sparse vector vecX must be aligned to 16 bytes

cusparseSpVV() has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the the sparse vector vecX indices are distinct

cusparseSpVV() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See cusparseStatus_t for the description of the return status.

Please visit cuSPARSE Library Samples - cusparseSpVV for a code example.

14.6.8. cusparseSpMV()

```
cusparseStatus_t
cusparseSpMV_bufferSize(cusparseHandle_t     handle,
                        cusparseOperation_t  opA,
                        const void*          alpha,
                        cusparseSpMatDescr_t matA, //const descriptor
                        cusparseDnVecDescr_t vecX, //const descriptor
                        const void*          beta,
                        cusparseDnVecDescr_t vecY,
                        cudaDataType         computeType,
                        cusparseSpMVAlg_t    alg,
                        size_t*              bufferSize)
```

```
cusparseStatus_t
cusparseSpMV(cusparseHandle_t     handle,
             cusparseOperation_t  opA,
             const void*          alpha,
             cusparseSpMatDescr_t matA, //const descriptor
             cusparseDnVecDescr_t vecX, //const descriptor
             const void*          beta,
             cusparseDnVecDescr_t vecY,
             cudaDataType         computeType,
             cusparseSpMVAlg_t    alg,
             void*                externalBuffer)
```

This function performs the multiplication of a sparse matrix `matA` and a dense vector `vecX`

$$\mathbf{Y} = \alpha op(\mathbf{A}) \cdot \mathbf{X} + \beta \mathbf{Y}$$

where

- `op(A)` is a sparse matrix of size $m \times k$
- `X` is a dense vector of size k
- `Y` is a dense vector of size m
- α and β are scalars

Also, for matrix `A`

$$\text{op}(A) = \begin{cases} A & \text{if op(A) == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if op(A) == CUSPARSE\_ OPERATION\_TRANSPOSE} \\ A^H & \text{if op(A) == CUSPARSE\_ OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

The function `cusparseSpMV_bufferSize()` returns the size of the workspace needed by `cusparseSpMV()`

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`opA`	HOST	IN	Operation `op(A)`
`alpha`	HOST or DEVICE	IN	α scalar used for multiplication of type `computeType`
`matA`	HOST	IN	Sparse matrix `A`
`vecX`	HOST	IN	Dense vector `X`
`beta`	HOST or DEVICE	IN	β scalar used for multiplication of type `computeType`
`vecY`	HOST	IN/OUT	Dense vector `Y`
`computeType`	HOST	IN	Datatype in which the computation is executed
`alg`	HOST	IN	Algorithm for the computation
`bufferSize`	HOST	OUT	Number of bytes of workspace needed by `cusparseSpMV`
`externalBuffer`	DEVICE	IN	Pointer to a workspace buffer of at least `bufferSize` bytes

The sparse matrix formats currently supported are listed below:

- `CUSPARSE_FORMAT_COO`
- `CUSPARSE_FORMAT_CSR`
- `CUSPARSE_FORMAT_CSC`

`cusparseSpMV` supports the following index type for representing the sparse matrix `matA`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseSpMV` supports the following data types:

Uniform-precision computation:

A/X/ Y/computeType
CUDA_R_32F
CUDA_R_64F
CUDA_C_32F
CUDA_C_64F

Mixed-precision computation:

<table>
<tr><th>A/X</th><th>Y</th><th>computeType</th></tr>
<tr><td>CUDA_R_8I</td><td>CUDA_R_32I</td><td>CUDA_R_32I</td></tr>
<tr><td>CUDA_R_8I</td><td rowspan="3">CUDA_R_32F</td><td rowspan="5">CUDA_R_32F</td></tr>
<tr><td>CUDA_R_16F</td></tr>
<tr><td>CUDA_R_16BF</td></tr>
<tr><td>CUDA_R_16F</td><td>CUDA_R_16F</td></tr>
<tr><td>CUDA_R_16BF</td><td>CUDA_R_16BF</td></tr>
<tr><td>CUDA_C_32F</td><td>CUDA_C_32F</td><td rowspan="3">CUDA_C_32F</td></tr>
<tr><td>CUDA_C_16F</td><td>CUDA_C_16F</td></tr>
<tr><td>CUDA_C_16BF</td><td>CUDA_C_16BF</td></tr>
</table>

Mixed Regular/Complex computation:

A	X/Y/computeType
CUDA_R_32F	CUDA_C_32F
CUDA_R_64F	CUDA_C_64F

NOTE: `CUDA_R_16F`, `CUDA_R_16BF`, `CUDA_C_16F`, and `CUDA_C_16BF` data types always imply mixed-precision computation.

`cusparseSpMV()` supports the following algorithms:

Algorithm	Notes
CUSPARSE_SPMV_ALG_DEFAULT	Default algorithm for any sparse matrix format
CUSPARSE_SPMV_COO_ALG1	Default algorithm for COO sparse matrix format. May produce slightly different results during different runs with the same input parameters
CUSPARSE_SPMV_COO_ALG2	Provides deterministic (bit-wise) results for each run. If `opA != CUSPARSE_OPERATION_NON_TRANSPOSE`, it is identical to `CUSPARSE_SPMV_COO_ALG1`

Algorithm	Notes
`CUSPARSE_SPMV_CSR_ALG1`	Default algorithm for CSR/CSC sparse matrix format. May produce slightly different results during different runs with the same input parameters
`CUSPARSE_SPMV_CSR_ALG2`	Provides deterministic (bit-wise) results for each run. If `opA != CUSPARSE_OPERATION_NON_TRANSPOSE`, it is identical to `CUSPARSE_SPMV_CSR_ALG1`

Performance notes:

- `CUSPARSE_SPMV_COO_ALG1` and `CUSPARSE_SPMV_CSR_ALG1` provide higher performance than `CUSPARSE_SPMV_COO_ALG2` and `CUSPARSE_SPMV_CSR_ALG2`.
- In general, `opA == CUSPARSE_OPERATION_NON_TRANSPOSE` is 3x faster than `opA != CUSPARSE_OPERATION_NON_TRANSPOSE`.

`cusparseSpMV()` has the following properties:

- The routine requires extra storage for CSR/CSC format (all algorithms) and for COO format with `CUSPARSE_SPMM_COO_ALG2` algorithm.
- Provides deterministic (bit-wise) results for each run only for `CUSPARSE_SPMM_COO_ALG2` and `CUSPARSE_SPMM_CSR_ALG2` algorithms, and `opA == CUSPARSE_OPERATION_NON_TRANSPOSE`.
- The routine supports asynchronous execution.
- compute-sanitizer could report false race conditions for this routine when `beta == 0`. This is for optimization purposes and does not affect the correctness of the computation.

`cusparseSpMV()` supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See `cusparseStatus_t` for the description of the return status.

Please visit cuSPARSE Library Samples - cusparseSpMV CSR and cusparseSpMV COO for a code example.

14.6.9. cusparseSpSV()

```
cusparseStatus_t
cusparseSpSV_createDescr(cusparseSpSVDescr_t* spsvDescr);

cusparseStatus_t
cusparseSpSV_destroyDescr(cusparseSpSVDescr_t spsvDescr);
```

```
cusparseStatus_t
cusparseSpSV_bufferSize(cusparseHandle_t     handle,
                        cusparseOperation_t  opA,
                        const void*          alpha,
```

```
                        cusparseSpMatDescr_t matA, //const descriptor
                        cusparseDnVecDescr_t vecX, //const descriptor
                        cusparseDnVecDescr_t vecY,
                        cudaDataType         computeType,
                        cusparseSpSVAlg_t    alg,
                        cusparseSpSVDescr_t  spsvDescr,
                        size_t*              bufferSize)
```

```
cusparseStatus_t
cusparseSpSV_analysis(cusparseHandle_t     handle,
                      cusparseOperation_t  opA,
                      const void*          alpha,
                      cusparseSpMatDescr_t matA, //const descriptor
                      cusparseDnVecDescr_t vecX, //const descriptor
                      cusparseDnVecDescr_t vecY,
                      cudaDataType         computeType,
                      cusparseSpSVAlg_t    alg,
                      cusparseSpSVDescr_t  spsvDescr
                      void*                externalBuffer)
```

```
cusparseStatus_t
cusparseSpSV_solve(cusparseHandle_t     handle,
                   cusparseOperation_t  opA,
                   const void*          alpha,
                   cusparseSpMatDescr_t matA, //const descriptor
                   cusparseDnVecDescr_t vecX, //const descriptor
                   cusparseDnVecDescr_t vecY,
                   cudaDataType         computeType,
                   cusparseSpSVAlg_t    alg,
                   cusparseSpSVDescr_t  spsvDescr)
```

The function solves a system of linear equations whose coefficients are represented in a sparse triangular matrix:

$$op(\mathbf{A}) \cdot \mathbf{Y} = \alpha\mathbf{X}$$

where

- `op(A)` is a sparse square matrix of size $m \times m$
- `X` is a dense vector of size m
- `Y` is a dense vector of size m
- α is a scalar

Also, for matrix `A`

$$\text{op}(A) = \begin{cases} A & \text{if op(A) == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if op(A) == CUSPARSE\_ OPERATION\_TRANSPOSE} \\ A^H & \text{if op(A) == CUSPARSE\_ OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

The function `cusparseSpSV_bufferSize()` returns the size of the workspace needed by `cusparseSpSV_analysis()` and `cusparseSpSV_solve()`. The function `cusparseSpSV_analysis()` performs the analysis phase, while `cusparseSpSV_solve()`

executes the solve phase for a sparse triangular linear system. The opaque data structure `spsvDescr` is used to share information among all functions.

The routine supports arbitrary sparsity for the input matrix, but only the upper or lower triangular part is taken into account in the computation.

NOTE: all parameters must be consistent across `cusparseSpSV` API calls and the matrix descriptions must not be modified between `cusparseSpSV_analysis()` and `cusparseSpSV_solve()`

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`opA`	HOST	IN	Operation `op(A)`
`alpha`	HOST or DEVICE	IN	α scalar used for multiplication of type `computeType`
`matA`	HOST	IN	Sparse matrix `A`
`vecX`	HOST	IN	Dense vector `X`
`vecY`	HOST	OUT	Dense vector `Y`
`computeType`	HOST	IN	Datatype in which the computation is executed
`alg`	HOST	IN	Algorithm for the computation
`bufferSize`	HOST	OUT	Number of bytes of workspace needed by `cusparseSpSV_analysis()` and `cusparseSpSV_solve()`
`externalBuffer`	DEVICE	IN	Pointer to a workspace buffer of at least `bufferSize` bytes. It is used by `cusparseSpSV_analysis` and `cusparseSpSV_solve()`
`spsvDescr`	HOST	IN/OUT	Opaque descriptor for storing internal data used across the three steps

The sparse matrix formats currently supported are listed below:

- `CUSPARSE_FORMAT_CSR`
- `CUSPARSE_FORMAT_COO`

The `cusparseSpSV()` supports the following shapes and properties:

- `CUSPARSE_FILL_MODE_LOWER` and `CUSPARSE_FILL_MODE_UPPER` fill modes
- `CUSPARSE_DIAG_TYPE_NON_UNIT` and `CUSPARSE_DIAG_TYPE_UNIT` diagonal types

The fill mode and diagonal type can be set by `cusparseSpMatSetAttribute()`

`cusparseSpSV()` supports the following index type for representing the sparse matrix `matA`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseSpSV()` supports the following data types:

Uniform-precision computation:

A/X/ Y/computeType
CUDA_R_32F
CUDA_R_64F
CUDA_C_32F
CUDA_C_64F

cusparseSpSV() supports the following algorithms:

Algorithm	Notes
CUSPARSE_SPSV_ALG_DEFAULT	Default algorithm

cusparseSpSV() has the following properties:

- The routine requires extra storage for the analysis phase which is proportional to number of non-zero entries of the sparse matrix
- Provides deterministic (bit-wise) results for each run for the solving phase cusparseSpSV_solve()
- The routine supports asynchronous execution

cusparseSpSV() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See cusparseStatus_t for the description of the return status

Please visit cuSPARSE Library Samples - cusparseSpSV CSR and cuSPARSE Library Samples - cusparseSpSV COO for code examples.

14.6.10. cusparseSpMM()

```
cusparseStatus_t
cusparseSpMM_bufferSize(cusparseHandle_t     handle,
                        cusparseOperation_t  opA,
                        cusparseOperation_t  opB,
                        const void*          alpha,
                        cusparseSpMatDescr_t matA, //const descriptor
                        cusparseDnMatDescr_t matB, //const descriptor
                        const void*          beta,
                        cusparseDnMatDescr_t matC,
                        cudaDataType         computeType,
                        cusparseSpMMAlg_t    alg,
                        size_t*              bufferSize)
```

```
cusparseStatus_t
cusparseSpMM_preprocess(cusparseHandle_t      handle,
                        cusparseOperation_t   opA,
                        cusparseOperation_t   opB,
```

```
                                const void*             alpha,
                                cusparseSpMatDescr_t  matA, //const descriptor
                                cusparseDnMatDescr_t  matB, //const descriptor
                                const void*             beta,
                                cusparseDnMatDescr_t  matC,
                                cudaDataType            computeType,
                                cusparseSpMMAlg_t     alg,
                                void*                   externalBuffer)
```

```
cusparseStatus_t
cusparseSpMM(cusparseHandle_t     handle,
             cusparseOperation_t  opA,
             cusparseOperation_t  opB,
             const void*          alpha,
             cusparseSpMatDescr_t matA, //const descriptor
             cusparseDnMatDescr_t matB, //const descriptor
             const void*          beta,
             cusparseDnMatDescr_t matC,
             cudaDataType         computeType,
             cusparseSpMMAlg_t    alg,
             void*                externalBuffer)
```

The function performs the multiplication of a sparse matrix `matA` and a dense matrix `matB`

$$\mathbf{C} = \alpha op(\mathbf{A}) \cdot op(\mathbf{B}) + \beta\mathbf{C}$$

where

- `op(A)` is a sparse matrix of size $m \times k$
- `op(B)` is a dense matrix of size $k \times n$
- `C` is a dense matrix of size $m \times n$
- α and β are scalars

The routine can be also used to perform the multiplication of a dense matrix and a sparse matrix by switching the dense matrices layout:

$$\mathbf{C}_C = \mathbf{B}_C \cdot \mathbf{A} + \beta\mathbf{C}_C \longrightarrow$$
$$\mathbf{C}_R = \mathbf{A}^T \cdot \mathbf{B}_R + \beta\mathbf{C}_R$$

where $\mathbf{B}_C$, $\mathbf{C}_C$ indicate column-major layout, while $\mathbf{B}_R$, $\mathbf{C}_R$ refer to row-major layout

Also, for matrix `A` and `B`

$$\text{op}(A) = \begin{cases} A & \text{if op(A) == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if op(A) == CUSPARSE\_ OPERATION\_TRANSPOSE} \\ A^H & \text{if op(A) == CUSPARSE\_ OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

$$\text{op}(B) = \begin{cases} B & \text{if op(B) == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ B^T & \text{if op(B) == CUSPARSE\_ OPERATION\_TRANSPOSE} \\ B^H & \text{if op(B) == CUSPARSE\_ OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

When using the (conjugate) transpose of the sparse matrix `A`, this routine may produce slightly different results during different runs with the same input parameters.

The function `cusparseSpMM_bufferSize()` returns the size of the workspace needed by `cusparseSpMM()`

The function `cusparseSpMM_preprocess()` can be called before `cusparseSpMM` to speedup the actual computation. It is useful when `cusparseSpMM` is called multiple times with the same sparsity pattern (`matA`). The values of the matrices (`matA`, `matB`, `matC`) can change arbitrarily. It provides performance advantages is used with `CUSPARSE_SPMM_CSR_ALG1` or `CUSPARSE_SPMM_CSR_ALG3`. For all other formats and algorithms have no effect.

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`opA`	HOST	IN	Operation `op(A)`
`opB`	HOST	IN	Operation `op(B)`
`alpha`	HOST or DEVICE	IN	α scalar used for multiplication of type `computeType`
`matA`	HOST	IN	Sparse matrix `A`
`matB`	HOST	IN	Dense matrix `B`
`beta`	HOST or DEVICE	IN	β scalar used for multiplication of type `computeType`
`matC`	HOST	IN/OUT	Dense matrix `C`
`computeType`	HOST	IN	Datatype in which the computation is executed
`alg`	HOST	IN	Algorithm for the computation
`bufferSize`	HOST	OUT	Number of bytes of workspace needed by `cusparseSpMM`
`externalBuffer`	DEVICE	IN	Pointer to workspace buffer of at least `bufferSize` bytes

`cusparseSpMM` supports the following sparse matrix formats:

- `CUSPARSE_FORMAT_COO`
- `CUSPARSE_FORMAT_CSR`
- `CUSPARSE_FORMAT_CSC`
- `CUSPARSE_FORMAT_BLOCKED_ELL`

(1)	**COO/CSR/CSC FORMATS**

`cusparseSpMM` supports the following index type for representing the sparse matrix `matA`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseSpMM` supports the following data types:

Uniform-precision computation:

A/B/ C/computeType
CUDA_R_32F
CUDA_R_64F
CUDA_C_32F
CUDA_C_64F

Mixed-precision computation:

A/B	C	computeType
CUDA_R_8I	CUDA_R_32I	CUDA_R_32I
CUDA_R_8I	CUDA_R_32F	CUDA_R_32F
CUDA_R_16F		
CUDA_R_16BF		
CUDA_R_16F	CUDA_R_16F	
CUDA_R_16BF	CUDA_R_16BF	
CUDA_C_16F	CUDA_C_16F	CUDA_C_32F
CUDA_C_16BF	CUDA_C_16BF	

NOTE: `CUDA_R_16F`, `CUDA_R_16BF`, `CUDA_C_16F`, and `CUDA_C_16BF` data types always imply mixed-precision computation.

`cusparseSpMM` supports the following algorithms:

Algorithm	Notes
CUSPARSE_SPMM_ALG_DEFAULT	Default algorithm for any sparse matrix format
CUSPARSE_SPMM_COO_ALG1	Algorithm 1 for COO sparse matrix format ▶ May provide better performance for small number of nnz ▶ Provide the best performance with column-major layout ▶ It supports batched computation ▶ May produce slightly different results during different runs with the same input parameters
CUSPARSE_SPMM_COO_ALG2	Algorithm 2 for COO sparse matrix format ▶ It provides deterministic result ▶ Provide the best performance with column-major layout ▶ In general, slower than Algorithm 1 ▶ It supports batched computation ▶ It requires additional memory

Algorithm	Notes
	▶ If `opA != CUSPARSE_OPERATION_NON_TRANSPOSE`, it is identical to `CUSPARSE_SPMM_COO_ALG1`
`CUSPARSE_SPMM_COO_ALG3`	Algorithm 3 for COO sparse matrix format ▶ May provide better performance for large number of nnz ▶ May produce slightly different results during different runs with the same input parameters
`CUSPARSE_SPMM_COO_ALG4`	Algorithm 4 for COO sparse matrix format ▶ Provide the best performance with row-major layout ▶ It supports batched computation ▶ May produce slightly different results during different runs with the same input parameters
`CUSPARSE_SPMM_CSR_ALG1`	Algorithm 1 for CSR/CSC sparse matrix format ▶ Provide the best performance with column-major layout ▶ It supports batched computation ▶ It requires additional memory ▶ May produce slightly different results during different runs with the same input parameters
`CUSPARSE_SPMM_CSR_ALG2`	Algorithm 2 for CSR/CSC sparse matrix format ▶ Provide the best performance with row-major layout ▶ It supports batched computation ▶ It requires additional memory ▶ May produce slightly different results during different runs with the same input parameters
`CUSPARSE_SPMM_CSR_ALG3`	Algorithm 3 for CSR/CSC sparse matrix format ▶ It provides deterministic result ▶ It requires additional memory ▶ It supports only `opA == CUSPARSE_OPERATION_NON_TRANSPOSE` (fallback to `CUSPARSE_SPMM_CSR_ALG2`) ▶ It does not support `CUDA_C_16F` and `CUDA_C_16BF` data types

Performance notes:

- Row-major layout provides higher performance than column-major

- CUSPARSE_SPMM_COO_ALG4 and CUSPARSE_SPMM_CSR_ALG2 should be used with row-major layout, while CUSPARSE_SPMM_COO_ALG1, CUSPARSE_SPMM_COO_ALG2, CUSPARSE_SPMM_COO_ALG3, and CUSPARSE_SPMM_CSR_ALG1 with column-major layout
- For beta != 1, the output matrix is scaled before the actual computation
- For n == 1, the routine uses cusparseSpMV() as fallback

cusparseSpMM() with all algorithms support the following batch modes except for CUSPARSE_SPMM_CSR_ALG3:

- $C_i = A \cdot B_i$
- $C_i = A_i \cdot B$
- $C_i = A_i \cdot B_i$

The number of batches and their strides can be set by using cusparseCooSetStridedBatch, cusparseCsrSetStridedBatch, and cusparseDnMatSetStridedBatch.

cusparseSpMM() has the following properties:

- The routine requires no extra storage for CUSPARSE_SPMM_COO_ALG1, CUSPARSE_SPMM_COO_ALG3, CUSPARSE_SPMM_COO_ALG4
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run only for CUSPARSE_SPMM_COO_ALG2 and CUSPARSE_SPMM_CSR_ALG3 algorithms
- compute-sanitizer could report false race conditions for this routine. This is for optimization purposes and does not affect the correctness of the computation.

cusparseSpMM() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

Please visit cuSPARSE Library Samples - cusparseSpMM CSR and cusparseSpMM COO for a code example. For batched computation please visit cusparseSpMM CSR Batched and cusparseSpMM COO Batched.

(2) BLOCKED-ELLPACK FORMAT

cusparseSpMM supports the following data types for CUSPARSE_FORMAT_BLOCKED_ELL format and the following GPU architectures for exploiting NVIDIA Tensor Cores:

A/B	C	computeType	opB	Compute Capability
CUDA_R_16F	CUDA_R_16F	CUDA_R_16F	N, T	≥ 70
CUDA_R_16F	CUDA_R_16F	CUDA_R_32F	N, T	≥ 70
CUDA_R_16F	CUDA_R_32F	CUDA_R_32F	N, T	≥ 70
CUDA_R_8I	CUDA_R_8I	CUDA_R_32I	N	≥ 75

A/B	C	computeType	opB	Compute Capability
CUDA_R_16BF	CUDA_R_16BF	CUDA_R_32F	N, T	≥ 80
CUDA_R_16BF	CUDA_R_32F	CUDA_R_32F	N, T	≥ 80
CUDA_R_32F	CUDA_R_32F	CUDA_R_32F	N, T	≥ 80
CUDA_R_64F	CUDA_R_64F	CUDA_R_64F	N, T	≥ 80

`cusparseSpMM` supports the following algorithms with `CUSPARSE_FORMAT_BLOCKED_ELL` format:

Algorithm	Notes
CUSPARSE_SPMM_ALG_DEFAULT	Default algorithm for any sparse matrix format
CUSPARSE_SPMM_BLOCKED_ELL_ALG1	Default algorithm for Blocked-ELL format

Performance notes:

- Blocked-ELL SpMM provides the best performance with Power-of-2 Block-Sizes
- Large Block-Sizes (e.g. ≥ 64) provide the best performance

The function has the following limitations:

- The pointer mode must be equal to `CUSPARSE_POINTER_MODE_HOST`
- Only `opA == CUSPARSE_OPERATION_NON_TRANSPOSE` is supported
- `opB == CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE` is not supported

Please visit cuSPARSE Library Samples - cusparseSpMM Blocked-ELL for a code example.

See `cusparseStatus_t` for the description of the return status

14.6.11. cusparseSpMMOp()

```
cusparseStatus_t CUSPARSEAPI
cusparseSpMMOp_createPlan(cusparseHandle_t      handle,
                          cusparseSpMMOpPlan_t* plan,
                          cusparseOperation_t   opA,
                          cusparseOperation_t   opB,
                          cusparseSpMatDescr_t  matA, //const descriptor
                          cusparseDnMatDescr_t  matB, //const descriptor
                          cusparseDnMatDescr_t  matC,
                          cudaDataType          computeType,
                          cusparseSpMMOpAlg_t   alg,
                          const void*           addOperationNvvmBuffer,
                          size_t                addOperationBufferSize,
                          const void*           mulOperationNvvmBuffer,
                          size_t                mulOperationBufferSize,
                          const void*           epilogueNvvmBuffer,
                          size_t                epilogueBufferSize,
                          size_t*               SpMMWorkspaceSize)
```

```
cusparseStatus_t
cusparseSpMMOp_destroyPlan(cusparseSpMMOpPlan_t plan)
```

```
cusparseStatus_t
cusparseSpMMOp(cusparseSpMMOpPlan_t plan,
               void*                externalBuffer)
```

NOTE 1: The routine requires CUDA driver ≥ 495.XX (CUDA 11.5).

NOTE 2: NVRTC and nvJitLink are not currently available on Arm64 Android platforms.

NOTE 3: The routine does not support Android and Tegra platforms except Judy (sm87).

Experimental: The function performs the multiplication of a sparse matrix `matA` and a dense matrix `matB` with custom operators

$$C'_{ij} = \text{epilogue}\left(\sum_k^{\oplus} op(A_{ik}) \otimes op(B_{kj}), C_{ij}\right)$$

where

- `op(A)` is a sparse matrix of size $m \times k$
- `op(B)` is a dense matrix of size $k \times n$
- `C` is a dense matrix of size $m \times n$
- $\oplus$, $\otimes$, and epilogue are custom **add**, **mul**, and **epilogue** operators respectively

Also, for matrix `A` and `B`

$$\text{op}(A) = \begin{cases} A & \text{if op(A) == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if op(A) == CUSPARSE\_ OPERATION\_TRANSPOSE} \end{cases}$$

$$\text{op}(B) = \begin{cases} B & \text{if op}(B) \text{ == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ B^T & \text{if op}(B) \text{ == CUSPARSE\_ OPERATION\_TRANSPOSE} \end{cases}$$

Only `opA == CUSPARSE_OPERATION_NON_TRANSPOSE` and `opB == CUSPARSE_OPERATION_NON_TRANSPOSE` is currently supported

The function `cusparseSpMMOp_createPlan()` returns the size of the workspace and the compiled kernel needed by `cusparseSpMMOp()`

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`opA`	HOST	IN	Operation `op(A)`
`opB`	HOST	IN	Operation `op(B)`
`matA`	HOST	IN	Sparse matrix `A`
`matB`	HOST	IN	Dense matrix `B`
`matC`	HOST	IN/OUT	Dense matrix `C`
`computeType`	HOST	IN	Datatype in which the computation is executed

Param.	Memory	In/out	Meaning
`alg`	HOST	IN	Algorithm for the computation
`addOperationNvvmBuffer`	HOST	IN	Pointer to the NVVM buffer containing the custom **add** operator
`addOperationBufferSize`	HOST	IN	Size in bytes of `addOperationNvvmBuffer`
`mulOperationNvvmBuffer`	HOST	IN	Pointer to the NVVM buffer containing the custom **mul** operator
`mulOperationBufferSize`	HOST	IN	Size in bytes of `mulOperationNvvmBuffer`
`epilogueNvvmBuffer`	HOST	IN	Pointer to the NVVM buffer containing the custom **epilogue** operator
`epilogueBufferSize`	HOST	IN	Size in bytes of `epilogueNvvmBuffer`
`SpMMWorkspaceSize`	HOST	OUT	Number of bytes of workspace needed by `cusparseSpMMOp`

The operators must have the following signature and return type

```
__device__ <computetype> add_op(<computetype> value1, <computetype> value2);

__device__ <computetype> mul_op(<computetype> value1, <computetype> value2);

__device__ <computetype> epilogue(<computetype> value1, <computetype>
 value2);
```

`<computetype>` is one of `float`, `double`, `cuComplex`, `cuDoubleComplex`, or `int`,

`cusparseSpMMOp` supports the following sparse matrix formats:

- `CUSPARSE_FORMAT_CSR`

`cusparseSpMMOp` supports the following index type for representing the sparse matrix `matA`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseSpMMOp` supports the following data types:

Uniform-precision computation:

A/B/ C/computeType
CUDA_R_32F
CUDA_R_64F
CUDA_C_32F
CUDA_C_64F

Mixed-precision computation:

A/B	C	computeType
CUDA_R_8I	CUDA_R_32I	CUDA_R_32I
CUDA_R_8I	CUDA_R_32F	CUDA_R_32F

A/B	C	computeType
CUDA_R_16F		
CUDA_R_16BF		
CUDA_R_16F	CUDA_R_16F	
CUDA_R_16BF	CUDA_R_16BF	

cusparseSpMMOp supports the following algorithms:

Algorithm	Notes
CUSPARSE_SPMM_OP_ALG_DEFAULT	Default algorithm for any sparse matrix format

Performance notes:

- Row-major layout provides higher performance than column-major.

cusparseSpMMOp() has the following properties:

- The routine requires extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run

cusparseSpMMOp() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

Please visit cuSPARSE Library Samples - cusparseSpMMOp

See cusparseStatus_t for the description of the return status

14.6.12. cusparseSpSM()

```
cusparseStatus_t
cusparseSpSM_createDescr(cusparseSpSMDescr_t* spsmDescr);
cusparseStatus_t
cusparseSpSM_destroyDescr(cusparseSpSMDescr_t spsmDescr);
```

```
cusparseStatus_t
cusparseSpSM_bufferSize(cusparseHandle_t     handle,
                        cusparseOperation_t  opA,
                        cusparseOperation_t  opB,
                        const void*          alpha,
                        cusparseSpMatDescr_t matA, //const descriptor
                        cusparseDnMatDescr_t matB, //const descriptor
                        cusparseDnMatDescr_t matC,
                        cudaDataType         computeType,
```

```
                        cusparseSpSMAlg_t    alg,
                        cusparseSpSMDescr_t  spsmDescr,
                        size_t*              bufferSize)
```

```
cusparseStatus_t
cusparseSpSM_analysis(cusparseHandle_t     handle,
                      cusparseOperation_t  opA,
                      cusparseOperation_t  opB,
                      const void*          alpha,
                      cusparseSpMatDescr_t matA, //const descriptor
                      cusparseDnMatDescr_t matB, //const descriptor
                      cusparseDnMatDescr_t matC,
                      cudaDataType         computeType,
                      cusparseSpSMAlg_t    alg,
                      cusparseSpSMDescr_t  spsmDescr,
                      void*                externalBuffer)
```

```
cusparseStatus_t
cusparseSpSM_solve(cusparseHandle_t     handle,
                   cusparseOperation_t  opA,
                   cusparseOperation_t  opB,
                   const void*          alpha,
                   cusparseSpMatDescr_t matA, //const descriptor
                   cusparseDnMatDescr_t matB, //const descriptor
                   cusparseDnMatDescr_t matC,
                   cudaDataType         computeType,
                   cusparseSpSMAlg_t    alg,
                   cusparseSpSMDescr_t  spsmDescr)
```

The function solves a system of linear equations whose coefficients are represented in a sparse triangular matrix:

$$op(\mathbf{A}) \cdot \mathbf{C} = \boldsymbol{\alpha} op(\mathbf{B})$$

where

- `op(A)` is a sparse square matrix of size $m \times m$
- `op(B)` is a dense matrix of size $m \times n$
- `C` is a dense matrix of size $m \times n$
- α is a scalar

Also, for matrix `A`

$$\mathrm{op}(A) = \begin{cases} A & \text{if op(A) == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if op(A) == CUSPARSE\_ OPERATION\_TRANSPOSE} \\ A^H & \text{if op(A) == CUSPARSE\_ OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

$$\mathrm{op}(B) = \begin{cases} B & \text{if op(B) == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ B^T & \text{if op(B) == CUSPARSE\_ OPERATION\_TRANSPOSE} \end{cases}$$

The function `cusparseSpSM_bufferSize()` returns the size of the workspace needed by `cusparseSpSM_analysis()` and `cusparseSpSM_solve()`. The function `cusparseSpSM_analysis()` performs the analysis phase, while `cusparseSpSM_solve()` executes the solve phase for a sparse triangular linear system. The opaque data structure `spsmDescr` is used to share information among all functions.

The routine supports arbitrary sparsity for the input matrix, but only the upper or lower triangular part is taken into account in the computation.

`cusparseSpSM_bufferSize()` requires a buffer size for the analysis phase which is proportional to number of non-zero entries of the sparse matrix

The `externalBuffer` is stored into `spsmDescr` and used by `cusparseSpSM_solve()`. For this reason, the device memory buffer must be deallocated only after `cusparseSpSM_solve()`

NOTE: all parameters must be consistent across `cusparseSpSM` API calls and the matrix descriptions must not be modified between `cusparseSpSM_analysis()` and `cusparseSpSM_solve()`

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`opA`	HOST	IN	Operation `op(A)`
`opB`	HOST	IN	Operation `op(B)`
`alpha`	HOST or DEVICE	IN	α scalar used for multiplication of type `computeType`
`matA`	HOST	IN	Sparse matrix `A`
`matB`	HOST	IN	Dense matrix `B`
`matC`	HOST	IN/OUT	Dense matrix `C`
`computeType`	HOST	IN	Datatype in which the computation is executed
`alg`	HOST	IN	Algorithm for the computation
`bufferSize`	HOST	OUT	Number of bytes of workspace needed by `cusparseSpSM_analysis()` and `cusparseSpSM_solve()`
`externalBuffer`	DEVICE	IN	Pointer to a workspace buffer of at least `bufferSize` bytes. It is used by `cusparseSpSM_analysis` and `cusparseSpSM_solve()`
`spsmDescr`	HOST	IN/OUT	Opaque descriptor for storing internal data used across the three steps

The sparse matrix formats currently supported are listed below:

- `CUSPARSE_FORMAT_CSR`
- `CUSPARSE_FORMAT_COO`

The `cusparseSpSM()` supports the following shapes and properties:

- `CUSPARSE_FILL_MODE_LOWER` and `CUSPARSE_FILL_MODE_UPPER` fill modes
- `CUSPARSE_DIAG_TYPE_NON_UNIT` and `CUSPARSE_DIAG_TYPE_UNIT` diagonal types

The fill mode and diagonal type can be set by `cusparseSpMatSetAttribute()`

`cusparseSpSM()` supports the following index type for representing the sparse matrix `matA`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseSpSM()` supports the following data types:

Uniform-precision computation:

A/B/ C/computeType
CUDA_R_32F
CUDA_R_64F
CUDA_C_32F
CUDA_C_64F

`cusparseSpSM()` supports the following algorithms:

Algorithm	Notes
CUSPARSE_SPSM_ALG_DEFAULT	Default algorithm

`cusparseSpSM()` has the following properties:

- The routine requires no extra storage
- Provides deterministic (bit-wise) results for each run for the solving phase `cusparseSpSM_solve()`
- The routine supports asynchronous execution

`cusparseSpSM()` supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See `cusparseStatus_t` for the description of the return status

Please visit cuSPARSE Library Samples - cusparseSpSM CSR and cuSPARSE Library Samples - cusparseSpSM COO for code examples.

14.6.13. cusparseSDDMM()

```
cusparseStatus_t
cusparseSDDMM_bufferSize(cusparseHandle_t     handle,
                         cusparseOperation_t  opA,
                         cusparseOperation_t  opB,
                         const void*          alpha,
                         cusparseDnMatDescr_t matA, //const descriptor
                         cusparseDnMatDescr_t matB, //const descriptor
                         const void*          beta,
```

```
                          cusparseSpMatDescr_t matC,
                          cudaDataType         computeType,
                          cusparseSDDMMAlg_t   alg,
                          size_t*              bufferSize)
```

```
cusparseStatus_t
cusparseSDDMM_preprocess(cusparseHandle_t     handle,
                          cusparseOperation_t  opA,
                          cusparseOperation_t  opB,
                          const void*          alpha,
                          cusparseDnMatDescr_t matA, //const descriptor
                          cusparseDnMatDescr_t matB, //const descriptor
                          const void*          beta,
                          cusparseSpMatDescr_t matC,
                          cudaDataType         computeType,
                          cusparseSDDMMAlg_t   alg,
                          void*                externalBuffer)
```

```
cusparseStatus_t
cusparseSDDMM(cusparseHandle_t     handle,
              cusparseOperation_t  opA,
              cusparseOperation_t  opB,
              const void*          alpha,
              cusparseDnMatDescr_t matA, //const descriptor
              cusparseDnMatDescr_t matB, //const descriptor
              const void*          beta,
              cusparseSpMatDescr_t matC,
              cudaDataType         computeType,
              cusparseSDDMMAlg_t   alg,
              void*                externalBuffer)
```

This function performs the multiplication of `matA` and `matB`, followed by an element-wise multiplication with the sparsity pattern of `matC`. Formally, it performs the following operation:

$$\mathbf{C} = \alpha(\mathrm{op}(\mathbf{A}) \cdot \mathrm{op}(\mathbf{B})) \circ \mathrm{spy}(\mathbf{C}) + \beta\mathbf{C}$$

where

- `op(A)` is a dense matrix of size $m \times k$
- `op(B)` is a dense matrix of size $k \times n$
- `C` is a sparse matrix of size $m \times n$
- α and β are scalars
- $\circ$ denotes the Hadamard (entry-wise) matrix product, and $\mathrm{spy}(\mathbf{C})$ is the sparsity pattern matrix of `C` defined as:

  $$\mathrm{spy}(\mathbf{C})_{ij} = \begin{cases} 0 & \text{if } \mathbf{C}_{ij} = 0 \\ 1 & \text{otherwise} \end{cases}$$

Also, for matrix `A` and `B`

$$\mathrm{op}(A) = \begin{cases} A & \text{if op(A) == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if op(A) == CUSPARSE\_ OPERATION\_TRANSPOSE} \\ A^H & \text{if op(A) == CUSPARSE\_ OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

$$\mathrm{op}(B) = \begin{cases} B & \text{if op(B) == CUSPARSE\_ OPERATION\_NON\_TRANSPOSE} \\ B^T & \text{if op(B) == CUSPARSE\_ OPERATION\_TRANSPOSE} \\ B^H & \text{if op(B) == CUSPARSE\_ OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

`CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE` is currently not supported.

The function `cusparseSDDMM_bufferSize()` returns the size of the workspace needed by `cusparseSDDMM` or `cusparseSDDMM_preprocess`.

The function `cusparseSDDMM_preprocess()` can be called before `cusparseSDDMM` to speedup the actual computation. It is useful when `cusparseSDDMM` is called multiple times with the same sparsity pattern (`matC`). The values of the dense matrices (`matA`, `matB`) can change arbitrarily.

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`opA`	HOST	IN	Operation `op(A)`
`opB`	HOST	IN	Operation `op(B)`
`alpha`	HOST or DEVICE	IN	α scalar used for multiplication of type `computeType`
`matA`	HOST	IN	Dense matrix `matA`
`matB`	HOST	IN	Dense matrix `matB`
`beta`	HOST or DEVICE	IN	β scalar used for multiplication of type `computeType`
`matC`	HOST	IN/OUT	Sparse matrix `matC`
`computeType`	HOST	IN	Datatype in which the computation is executed
`alg`	HOST	IN	Algorithm for the computation
`bufferSize`	HOST	OUT	Number of bytes of workspace needed by `cusparseSDDMM`
`externalBuffer`	DEVICE	IN	Pointer to a workspace buffer of at least `bufferSize` bytes

Currently supported sparse matrix formats:

- `CUSPARSE_FORMAT_CSR`

`cusparseSDDMM()` supports the following index type for representing the sparse matrix `matA`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

The data types combinations currently supported for `cusparseSDDMM` are listed below :

Uniform-precision computation:

A/X/ Y/computeType
CUDA_R_32F
CUDA_R_64F
CUDA_C_32F
CUDA_C_64F

cusparseSDDMM() supports the following algorithms:

Algorithm	Notes
CUSPARSE_SDDMM_ALG_DEFAULT	Default algorithm. It supports batched computation.

Performance notes: cusparseSDDMM() provides the best performance when matA and matB satisfy:

- matA is in row-major order and opA is CUSPARSE_OPERATION_NON_TRANSPOSE, or is in col-major order and opA is not CUSPARSE_OPERATION_NON_TRANSPOSE
- matB is in col-major order and opB is CUSPARSE_OPERATION_NON_TRANSPOSE, or is in row-major order and opB is not CUSPARSE_OPERATION_NON_TRANSPOSE

cusparseSDDMM() supports the following batch modes:

- $C_i = (A \cdot B) \circ C_i$
- $C_i = (A_i \cdot B) \circ C_i$
- $C_i = (A \cdot B_i) \circ C_i$
- $C_i = (A_i \cdot B_i) \circ C_i$

The number of batches and their strides can be set by using cusparseCsrSetStridedBatch and cusparseDnMatSetStridedBatch.

cusparseSDDMM() has the following properties:

- The routine requires no extra storage
- Provides deterministic (bit-wise) results for each run
- The routine supports asynchronous execution

cusparseSDDMM() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See cusparseStatus_t for the description of the return status

Please visit cuSPARSE Library Samples - cusparseSDDMM for a code example. For batched computation please visit cusparseSDDMM CSR Batched.

14.6.14. cusparseSpGEMM()

```
cusparseStatus_t
cusparseSpGEMM_createDescr(cusparseSpGEMMDescr_t* descr)

cusparseStatus_t
cusparseSpGEMM_destroyDescr(cusparseSpGEMMDescr_t descr)
```

```
cusparseStatus_t
cusparseSpGEMM_workEstimation(cusparseHandle_t      handle,
                              cusparseOperation_t   opA,
                              cusparseOperation_t   opB,
                              const void*           alpha,
                              cusparseSpMatDescr_t  matA, //const descriptor
                              cusparseSpMatDescr_t  matB, //const descriptor
                              const void*           beta,
                              cusparseSpMatDescr_t  matC,
                              cudaDataType          computeType,
                              cusparseSpGEMMAlg_t   alg,
                              cusparseSpGEMMDescr_t spgemmDescr,
                              size_t*               bufferSize1,
                              void*                 externalBuffer1)

cusparseStatus_t
cusparseSpGEMM_getNumProducts(cusparseSpGEMMDescr_t spgemmDescr,
                              int64_t*              num_prods)

cusparseStatus_t
cusparseSpGEMM_estimateMemory(cusparseHandle_t          handle,
                              cusparseOperation_t       opA,
                              cusparseOperation_t       opB,
                              const void*               alpha,
                              cusparseConstSpMatDescr_t matA, //const
 descriptor
                              cusparseConstSpMatDescr_t matB, //const
 descriptor
                              const void*               beta,
                              cusparseSpMatDescr_t      matC,
                              cudaDataType              computeType,
                              cusparseSpGEMMAlg_t       alg,
                              cusparseSpGEMMDescr_t     spgemmDescr,
                              float                     chunk_fraction,
                              size_t*                   bufferSize3,
                              void*                     externalBuffer3,
                              size_t*                   bufferSize2)

cusparseStatus_t
cusparseSpGEMM_compute(cusparseHandle_t      handle,
                       cusparseOperation_t   opA,
                       cusparseOperation_t   opB,
                       const void*           alpha,
                       cusparseSpMatDescr_t  matA, //const descriptor
                       cusparseSpMatDescr_t  matB, //const descriptor
                       const void*           beta,
                       cusparseSpMatDescr_t  matC,
                       cudaDataType          computeType,
                       cusparseSpGEMMAlg_t   alg,
```

```
                        cusparseSpGEMMDescr_t spgemmDescr,
                        size_t*               bufferSize2,
                        void*                 externalBuffer2)

cusparseStatus_t
cusparseSpGEMM_copy(cusparseHandle_t      handle,
                    cusparseOperation_t   opA,
                    cusparseOperation_t   opB,
                    const void*           alpha,
                    cusparseSpMatDescr_t  matA, //const descriptor
                    cusparseSpMatDescr_t  matB, //const descriptor
                    const void*           beta,
                    cusparseSpMatDescr_t  matC,
                    cudaDataType          computeType,
                    cusparseSpGEMMAlg_t   alg,
                    cusparseSpGEMMDescr_t spgemmDescr)
```

This function performs the multiplication of two sparse matrices `matA` and `matB`

$$\mathbf{C'} = \alpha op(\mathbf{A}) \cdot op(\mathbf{B}) + \beta \mathbf{C}$$

where α, β are scalars, and $\mathbf{C}, \mathbf{C'}$ have the same sparsity pattern.

The functions `cusparseSpGEMM_workEstimation()`, `cusparseSpGEMM_estimateMemory()` and `cusparseSpGEMM_compute()` are used for both determining the buffer size and performing the actual computation

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`opA`	HOST	IN	Operation `op(A)`
`opB`	HOST	IN	Operation `op(B)`
`alpha`	HOST or DEVICE	IN	α scalar used for multiplication
`matA`	HOST	IN	Sparse matrix `A`
`matB`	HOST	IN	Sparse matrix `B`
`beta`	HOST or DEVICE	IN	β scalar used for multiplication
`matC`	HOST	IN/OUT	Sparse matrix `C`
`computeType`	HOST	IN	Enumerator specifying the datatype in which the computation is executed
`alg`	HOST	IN	Enumerator specifying the algorithm for the computation
`spgemmDescr`	HOST	IN/OUT	Opaque descriptor for storing internal data used across the three steps
`num_prods`	HOST	OUT	Pointer to a 64-bit integer that stores the number of intermediate products returned by `cusparseSpGEMM_estimateMemory`
`chunk_fraction`	HOST	IN	The fraction of total intermediate products being computed in a chunk. Used by `CUSPARSE_SPGEMM_ALG3` only. Value is in range [0,1].

Param.	Memory	In/out	Meaning
bufferSize1	HOST	IN/OUT	Number of bytes of workspace requested by cusparseSpGEMM_workEstimation
bufferSize2	HOST	IN/OUT	Number of bytes of workspace requested by cusparseSpGEMM_compute
bufferSize3	HOST	IN/OUT	Number of bytes of workspace requested by cusparseSpGEMM_estimateMemory
externalBuffer1	DEVICE	IN	Pointer to workspace buffer needed by cusparseSpGEMM_workEstimation and cusparseSpGEMM_compute
externalBuffer2	DEVICE	IN	Pointer to workspace buffer needed by cusparseSpGEMM_compute and cusparseSpGEMM_copy
externalBuffer3	DEVICE	IN	Pointer to workspace buffer needed by cusparseSpGEMM_estimateMemory

Currently, the function has the following limitations:

- Only 32-bit indices `CUSPARSE_INDEX_32I` is supported
- Only CSR format `CUSPARSE_FORMAT_CSR` is supported
- Only `opA`, `opB` equal to `CUSPARSE_OPERATION_NON_TRANSPOSE` are supported

The data types combinations currently supported for `cusparseSpGEMM` are listed below :

Uniform-precision computation:

A/B/ C/computeType
CUDA_R_16F
CUDA_R_16BF
CUDA_R_32F
CUDA_R_64F
CUDA_C_16F
CUDA_C_16BF
CUDA_C_32F
CUDA_C_64F

`cusparseSpGEMM` routine runs for the following algorithm:

Algorithm	Notes
CUSPARSE_SPGEMM_DEFAULT	Default algorithm. Currently, it is CUSPARSE_SPGEMM_ALG1.
CUSPARSE_SPGEMM_ALG1	Algorithm 1 ▶ Invokes cusparseSpGEMM_compute twice. The first invocation provides an *upper bound* of the memory required for the computation. ▶ The required memory is generally several times larger of the actual memory used.

Algorithm	Notes
	▶ The user can provide an arbitrary buffer size `bufferSize2` in the second invocation. If it is not sufficient, the routine will returns `CUSPARSE_STATUS_INSUFFICIENT_RESOURCES` status. ▶ Provides better performance than other algorithms ▶ Provides deterministic (bit-wise) results for each run
`CUSPARSE_SPGEMM_ALG2`	Algorithm 2 ▶ Invokes `cusparseSpGEMM_estimateMemory` to get the amount of the memory required for the computation. ▶ Requires less memory for the computation than Algorithm 1 ▶ Performance is lower than Algorithm 1, higher than Algorithm 3 ▶ Provides deterministic (bit-wise) results for each run
`CUSPARSE_SPGEMM_ALG3`	Algorithm 3 ▶ Computes the intermediate products in chunks, one chunk at a time ▶ Invokes `cusparseSpGEMM_estimateMemory` to get the amount of the memory required for the computation. ▶ The user can control the amount of required memory by changing the chunk size via `chunk_fraction` ▶ The chunk size is a fraction of total intermediate products: `chunk_fraction * (*num_prods)` ▶ Provides deterministic (bit-wise) results for each run

`cusparseSpGEMM()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports does **not** support CUDA graph capture

`cusparseSpGEMM()` supports the following optimizations:

- Hardware Memory Compression

See `cusparseStatus_t` for the description of the return status

Please visit cuSPARSE Library Samples - cusparseSpGEMM for a code example.

14.6.15. cusparseSpGEMMreuse()

```
cusparseStatus_t
```

```
cusparseSpGEMM_createDescr(cusparseSpGEMMDescr_t* descr)

cusparseStatus_t
cusparseSpGEMM_destroyDescr(cusparseSpGEMMDescr_t descr)
```

```
cusparseStatus_t
cusparseSpGEMMreuse_workEstimation(cusparseHandle_t      handle,
                                   cusparseOperation_t   opA,
                                   cusparseOperation_t   opB,
                                   cusparseSpMatDescr_t  matA, //const
 descriptor
                                   cusparseSpMatDescr_t  matB, //const
 descriptor
                                   cusparseSpMatDescr_t  matC,
                                   cusparseSpGEMMAlg_t   alg,
                                   cusparseSpGEMMDescr_t spgemmDescr,
                                   size_t*               bufferSize1,
                                   void*                 externalBuffer1)

cusparseStatus_t
cusparseSpGEMMreuse_nnz(cusparseHandle_t      handle,
                        cusparseOperation_t   opA,
                        cusparseOperation_t   opB,
                        cusparseSpMatDescr_t  matA, //const descriptor
                        cusparseSpMatDescr_t  matB, //const descriptor
                        cusparseSpMatDescr_t  matC,
                        cusparseSpGEMMAlg_t   alg,
                        cusparseSpGEMMDescr_t spgemmDescr,
                        size_t*               bufferSize2,
                        void*                 externalBuffer2,
                        size_t*               bufferSize3,
                        void*                 externalBuffer3,
                        size_t*               bufferSize4,
                        void*                 externalBuffer4)

cusparseStatus_t CUSPARSEAPI
cusparseSpGEMMreuse_copy(cusparseHandle_t      handle,
                         cusparseOperation_t   opA,
                         cusparseOperation_t   opB,
                         cusparseSpMatDescr_t  matA, //const descriptor
                         cusparseSpMatDescr_t  matB, //const descriptor
                         cusparseSpMatDescr_t  matC,
                         cusparseSpGEMMAlg_t   alg,
                         cusparseSpGEMMDescr_t spgemmDescr,
                         size_t*               bufferSize5,
                         void*                 externalBuffer5)

cusparseStatus_t CUSPARSEAPI
cusparseSpGEMMreuse_compute(cusparseHandle_t      handle,
                            cusparseOperation_t   opA,
                            cusparseOperation_t   opB,
                            const void*           alpha,
                            cusparseSpMatDescr_t  matA, //const descriptor
                            cusparseSpMatDescr_t  matB, //const descriptor
                            const void*           beta,
                            cusparseSpMatDescr_t  matC,
                            cudaDataType          computeType,
                            cusparseSpGEMMAlg_t   alg,
                            cusparseSpGEMMDescr_t spgemmDescr)
```

This function performs the multiplication of two sparse matrices `matA` and `matB` where the structure of the output matrix `matC` can be reused for multiple computations with different values.

$$\mathbf{C}' = \alpha op(\mathbf{A}) \cdot op(\mathbf{B}) + \beta \mathbf{C}$$

where α and β are scalars.

The functions `cusparseSpGEMMreuse_workEstimation()`, `cusparseSpGEMMreuse_nnz()`, and `cusparseSpGEMMreuse_copy()` are used for determining the buffer size and performing the actual computation.

Note: `cusparseSpGEMMreuse()` output CSR matrix (`matC`) is sorted by column indices.

Param.	Memory	In/out	Meaning
`handle`	HOST	IN	Handle to the cuSPARSE library context
`opA`	HOST	IN	Operation `op(A)`
`opB`	HOST	IN	Operation `op(B)`
`alpha`	HOST or DEVICE	IN	α scalar used for multiplication
`matA`	HOST	IN	Sparse matrix `A`
`matB`	HOST	IN	Sparse matrix `B`
`beta`	HOST or DEVICE	IN	β scalar used for multiplication
`matC`	HOST	IN/OUT	Sparse matrix `C`
`computeType`	HOST	IN	Enumerator specifying the datatype in which the computation is executed
`alg`	HOST	IN	Enumerator specifying the algorithm for the computation
`spgemmDescr`	HOST	IN/OUT	Opaque descriptor for storing internal data used across the three steps
`bufferSize1`	HOST	IN/OUT	Number of bytes of workspace requested by `cusparseSpGEMMreuse_workEstimation`

`bufferSize2`
`bufferSize3`
`bufferSize4`
	HOST	IN/OUT	Number of bytes of workspace requested by `cusparseSpGEMMreuse_nnz`

`bufferSize5`	HOST	IN/OUT	Number of bytes of workspace requested by `cusparseSpGEMMreuse_copy`
`externalBuffer1`	DEVICE	IN	Pointer to workspace buffer needed by `cusparseSpGEMMreuse_workEstimation` and `cusparseSpGEMMreuse_nnz`
`externalBuffer2`	DEVICE	IN	Pointer to workspace buffer needed by `cusparseSpGEMMreuse_nnz`
`externalBuffer3`	DEVICE	IN	Pointer to workspace buffer needed by `cusparseSpGEMMreuse_nnz` and `cusparseSpGEMMreuse_copy`

Param.	Memory	In/out	Meaning
externalBuffer4	DEVICE	IN	Pointer to workspace buffer needed by cusparseSpGEMMreuse_nnz and cusparseSpGEMMreuse_compute
externalBuffer5	DEVICE	IN	Pointer to workspace buffer needed by cusparseSpGEMMreuse_copy and cusparseSpGEMMreuse_compute

MEMORY REQUIREMENT: cusparseSpGEMMreuse requires to keep in memory all intermediate products to reuse the structure of the output matrix. On the other hand, the number of intermediate products is orders of magnitude higher than the number of non-zero entries in general. In order to minimize the memory requirements, the routine uses multiple buffers that can be deallocated after they are no more needed. If the number of intermediate product exceeds 2^31-1, the routine will returns CUSPARSE_STATUS_INSUFFICIENT_RESOURCES status.

Currently, the function has the following limitations:

- Only 32-bit indices CUSPARSE_INDEX_32I is supported
- Only CSR format CUSPARSE_FORMAT_CSR is supported
- Only opA, opB equal to CUSPARSE_OPERATION_NON_TRANSPOSE are supported

The data types combinations currently supported for cusparseSpGEMMreuse are listed below.

Uniform-precision computation:

A/B/ C/computeType
CUDA_R_32F
CUDA_R_64F
CUDA_C_16F
CUDA_C_16BF
CUDA_C_32F
CUDA_C_64F

Mixed-precision computation:

A/B	C	computeType
CUDA_R_16F	CUDA_R_16F	CUDA_R_32F
CUDA_R_16BF	CUDA_R_16BF	CUDA_R_32F

cusparseSpGEMMreuse routine runs for the following algorithm:

Algorithm	Notes
CUSPARSE_SPGEMM_DEFAULT CUSPARSE_SPGEMM_CSR_ALG_NONDETERMINITIC	Default algorithm. Provides deterministic (bit-wise) structure for the output matrix for each run, while value computation is not deterministic
CUSPARSE_SPGEMM_CSR_ALG_DETERMINITIC	Provides deterministic (bit-wise) structure for the output matrix and value computation for each run

`cusparseSpGEMMreuse()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine does **not** support CUDA graph capture

`cusparseSpGEMMreuse()` supports the following optimizations:

- Hardware Memory Compression

See `cusparseStatus_t` for the description of the return status.

Please visit cuSPARSE Library Samples - cusparseSpGEMMreuse for a code example.

Chapter 15. Appendix A: cuSPARSE Fortran Bindings

The cuSPARSE library is implemented using the C-based CUDA toolchain, and it thus provides a C-style API that makes interfacing to applications written in C or C++ trivial. There are also many applications implemented in Fortran that would benefit from using cuSPARSE, and therefore a cuSPARSE Fortran interface has been developed.

Unfortunately, Fortran-to-C calling conventions are not standardized and differ by platform and toolchain. In particular, differences may exist in the following areas:

Symbol names (capitalization, name decoration)

Argument passing (by value or reference)

Passing of pointer arguments (size of the pointer)

To provide maximum flexibility in addressing those differences, the cuSPARSE Fortran interface is provided in the form of wrapper functions, which are written in C and are located in the file `cusparse_fortran.c`. This file also contains a few additional wrapper functions (for `cudaMalloc()`, `cudaMemset`, and so on) that can be used to allocate memory on the GPU.

The cuSPARSE Fortran wrapper code is provided as an example only and needs to be compiled into an application for it to call the cuSPARSE API functions. Providing this source code allows users to make any changes necessary for a particular platform and toolchain.

The cuSPARSE Fortran wrapper code has been used to demonstrate interoperability with the compilers g95 0.91 (on 32-bit and 64-bit Linux) and g95 0.92 (on 32-bit and 64-bit Mac OS X). In order to use other compilers, users have to make any changes to the wrapper code that may be required.

The direct wrappers, intended for production code, substitute device pointers for vector and matrix arguments in all cuSPARSE functions. To use these interfaces, existing applications need to be modified slightly to allocate and deallocate data structures in GPU memory space (using `CUDA_MALLOC()` and `CUDA_FREE()`) and to copy data between GPU and CPU memory spaces (using the `CUDA_MEMCPY()` routines). The sample wrappers provided in `cusparse_fortran.c` map device pointers to the OS-dependent type `size_t`, which is 32 bits wide on 32-bit platforms and 64 bits wide on a 64-bit platforms.

One approach to dealing with index arithmetic on device pointers in Fortran code is to use C-style macros and to use the C preprocessor to expand them. On Linux and Mac OS X, preprocessing can be done by using the option `'-cpp'` with g95 or gfortran. The function

`GET_SHIFTED_ADDRESS()`, provided with the cuSPARSE Fortran wrappers, can also be used, as shown in example B.

Example B shows the the C++ of example A implemented in Fortran 77 on the host. This example should be compiled with `ARCH_64` defined as 1 on a 64-bit OS system and as undefined on a 32-bit OS system. For example, on g95 or gfortran, it can be done directly on the command line using the option `-cpp -DARCH_64=1`.

15.1. Fortran Application

```
c     #define ARCH_64 0
c     #define ARCH_64 1

      program cusparse_fortran_example
      implicit none
      integer cuda_malloc
      external cuda_free
      integer cuda_memcpy_c2fort_int
      integer cuda_memcpy_c2fort_real
      integer cuda_memcpy_fort2c_int
      integer cuda_memcpy_fort2c_real
      integer cuda_memset
      integer cusparse_create
      external cusparse_destroy
      integer cusparse_get_version
      integer cusparse_create_mat_descr
      external cusparse_destroy_mat_descr
      integer cusparse_set_mat_type
      integer cusparse_get_mat_type
      integer cusparse_get_mat_fill_mode
      integer cusparse_get_mat_diag_type
      integer cusparse_set_mat_index_base
      integer cusparse_get_mat_index_base
      integer cusparse_xcoo2csr
      integer cusparse_dsctr
      integer cusparse_dcsrmv
      integer cusparse_dcsrmm
      external get_shifted_address
#if ARCH_64
      integer*8 handle
      integer*8 descrA
      integer*8 cooRowIndex
      integer*8 cooColIndex
      integer*8 cooVal
      integer*8 xInd
      integer*8 xVal
      integer*8 y
      integer*8 z
      integer*8 csrRowPtr
      integer*8 ynp1
#else
      integer*4 handle
      integer*4 descrA
      integer*4 cooRowIndex
      integer*4 cooColIndex
      integer*4 cooVal
      integer*4 xInd
      integer*4 xVal
      integer*4 y
      integer*4 z
      integer*4 csrRowPtr
      integer*4 ynp1
```

```
#endif
      integer status
      integer cudaStat1,cudaStat2,cudaStat3
      integer cudaStat4,cudaStat5,cudaStat6
      integer n, nnz, nnz_vector
      parameter (n=4, nnz=9, nnz_vector=3)
      integer cooRowIndexHostPtr(nnz)
      integer cooColIndexHostPtr(nnz)
      real*8  cooValHostPtr(nnz)
      integer xIndHostPtr(nnz_vector)
      real*8  xValHostPtr(nnz_vector)
      real*8  yHostPtr(2*n)
      real*8  zHostPtr(2*(n+1))
      integer i, j
      integer version, mtype, fmode, dtype, ibase
      real*8  dzero,dtwo,dthree,dfive
      real*8  epsilon


      write(*,*) "testing fortran example"

c     predefined constants (need to be careful with them)
      dzero = 0.0
      dtwo  = 2.0
      dthree= 3.0
      dfive = 5.0
c     create the following sparse test matrix in COO format
c     (notice one-based indexing)
c     |1.0     2.0 3.0|
c     |    4.0        |
c     |5.0     6.0 7.0|
c     |    8.0     9.0|
      cooRowIndexHostPtr(1)=1
      cooColIndexHostPtr(1)=1
      cooValHostPtr(1)     =1.0
      cooRowIndexHostPtr(2)=1
      cooColIndexHostPtr(2)=3
      cooValHostPtr(2)     =2.0
      cooRowIndexHostPtr(3)=1
      cooColIndexHostPtr(3)=4
      cooValHostPtr(3)     =3.0
      cooRowIndexHostPtr(4)=2
      cooColIndexHostPtr(4)=2
      cooValHostPtr(4)     =4.0
      cooRowIndexHostPtr(5)=3
      cooColIndexHostPtr(5)=1
      cooValHostPtr(5)     =5.0
      cooRowIndexHostPtr(6)=3
      cooColIndexHostPtr(6)=3
      cooValHostPtr(6)     =6.0
      cooRowIndexHostPtr(7)=3
      cooColIndexHostPtr(7)=4
      cooValHostPtr(7)     =7.0
      cooRowIndexHostPtr(8)=4
      cooColIndexHostPtr(8)=2
      cooValHostPtr(8)     =8.0
      cooRowIndexHostPtr(9)=4
      cooColIndexHostPtr(9)=4
      cooValHostPtr(9)     =9.0
c     print the matrix
      write(*,*) "Input data:"
      do i=1,nnz
         write(*,*) "cooRowIndexHostPtr[",i,"]=",cooRowIndexHostPtr(i)
         write(*,*) "cooColIndexHostPtr[",i,"]=",cooColIndexHostPtr(i)
         write(*,*) "cooValHostPtr[",     i,"]=",cooValHostPtr(i)
      enddo
```

```
c     create a sparse and dense vector
c     xVal= [100.0 200.0 400.0]   (sparse)
c     xInd= [0     1     3    ]
c     y   = [10.0 20.0 30.0 40.0 | 50.0 60.0 70.0 80.0] (dense)
c     (notice one-based indexing)
      yHostPtr(1) = 10.0
      yHostPtr(2) = 20.0
      yHostPtr(3) = 30.0
      yHostPtr(4) = 40.0
      yHostPtr(5) = 50.0
      yHostPtr(6) = 60.0
      yHostPtr(7) = 70.0
      yHostPtr(8) = 80.0
      xIndHostPtr(1)=1
      xValHostPtr(1)=100.0
      xIndHostPtr(2)=2
      xValHostPtr(2)=200.0
      xIndHostPtr(3)=4
      xValHostPtr(3)=400.0
c     print the vectors
      do j=1,2
         do i=1,n
            write(*,*) "yHostPtr[",i,",",j,"]=",yHostPtr(i+n*(j-1))
         enddo
      enddo
      do i=1,nnz_vector
         write(*,*) "xIndHostPtr[",i,"]=",xIndHostPtr(i)
         write(*,*) "xValHostPtr[",i,"]=",xValHostPtr(i)
      enddo

c     allocate GPU memory and copy the matrix and vectors into it
c     cudaSuccess=0
c     cudaMemcpyHostToDevice=1
      cudaStat1 = cuda_malloc(cooRowIndex,nnz*4)
      cudaStat2 = cuda_malloc(cooColIndex,nnz*4)
      cudaStat3 = cuda_malloc(cooVal,     nnz*8)
      cudaStat4 = cuda_malloc(y,          2*n*8)
      cudaStat5 = cuda_malloc(xInd,nnz_vector*4)
      cudaStat6 = cuda_malloc(xVal,nnz_vector*8)
      if ((cudaStat1 /= 0) .OR.
     $    (cudaStat2 /= 0) .OR.
     $    (cudaStat3 /= 0) .OR.
     $    (cudaStat4 /= 0) .OR.
     $    (cudaStat5 /= 0) .OR.
     $    (cudaStat6 /= 0)) then
         write(*,*) "Device malloc failed"
         write(*,*) "cudaStat1=",cudaStat1
         write(*,*) "cudaStat2=",cudaStat2
         write(*,*) "cudaStat3=",cudaStat3
         write(*,*) "cudaStat4=",cudaStat4
         write(*,*) "cudaStat5=",cudaStat5
         write(*,*) "cudaStat6=",cudaStat6
         stop 2
      endif
      cudaStat1 = cuda_memcpy_fort2c_int(cooRowIndex,cooRowIndexHostPtr,
     $                                   nnz*4,1)
      cudaStat2 = cuda_memcpy_fort2c_int(cooColIndex,cooColIndexHostPtr,
     $                                   nnz*4,1)
      cudaStat3 = cuda_memcpy_fort2c_real(cooVal,    cooValHostPtr,
     $                                    nnz*8,1)
      cudaStat4 = cuda_memcpy_fort2c_real(y,      yHostPtr,
     $                                    2*n*8,1)
      cudaStat5 = cuda_memcpy_fort2c_int(xInd,       xIndHostPtr,
     $                                   nnz_vector*4,1)
      cudaStat6 = cuda_memcpy_fort2c_real(xVal,      xValHostPtr,
     $                                    nnz_vector*8,1)
      if ((cudaStat1 /= 0) .OR.
```

```
     $     (cudaStat2 /= 0) .OR.
     $     (cudaStat3 /= 0) .OR.
     $     (cudaStat4 /= 0) .OR.
     $     (cudaStat5 /= 0) .OR.
     $     (cudaStat6 /= 0)) then
           write(*,*) "Memcpy from Host to Device failed"
           write(*,*) "cudaStat1=",cudaStat1
           write(*,*) "cudaStat2=",cudaStat2
           write(*,*) "cudaStat3=",cudaStat3
           write(*,*) "cudaStat4=",cudaStat4
           write(*,*) "cudaStat5=",cudaStat5
           write(*,*) "cudaStat6=",cudaStat6
           call cuda_free(cooRowIndex)
           call cuda_free(cooColIndex)
           call cuda_free(cooVal)
           call cuda_free(xInd)
           call cuda_free(xVal)
           call cuda_free(y)
           stop 1
      endif

c     initialize cusparse library
c     CUSPARSE_STATUS_SUCCESS=0
      status = cusparse_create(handle)
      if (status /= 0) then
         write(*,*) "CUSPARSE Library initialization failed"
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         stop 1
      endif
c     get version
c     CUSPARSE_STATUS_SUCCESS=0
      status = cusparse_get_version(handle,version)
      if (status /= 0) then
         write(*,*) "CUSPARSE Library initialization failed"
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cusparse_destroy(handle)
         stop 1
      endif
      write(*,*) "CUSPARSE Library version",version

c     create and setup the matrix descriptor
c     CUSPARSE_STATUS_SUCCESS=0
c     CUSPARSE_MATRIX_TYPE_GENERAL=0
c     CUSPARSE_INDEX_BASE_ONE=1
      status= cusparse_create_mat_descr(descrA)
      if (status /= 0) then
         write(*,*) "Creating matrix descriptor failed"
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cusparse_destroy(handle)
         stop 1
      endif
      status = cusparse_set_mat_type(descrA,0)
```

```
      status = cusparse_set_mat_index_base(descrA,1)
c     print the matrix descriptor
      mtype = cusparse_get_mat_type(descrA)
      fmode = cusparse_get_mat_fill_mode(descrA)
      dtype = cusparse_get_mat_diag_type(descrA)
      ibase = cusparse_get_mat_index_base(descrA)
      write (*,*) "matrix descriptor:"
      write (*,*) "t=",mtype,"m=",fmode,"d=",dtype,"b=",ibase

c     exercise conversion routines (convert matrix from COO 2 CSR format)
c     cudaSuccess=0
c     CUSPARSE_STATUS_SUCCESS=0
c     CUSPARSE_INDEX_BASE_ONE=1
      cudaStat1 = cuda_malloc(csrRowPtr,(n+1)*4)
      if (cudaStat1 /= 0) then
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cusparse_destroy_mat_descr(descrA)
         call cusparse_destroy(handle)
         write(*,*) "Device malloc failed (csrRowPtr)"
         stop 2
      endif
      status= cusparse_xcoo2csr(handle,cooRowIndex,nnz,n,
     $                          csrRowPtr,1)
      if (status /= 0) then
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cuda_free(csrRowPtr)
         call cusparse_destroy_mat_descr(descrA)
         call cusparse_destroy(handle)
         write(*,*) "Conversion from COO to CSR format failed"
         stop 1
      endif
c     csrRowPtr = [0 3 4 7 9]

c     exercise Level 1 routines (scatter vector elements)
c     CUSPARSE_STATUS_SUCCESS=0
c     CUSPARSE_INDEX_BASE_ONE=1
      call get_shifted_address(y,n*8,ynp1)
      status= cusparse_dsctr(handle, nnz_vector, xVal, xInd,
     $                       ynp1, 1)
      if (status /= 0) then
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cuda_free(csrRowPtr)
         call cusparse_destroy_mat_descr(descrA)
         call cusparse_destroy(handle)
         write(*,*) "Scatter from sparse to dense vector failed"
         stop 1
      endif
c     y = [10 20 30 40 | 100 200 70 400]

c     exercise Level 2 routines (csrmv)
c     CUSPARSE_STATUS_SUCCESS=0
c     CUSPARSE_OPERATION_NON_TRANSPOSE=0
```

```
      status= cusparse_dcsrmv(handle, 0, n, n, nnz, dtwo,
     $                        descrA, cooVal, csrRowPtr, cooColIndex,
     $                        y, dthree, ynp1)
      if (status /= 0) then
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cuda_free(csrRowPtr)
         call cusparse_destroy_mat_descr(descrA)
         call cusparse_destroy(handle)
         write(*,*) "Matrix-vector multiplication failed"
         stop 1
      endif

c     print intermediate results (y)
c     y = [10 20 30 40 | 680 760 1230 2240]
c     cudaSuccess=0
c     cudaMemcpyDeviceToHost=2
      cudaStat1 = cuda_memcpy_c2fort_real(yHostPtr, y, 2*n*8, 2)
      if (cudaStat1 /= 0) then
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cuda_free(csrRowPtr)
         call cusparse_destroy_mat_descr(descrA)
         call cusparse_destroy(handle)
         write(*,*) "Memcpy from Device to Host failed"
         stop 1
      endif
      write(*,*) "Intermediate results:"
      do j=1,2
         do i=1,n
             write(*,*) "yHostPtr[",i,",",j,"]=",yHostPtr(i+n*(j-1))
         enddo
      enddo

c     exercise Level 3 routines (csrmm)
c     cudaSuccess=0
c     CUSPARSE_STATUS_SUCCESS=0
c     CUSPARSE_OPERATION_NON_TRANSPOSE=0
      cudaStat1 = cuda_malloc(z, 2*(n+1)*8)
      if (cudaStat1 /= 0) then
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cuda_free(csrRowPtr)
         call cusparse_destroy_mat_descr(descrA)
         call cusparse_destroy(handle)
         write(*,*) "Device malloc failed (z)"
         stop 2
      endif
      cudaStat1 = cuda_memset(z, 0, 2*(n+1)*8)
      if (cudaStat1 /= 0) then
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
```

```
           call cuda_free(y)
           call cuda_free(z)
           call cuda_free(csrRowPtr)
           call cusparse_destroy_mat_descr(descrA)
           call cusparse_destroy(handle)
           write(*,*) "Memset on Device failed"
           stop 1
        endif
        status= cusparse_dcsrmm(handle, 0, n, 2, n, nnz, dfive,
      $                         descrA, cooVal, csrRowPtr, cooColIndex,
      $                         y, n, dzero, z, n+1)
        if (status /= 0) then
           call cuda_free(cooRowIndex)
           call cuda_free(cooColIndex)
           call cuda_free(cooVal)
           call cuda_free(xInd)
           call cuda_free(xVal)
           call cuda_free(y)
           call cuda_free(z)
           call cuda_free(csrRowPtr)
           call cusparse_destroy_mat_descr(descrA)
           call cusparse_destroy(handle)
           write(*,*) "Matrix-matrix multiplication failed"
           stop 1
        endif

c       print final results (z)
c       cudaSuccess=0
c       cudaMemcpyDeviceToHost=2
        cudaStat1 = cuda_memcpy_c2fort_real(zHostPtr, z, 2*(n+1)*8, 2)
        if (cudaStat1 /= 0) then
           call cuda_free(cooRowIndex)
           call cuda_free(cooColIndex)
           call cuda_free(cooVal)
           call cuda_free(xInd)
           call cuda_free(xVal)
           call cuda_free(y)
           call cuda_free(z)
           call cuda_free(csrRowPtr)
           call cusparse_destroy_mat_descr(descrA)
           call cusparse_destroy(handle)
           write(*,*) "Memcpy from Device to Host failed"
           stop 1
        endif
c       z = [950 400 2550 2600 0 | 49300 15200 132300 131200 0]
        write(*,*) "Final results:"
        do j=1,2
           do i=1,n+1
              write(*,*) "z[",i,",",j,"]=",zHostPtr(i+(n+1)*(j-1))
           enddo
        enddo

c       check the results
        epsilon = 0.00000000000001
        if ((DABS(zHostPtr(1) - 950.0)   .GT. epsilon)  .OR.
      $     (DABS(zHostPtr(2) - 400.0)   .GT. epsilon)  .OR.
      $     (DABS(zHostPtr(3) - 2550.0)  .GT. epsilon)  .OR.
      $     (DABS(zHostPtr(4) - 2600.0)  .GT. epsilon)  .OR.
      $     (DABS(zHostPtr(5) - 0.0)     .GT. epsilon)  .OR.
      $     (DABS(zHostPtr(6) - 49300.0) .GT. epsilon)  .OR.
      $     (DABS(zHostPtr(7) - 15200.0) .GT. epsilon)  .OR.
      $     (DABS(zHostPtr(8) - 132300.0).GT. epsilon)  .OR.
      $     (DABS(zHostPtr(9) - 131200.0).GT. epsilon)  .OR.
      $     (DABS(zHostPtr(10) - 0.0)    .GT. epsilon)  .OR.
      $     (DABS(yHostPtr(1) - 10.0)    .GT. epsilon)  .OR.
      $     (DABS(yHostPtr(2) - 20.0)    .GT. epsilon)  .OR.
      $     (DABS(yHostPtr(3) - 30.0)    .GT. epsilon)  .OR.
```

```
     $    (DABS(yHostPtr(4) - 40.0)    .GT. epsilon)  .OR.
     $    (DABS(yHostPtr(5) - 680.0)   .GT. epsilon)  .OR.
     $    (DABS(yHostPtr(6) - 760.0)   .GT. epsilon)  .OR.
     $    (DABS(yHostPtr(7) - 1230.0)  .GT. epsilon)  .OR.
     $    (DABS(yHostPtr(8) - 2240.0)  .GT. epsilon)) then
         write(*,*) "fortran example test FAILED"
      else
         write(*,*) "fortran example test PASSED"
      endif

c     deallocate GPU memory and exit
      call cuda_free(cooRowIndex)
      call cuda_free(cooColIndex)
      call cuda_free(cooVal)
      call cuda_free(xInd)
      call cuda_free(xVal)
      call cuda_free(y)
      call cuda_free(z)
      call cuda_free(csrRowPtr)
      call cusparse_destroy_mat_descr(descrA)
      call cusparse_destroy(handle)

      stop 0
      end
```

Chapter 16. Appendix B: Examples of prune

16.1. Prune Dense to Sparse

This section provides a simple example in the C programming language of pruning a dense matrix to a sparse matrix of CSR format.

A is a 4x4 dense matrix,

$$A = \begin{pmatrix} 1.0 & 0.0 & 2.0 & -3.0 \\ 0.0 & 4.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 6.0 & 7.0 \\ 0.0 & 8.0 & 0.0 & 9.0 \end{pmatrix}$$

```
/*
 * How to compile (assume cuda is installed at /usr/local/cuda/)
 *   nvcc -c -I/usr/local/cuda/include prunedense_example.cpp
 *   g++ -o prunedense_example.cpp prunedense_example.o -L/usr/local/cuda/lib64 -
lcusparse -lcudart
 */
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <cuda_runtime.h>
#include <cusparse.h>

void printMatrix(int m, int n, const float*A, int lda, const char* name)
{
    for(int row = 0 ; row < m ; row++){
        for(int col = 0 ; col < n ; col++){
            float Areg = A[row + col*lda];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

void printCsr(
    int m,
    int n,
    int nnz,
    const cusparseMatDescr_t descrA,
    const float *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
```

```
    const char* name)
{
    const int base = (cusparseGetMatIndexBase(descrA) != CUSPARSE_INDEX_BASE_ONE)?
 0:1 ;

    printf("matrix %s is %d-by-%d, nnz=%d, base=%d\n", name, m, n, nnz, base);
    for(int row = 0 ; row < m ; row++){
        const int start = csrRowPtrA[row  ] - base;
        const int end   = csrRowPtrA[row+1] - base;
        for(int colidx = start ; colidx < end ; colidx++){
            const int col = csrColIndA[colidx] - base;
            const float Areg = csrValA[colidx];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrC = NULL;
```

```
    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    cudaError_t cudaStat2 = cudaSuccess;
    cudaError_t cudaStat3 = cudaSuccess;
    cudaError_t cudaStat4 = cudaSuccess;
    cudaError_t cudaStat5 = cudaSuccess;
    const int m = 4;
    const int n = 4;
    const int lda = m;
/*
 *       |    1     0     2     -3   |
 *       |    0     4     0     0    |
 *   A = |    5     0     6     7    |
 *       |    0     8     0     9    |
 *
 */
    const float A[lda*n] = {1, 0, 5, 0, 0, 4, 0, 8, 2, 0, 6, 0, -3, 0, 7, 9};
    int* csrRowPtrC = NULL;
    int* csrColIndC = NULL;
    float* csrValC  = NULL;

    float *d_A = NULL;
    int *d_csrRowPtrC = NULL;
    int *d_csrColIndC = NULL;
    float *d_csrValC = NULL;

    size_t lworkInBytes = 0;
    char *d_work = NULL;

    int nnzC = 0;

    float threshold = 4.1; /* remove Aij <= 4.1 */
//    float threshold = 0; /* remove zeros */

    printf("example of pruneDense2csr \n");

    printf("prune |A(i,j)| <= threshold \n");
    printf("threshold = %E \n", threshold);

    printMatrix(m, n, A, lda, "A");
```

```
/* step 1: create cusparse handle, bind a stream */
    cudaStat1 = cudaStreamCreateWithFlags(&stream, cudaStreamNonBlocking);
    assert(cudaSuccess == cudaStat1);

    status = cusparseCreate(&handle);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    status = cusparseSetStream(handle, stream);
    assert(CUSPARSE_STATUS_SUCCESS == status);
```

```
/* step 2: configuration of matrix C */
    status = cusparseCreateMatDescr(&descrC);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    cusparseSetMatIndexBase(descrC,CUSPARSE_INDEX_BASE_ZERO);
    cusparseSetMatType(descrC, CUSPARSE_MATRIX_TYPE_GENERAL );

    cudaStat1 = cudaMalloc ((void**)&d_A         , sizeof(float)*lda*n );
    cudaStat2 = cudaMalloc ((void**)&d_csrRowPtrC, sizeof(int)*(m+1) );
    assert(cudaSuccess == cudaStat1);
    assert(cudaSuccess == cudaStat2);

/* step 3: query workspace */
    cudaStat1 = cudaMemcpy(d_A, A, sizeof(float)*lda*n, cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);

    status = cusparseSpruneDense2csr_bufferSizeExt(
        handle,
        m,
        n,
        d_A,
        lda,
        &threshold,
        descrC,
        d_csrValC,
        d_csrRowPtrC,
        d_csrColIndC,
        &lworkInBytes);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    printf("lworkInBytes (prune) = %lld \n", (long long)lworkInBytes);

    if (NULL != d_work) { cudaFree(d_work); }
    cudaStat1 = cudaMalloc((void**)&d_work, lworkInBytes);
    assert(cudaSuccess == cudaStat1);

/* step 4: compute csrRowPtrC and nnzC */
    status = cusparseSpruneDense2csrNnz(
        handle,
        m,
        n,
        d_A,
        lda,
        &threshold,
        descrC,
        d_csrRowPtrC,
        &nnzC, /* host */
        d_work);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    cudaStat1 = cudaDeviceSynchronize();
    assert(cudaSuccess == cudaStat1);
```

```
    printf("nnzC = %d\n", nnzC);
    if (0 == nnzC ){
        printf("C is empty \n");
        return 0;
    }
```

```
/* step 5: compute csrColIndC and csrValC */
    cudaStat1 = cudaMalloc ((void**)&d_csrColIndC, sizeof(int  ) * nnzC );
    cudaStat2 = cudaMalloc ((void**)&d_csrValC   , sizeof(float) * nnzC );
    assert(cudaSuccess == cudaStat1);
    assert(cudaSuccess == cudaStat2);

    status = cusparseSpruneDense2csr(
        handle,
        m,
        n,
        d_A,
        lda,
        &threshold,
        descrC,
        d_csrValC,
        d_csrRowPtrC,
        d_csrColIndC,
        d_work);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    cudaStat1 = cudaDeviceSynchronize();
    assert(cudaSuccess == cudaStat1);

/* step 6: output C */
    csrRowPtrC = (int*  )malloc(sizeof(int  )*(m+1));
    csrColIndC = (int*  )malloc(sizeof(int  )*nnzC);
    csrValC    = (float*)malloc(sizeof(float)*nnzC);
    assert( NULL != csrRowPtrC);
    assert( NULL != csrColIndC);
    assert( NULL != csrValC);

    cudaStat1 = cudaMemcpy(csrRowPtrC, d_csrRowPtrC, sizeof(int  )*(m+1),
 cudaMemcpyDeviceToHost);
    cudaStat2 = cudaMemcpy(csrColIndC, d_csrColIndC, sizeof(int  )*nnzC ,
 cudaMemcpyDeviceToHost);
    cudaStat3 = cudaMemcpy(csrValC   , d_csrValC   , sizeof(float)*nnzC ,
 cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    assert(cudaSuccess == cudaStat2);
    assert(cudaSuccess == cudaStat3);

    printCsr(m, n, nnzC, descrC, csrValC, csrRowPtrC, csrColIndC, "C");

/* free resources */
    if (d_A            ) cudaFree(d_A);
    if (d_csrRowPtrC   ) cudaFree(d_csrRowPtrC);
    if (d_csrColIndC   ) cudaFree(d_csrColIndC);
    if (d_csrValC      ) cudaFree(d_csrValC);

    if (csrRowPtrC  ) free(csrRowPtrC);
    if (csrColIndC  ) free(csrColIndC);
    if (csrValC     ) free(csrValC);

    if (handle         ) cusparseDestroy(handle);
    if (stream         ) cudaStreamDestroy(stream);
    if (descrC         ) cusparseDestroyMatDescr(descrC);

    cudaDeviceReset();
    return 0;
}
```

16.2. Prune Sparse to Sparse

This section provides a simple example in the C programming language of pruning a sparse matrix to a sparse matrix of CSR format.

A is a 4x4 sparse matrix,

$$A = \begin{pmatrix} 1.0 & 0.0 & 2.0 & -3.0 \\ 0.0 & 4.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 6.0 & 7.0 \\ 0.0 & 8.0 & 0.0 & 9.0 \end{pmatrix}$$

```
/*
 * How to compile (assume cuda is installed at /usr/local/cuda/)
 *   nvcc -c -I/usr/local/cuda/include prunecsr_example.cpp
 *   g++ -o prunecsr_example.cpp prunecsr_example.o -L/usr/local/cuda/lib64 -
lcusparse -lcudart
 */
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <cuda_runtime.h>
#include <cusparse.h>

void printCsr(
    int m,
    int n,
    int nnz,
    const cusparseMatDescr_t descrA,
    const float *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    const char* name)
{
    const int base = (cusparseGetMatIndexBase(descrA) != CUSPARSE_INDEX_BASE_ONE)?
 0:1 ;

    printf("matrix %s is %d-by-%d, nnz=%d, base=%d, output base-1\n", name, m, n,
 nnz, base);
    for(int row = 0 ; row < m ; row++){
        const int start = csrRowPtrA[row  ] - base;
        const int end   = csrRowPtrA[row+1] - base;
        for(int colidx = start ; colidx < end ; colidx++){
            const int col = csrColIndA[colidx] - base;
            const float Areg = csrValA[colidx];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrA = NULL;
    cusparseMatDescr_t descrC = NULL;

    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    const int m = 4;
```

```
    const int n = 4;
    const int nnzA = 9;
/*
 *      |    1     0     2     -3  |
 *      |    0     4     0     0   |
 *  A = |    5     0     6     7   |
 *      |    0     8     0     9   |
 *
 */
```

```
    const int csrRowPtrA[m+1] = { 1, 4, 5, 8, 10};
    const int csrColIndA[nnzA] = { 1, 3, 4, 2, 1, 3, 4, 2, 4};
    const float csrValA[nnzA] = {1, 2, -3, 4, 5, 6, 7, 8, 9};

    int* csrRowPtrC = NULL;
    int* csrColIndC = NULL;
    float* csrValC  = NULL;

    int *d_csrRowPtrA = NULL;
    int *d_csrColIndA = NULL;
    float *d_csrValA = NULL;

    int *d_csrRowPtrC = NULL;
    int *d_csrColIndC = NULL;
    float *d_csrValC = NULL;

    size_t lworkInBytes = 0;
    char *d_work = NULL;

    int nnzC = 0;

    float threshold = 4.1; /* remove Aij <= 4.1 */
//    float threshold = 0; /* remove zeros */

    printf("example of pruneCsr2csr \n");

    printf("prune |A(i,j)| <= threshold \n");
    printf("threshold = %E \n", threshold);

/* step 1: create cusparse handle, bind a stream */
    cudaStat1 = cudaStreamCreateWithFlags(&stream, cudaStreamNonBlocking);
    assert(cudaSuccess == cudaStat1);

    status = cusparseCreate(&handle);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    status = cusparseSetStream(handle, stream);
    assert(CUSPARSE_STATUS_SUCCESS == status);

/* step 2: configuration of matrix A and C */
    status = cusparseCreateMatDescr(&descrA);
    assert(CUSPARSE_STATUS_SUCCESS == status);
/* A is base-1*/
    cusparseSetMatIndexBase(descrA,CUSPARSE_INDEX_BASE_ONE);
    cusparseSetMatType(descrA, CUSPARSE_MATRIX_TYPE_GENERAL );

    status = cusparseCreateMatDescr(&descrC);
    assert(CUSPARSE_STATUS_SUCCESS == status);
/* C is base-0 */
    cusparseSetMatIndexBase(descrC,CUSPARSE_INDEX_BASE_ZERO);
    cusparseSetMatType(descrC, CUSPARSE_MATRIX_TYPE_GENERAL );

    printCsr(m, n, nnzA, descrA, csrValA, csrRowPtrA, csrColIndA, "A");
```

```
    cudaStat1 = cudaMalloc ((void**)&d_csrRowPtrA, sizeof(int)*(m+1) );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrColIndA, sizeof(int)*nnzA );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrValA   , sizeof(float)*nnzA );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrRowPtrC, sizeof(int)*(m+1) );
    assert(cudaSuccess == cudaStat1);

    cudaStat1 = cudaMemcpy(d_csrRowPtrA, csrRowPtrA, sizeof(int)*(m+1),
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(d_csrColIndA, csrColIndA, sizeof(int)*nnzA,
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(d_csrValA   , csrValA   , sizeof(float)*nnzA,
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);

/* step 3: query workspace */
    status = cusparseSpruneCsr2csr_bufferSizeExt(
        handle,
        m,
        n,
        nnzA,
        descrA,
        d_csrValA,
        d_csrRowPtrA,
        d_csrColIndA,
        &threshold,
        descrC,
        d_csrValC,
        d_csrRowPtrC,
        d_csrColIndC,
        &lworkInBytes);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    printf("lworkInBytes (prune) = %lld \n", (long long)lworkInBytes);

    if (NULL != d_work) { cudaFree(d_work); }
    cudaStat1 = cudaMalloc((void**)&d_work, lworkInBytes);
    assert(cudaSuccess == cudaStat1);

/* step 4: compute csrRowPtrC and nnzC */
    status = cusparseSpruneCsr2csrNnz(
        handle,
        m,
        n,
        nnzA,
        descrA,
        d_csrValA,
        d_csrRowPtrA,
        d_csrColIndA,
        &threshold,
        descrC,
        d_csrRowPtrC,
        &nnzC, /* host */
        d_work);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    cudaStat1 = cudaDeviceSynchronize();
    assert(cudaSuccess == cudaStat1);
```

```
    printf("nnzC = %d\n", nnzC);
    if (0 == nnzC ){
        printf("C is empty \n");
        return 0;
    }
/* step 5: compute csrColIndC and csrValC */
    cudaStat1 = cudaMalloc ((void**)&d_csrColIndC, sizeof(int  ) * nnzC );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrValC   , sizeof(float) * nnzC );
    assert(cudaSuccess == cudaStat1);

    status = cusparseSpruneCsr2csr(
        handle,
        m,
        n,
        nnzA,
        descrA,
        d_csrValA,
        d_csrRowPtrA,
        d_csrColIndA,
        &threshold,
        descrC,
        d_csrValC,
        d_csrRowPtrC,
        d_csrColIndC,
        d_work);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    cudaStat1 = cudaDeviceSynchronize();
    assert(cudaSuccess == cudaStat1);

/* step 6: output C */
    csrRowPtrC = (int*  )malloc(sizeof(int  )*(m+1));
    csrColIndC = (int*  )malloc(sizeof(int  )*nnzC);
    csrValC    = (float*)malloc(sizeof(float)*nnzC);
    assert( NULL != csrRowPtrC);
    assert( NULL != csrColIndC);
    assert( NULL != csrValC);
    cudaStat1 = cudaMemcpy(csrRowPtrC, d_csrRowPtrC, sizeof(int  )*(m+1),
 cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(csrColIndC, d_csrColIndC, sizeof(int  )*nnzC ,
 cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(csrValC   , d_csrValC   , sizeof(float)*nnzC ,
 cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    printCsr(m, n, nnzC, descrC, csrValC, csrRowPtrC, csrColIndC, "C");
/* free resources */
    if (d_csrRowPtrA  ) cudaFree(d_csrRowPtrA);
    if (d_csrColIndA  ) cudaFree(d_csrColIndA);
    if (d_csrValA     ) cudaFree(d_csrValA);
    if (d_csrRowPtrC  ) cudaFree(d_csrRowPtrC);
    if (d_csrColIndC  ) cudaFree(d_csrColIndC);
    if (d_csrValC     ) cudaFree(d_csrValC);
    if (csrRowPtrC  ) free(csrRowPtrC);
    if (csrColIndC  ) free(csrColIndC);
    if (csrValC     ) free(csrValC);
    if (handle        ) cusparseDestroy(handle);
    if (stream        ) cudaStreamDestroy(stream);
    if (descrA        ) cusparseDestroyMatDescr(descrA);
    if (descrC        ) cusparseDestroyMatDescr(descrC);
    cudaDeviceReset();
    return 0;
}
```

16.3. Prune Dense to Sparse by Percentage

This section provides a simple example in the C programming language of pruning a dense matrix to a sparse matrix by percentage.

A is a 4x4 dense matrix,

$$A = \begin{pmatrix} 1.0 & 0.0 & 2.0 & -3.0 \\ 0.0 & 4.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 6.0 & 7.0 \\ 0.0 & 8.0 & 0.0 & 9.0 \end{pmatrix}$$

The percentage is 50, which means to prune 50 percent of the dense matrix. The matrix has 16 elements, so 8 out of 16 must be pruned out. Therefore 7 zeros are pruned out, and value 1.0 is also out because it is the smallest among 9 nonzero elements.

```
/*
 * How to compile (assume cuda is installed at /usr/local/cuda/)
 *   nvcc -c -I/usr/local/cuda/include prunedense2csrbyP.cpp
 *   g++ -o prunedense2csrbyP.cpp prunedense2csrbyP.o -L/usr/local/cuda/lib64 -
lcusparse -lcudart
 */
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <cuda_runtime.h>
#include <cusparse.h>

void printMatrix(int m, int n, const float*A, int lda, const char* name)
{
    for(int row = 0 ; row < m ; row++){
        for(int col = 0 ; col < n ; col++){
            float Areg = A[row + col*lda];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

void printCsr(
    int m,
    int n,
    int nnz,
    const cusparseMatDescr_t descrA,
    const float *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    const char* name)
{
    const int base = (cusparseGetMatIndexBase(descrA) != CUSPARSE_INDEX_BASE_ONE)?
 0:1 ;

    printf("matrix %s is %d-by-%d, nnz=%d, base=%d, output base-1\n", name, m, n,
 nnz, base);
    for(int row = 0 ; row < m ; row++){
        const int start = csrRowPtrA[row  ] - base;
        const int end   = csrRowPtrA[row+1] - base;
        for(int colidx = start ; colidx < end ; colidx++){
```

```
            const int col = csrColIndA[colidx] - base;
            const float Areg = csrValA[colidx];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrC = NULL;
    pruneInfo_t info = NULL;

    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    cudaError_t cudaStat2 = cudaSuccess;
    cudaError_t cudaStat3 = cudaSuccess;
    cudaError_t cudaStat4 = cudaSuccess;
    cudaError_t cudaStat5 = cudaSuccess;
    const int m = 4;
    const int n = 4;
    const int lda = m;
```

```
/*
 *       |    1     0     2     -3   |
 *       |    0     4     0     0    |
 *   A = |    5     0     6     7    |
 *       |    0     8     0     9    |
 *
 */
    const float A[lda*n] = {1, 0, 5, 0, 0, 4, 0, 8, 2, 0, 6, 0, -3, 0, 7, 9};
    int* csrRowPtrC = NULL;
    int* csrColIndC = NULL;
    float* csrValC  = NULL;

    float *d_A = NULL;
    int *d_csrRowPtrC = NULL;
    int *d_csrColIndC = NULL;
    float *d_csrValC = NULL;

    size_t lworkInBytes = 0;
    char *d_work = NULL;

    int nnzC = 0;

    float percentage = 50; /* 50% of nnz */

    printf("example of pruneDense2csrByPercentage \n");

    printf("prune out %.1f percentage of A \n", percentage);

    printMatrix(m, n, A, lda, "A");

/* step 1: create cusparse handle, bind a stream */
    cudaStat1 = cudaStreamCreateWithFlags(&stream, cudaStreamNonBlocking);
    assert(cudaSuccess == cudaStat1);

    status = cusparseCreate(&handle);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    status = cusparseSetStream(handle, stream);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    status = cusparseCreatePruneInfo(&info);
```

```
    assert(CUSPARSE_STATUS_SUCCESS == status);

/* step 2: configuration of matrix C */
    status = cusparseCreateMatDescr(&descrC);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    cusparseSetMatIndexBase(descrC,CUSPARSE_INDEX_BASE_ZERO);
    cusparseSetMatType(descrC, CUSPARSE_MATRIX_TYPE_GENERAL );

    cudaStat1 = cudaMalloc ((void**)&d_A         , sizeof(float)*lda*n );
    cudaStat2 = cudaMalloc ((void**)&d_csrRowPtrC, sizeof(int)*(m+1) );
    assert(cudaSuccess == cudaStat1);
    assert(cudaSuccess == cudaStat2);

    cudaStat1 = cudaMemcpy(d_A, A, sizeof(float)*lda*n, cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
```

```
/* step 3: query workspace */
    status = cusparseSpruneDense2csrByPercentage_bufferSizeExt(
        handle,
        m,
        n,
        d_A,
        lda,
        percentage,
        descrC,
        d_csrValC,
        d_csrRowPtrC,
        d_csrColIndC,
        info,
        &lworkInBytes);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    printf("lworkInBytes = %lld \n", (long long)lworkInBytes);

    if (NULL != d_work) { cudaFree(d_work); }
    cudaStat1 = cudaMalloc((void**)&d_work, lworkInBytes);
    assert(cudaSuccess == cudaStat1);

/* step 4: compute csrRowPtrC and nnzC */
    status = cusparseSpruneDense2csrNnzByPercentage(
        handle,
        m,
        n,
        d_A,
        lda,
        percentage,
        descrC,
        d_csrRowPtrC,
        &nnzC, /* host */
        info,
        d_work);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    cudaStat1 = cudaDeviceSynchronize();
    assert(cudaSuccess == cudaStat1);

    printf("nnzC = %d\n", nnzC);
    if (0 == nnzC ){
        printf("C is empty \n");
        return 0;
    }

/* step 5: compute csrColIndC and csrValC */
    cudaStat1 = cudaMalloc ((void**)&d_csrColIndC, sizeof(int  ) * nnzC );
    cudaStat2 = cudaMalloc ((void**)&d_csrValC   , sizeof(float) * nnzC );
```

```
    assert(cudaSuccess == cudaStat1);
    assert(cudaSuccess == cudaStat2);
```

```
    status = cusparseSpruneDense2csrByPercentage(
        handle,
        m,
        n,
        d_A,
        lda,
        percentage,
        descrC,
        d_csrValC,
        d_csrRowPtrC,
        d_csrColIndC,
        info,
        d_work);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    cudaStat1 = cudaDeviceSynchronize();
    assert(cudaSuccess == cudaStat1);

/* step 7: output C */
    csrRowPtrC = (int*  )malloc(sizeof(int  )*(m+1));
    csrColIndC = (int*  )malloc(sizeof(int  )*nnzC);
    csrValC    = (float*)malloc(sizeof(float)*nnzC);
    assert( NULL != csrRowPtrC);
    assert( NULL != csrColIndC);
    assert( NULL != csrValC);

    cudaStat1 = cudaMemcpy(csrRowPtrC, d_csrRowPtrC, sizeof(int  )*(m+1),
 cudaMemcpyDeviceToHost);
    cudaStat2 = cudaMemcpy(csrColIndC, d_csrColIndC, sizeof(int  )*nnzC ,
 cudaMemcpyDeviceToHost);
    cudaStat3 = cudaMemcpy(csrValC   , d_csrValC   , sizeof(float)*nnzC ,
 cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    assert(cudaSuccess == cudaStat2);
    assert(cudaSuccess == cudaStat3);

    printCsr(m, n, nnzC, descrC, csrValC, csrRowPtrC, csrColIndC, "C");

/* free resources */
    if (d_A         ) cudaFree(d_A);
    if (d_csrRowPtrC) cudaFree(d_csrRowPtrC);
    if (d_csrColIndC) cudaFree(d_csrColIndC);
    if (d_csrValC   ) cudaFree(d_csrValC);

    if (csrRowPtrC  ) free(csrRowPtrC);
    if (csrColIndC  ) free(csrColIndC);
    if (csrValC     ) free(csrValC);

    if (handle      ) cusparseDestroy(handle);
    if (stream      ) cudaStreamDestroy(stream);
    if (descrC      ) cusparseDestroyMatDescr(descrC);
    if (info        ) cusparseDestroyPruneInfo(info);

    cudaDeviceReset();

    return 0;
}
```

16.4. Prune Sparse to Sparse by Percentage

This section provides a simple example in the C programming language of pruning a sparse matrix to a sparse matrix by percentage.

A is a 4x4 sparse matrix,

$$A = \begin{pmatrix} 1.0 & 0.0 & 2.0 & -3.0 \\ 0.0 & 4.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 6.0 & 7.0 \\ 0.0 & 8.0 & 0.0 & 9.0 \end{pmatrix}$$

The percentage is 20, which means to prune 20 percent of the nonzeros. The sparse matrix has 9 nonzero elements, so 1.4 elements must be pruned out. The function removes 1.0 and 2.0 which are first two smallest numbers of nonzeros.

```
/*
 * How to compile (assume cuda is installed at /usr/local/cuda/)
 *   nvcc -c -I/usr/local/cuda/include prunecsr2csrByP.cpp
 *   g++ -o prunecsr2csrByP.cpp prunecsr2csrByP.o -L/usr/local/cuda/lib64 -lcusparse
 -lcudart
 */
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <cuda_runtime.h>
#include <cusparse.h>

void printCsr(
    int m,
    int n,
    int nnz,
    const cusparseMatDescr_t descrA,
    const float *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    const char* name)
{
    const int base = (cusparseGetMatIndexBase(descrA) != CUSPARSE_INDEX_BASE_ONE)?
 0:1 ;

    printf("matrix %s is %d-by-%d, nnz=%d, base=%d, output base-1\n", name, m, n,
 nnz, base);
    for(int row = 0 ; row < m ; row++){
        const int start = csrRowPtrA[row  ] - base;
        const int end   = csrRowPtrA[row+1] - base;
        for(int colidx = start ; colidx < end ; colidx++){
            const int col = csrColIndA[colidx] - base;
            const float Areg = csrValA[colidx];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
```

```
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrA = NULL;
    cusparseMatDescr_t descrC = NULL;
    pruneInfo_t info = NULL;

    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    const int m = 4;
    const int n = 4;
    const int nnzA = 9;
/*
 *      |    1     0     2     -3  |
 *      |    0     4     0     0   |
 *  A = |    5     0     6     7   |
 *      |    0     8     0     9   |
 *
 */
```

```
    const int csrRowPtrA[m+1] = { 1, 4, 5, 8, 10};
    const int csrColIndA[nnzA] = { 1, 3, 4, 2, 1, 3, 4, 2, 4};
    const float csrValA[nnzA] = {1, 2, -3, 4, 5, 6, 7, 8, 9};

    int* csrRowPtrC = NULL;
    int* csrColIndC = NULL;
    float* csrValC  = NULL;

    int *d_csrRowPtrA = NULL;
    int *d_csrColIndA = NULL;
    float *d_csrValA = NULL;

    int *d_csrRowPtrC = NULL;
    int *d_csrColIndC = NULL;
    float *d_csrValC = NULL;

    size_t lworkInBytes = 0;
    char *d_work = NULL;

    int nnzC = 0;

    float percentage = 20; /* remove 20% of nonzeros */

    printf("example of pruneCsr2csrByPercentage \n");

    printf("prune %.1f percent of nonzeros \n", percentage);

/* step 1: create cusparse handle, bind a stream */
    cudaStat1 = cudaStreamCreateWithFlags(&stream, cudaStreamNonBlocking);
    assert(cudaSuccess == cudaStat1);

    status = cusparseCreate(&handle);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    status = cusparseSetStream(handle, stream);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    status = cusparseCreatePruneInfo(&info);
    assert(CUSPARSE_STATUS_SUCCESS == status);

/* step 2: configuration of matrix C */
    status = cusparseCreateMatDescr(&descrA);
    assert(CUSPARSE_STATUS_SUCCESS == status);
/* A is base-1*/
    cusparseSetMatIndexBase(descrA,CUSPARSE_INDEX_BASE_ONE);
    cusparseSetMatType(descrA, CUSPARSE_MATRIX_TYPE_GENERAL );
```

```
    status = cusparseCreateMatDescr(&descrC);
    assert(CUSPARSE_STATUS_SUCCESS == status);
/* C is base-0 */
    cusparseSetMatIndexBase(descrC,CUSPARSE_INDEX_BASE_ZERO);
    cusparseSetMatType(descrC, CUSPARSE_MATRIX_TYPE_GENERAL );

    printCsr(m, n, nnzA, descrA, csrValA, csrRowPtrA, csrColIndA, "A");
```

```
    cudaStat1 = cudaMalloc ((void**)&d_csrRowPtrA, sizeof(int)*(m+1) );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrColIndA, sizeof(int)*nnzA );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrValA   , sizeof(float)*nnzA );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrRowPtrC, sizeof(int)*(m+1) );
    assert(cudaSuccess == cudaStat1);

    cudaStat1 = cudaMemcpy(d_csrRowPtrA, csrRowPtrA, sizeof(int)*(m+1),
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(d_csrColIndA, csrColIndA, sizeof(int)*nnzA,
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(d_csrValA   , csrValA   , sizeof(float)*nnzA,
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);

/* step 3: query workspace */
    status = cusparseSpruneCsr2csrByPercentage_bufferSizeExt(
        handle,
        m,
        n,
        nnzA,
        descrA,
        d_csrValA,
        d_csrRowPtrA,
        d_csrColIndA,
        percentage,
        descrC,
        d_csrValC,
        d_csrRowPtrC,
        d_csrColIndC,
        info,
        &lworkInBytes);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    printf("lworkInBytes = %lld \n", (long long)lworkInBytes);

    if (NULL != d_work) { cudaFree(d_work); }
    cudaStat1 = cudaMalloc((void**)&d_work, lworkInBytes);
    assert(cudaSuccess == cudaStat1);

/* step 4: compute csrRowPtrC and nnzC */
    status = cusparseSpruneCsr2csrNnzByPercentage(
        handle,
        m,
        n,
        nnzA,
        descrA,
        d_csrValA,
        d_csrRowPtrA,
        d_csrColIndA,
        percentage,
```

```
        descrC,
        d_csrRowPtrC,
        &nnzC, /* host */
        info,
        d_work);
```

```
    assert(CUSPARSE_STATUS_SUCCESS == status);
    cudaStat1 = cudaDeviceSynchronize();
    assert(cudaSuccess == cudaStat1);

    printf("nnzC = %d\n", nnzC);
    if (0 == nnzC ){
        printf("C is empty \n");
        return 0;
    }

/* step 5: compute csrColIndC and csrValC */
    cudaStat1 = cudaMalloc ((void**)&d_csrColIndC, sizeof(int  ) * nnzC );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrValC   , sizeof(float) * nnzC );
    assert(cudaSuccess == cudaStat1);

    status = cusparseSpruneCsr2csrByPercentage(
        handle,
        m,
        n,
        nnzA,
        descrA,
        d_csrValA,
        d_csrRowPtrA,
        d_csrColIndA,
        percentage,
        descrC,
        d_csrValC,
        d_csrRowPtrC,
        d_csrColIndC,
        info,
        d_work);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    cudaStat1 = cudaDeviceSynchronize();
    assert(cudaSuccess == cudaStat1);

/* step 6: output C */
    csrRowPtrC = (int*   )malloc(sizeof(int  )*(m+1));
    csrColIndC = (int*   )malloc(sizeof(int  )*nnzC);
    csrValC    = (float*)malloc(sizeof(float)*nnzC);
    assert( NULL != csrRowPtrC);
    assert( NULL != csrColIndC);
    assert( NULL != csrValC);

    cudaStat1 = cudaMemcpy(csrRowPtrC, d_csrRowPtrC, sizeof(int  )*(m+1),
 cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(csrColIndC, d_csrColIndC, sizeof(int  )*nnzC ,
 cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(csrValC   , d_csrValC   , sizeof(float)*nnzC ,
 cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);

    printCsr(m, n, nnzC, descrC, csrValC, csrRowPtrC, csrColIndC, "C");
```

```
/* free resources */
    if (d_csrRowPtrA) cudaFree(d_csrRowPtrA);
    if (d_csrColIndA) cudaFree(d_csrColIndA);
    if (d_csrValA   ) cudaFree(d_csrValA);
    if (d_csrRowPtrC) cudaFree(d_csrRowPtrC);
    if (d_csrColIndC) cudaFree(d_csrColIndC);
    if (d_csrValC   ) cudaFree(d_csrValC);

    if (csrRowPtrC  ) free(csrRowPtrC);
    if (csrColIndC  ) free(csrColIndC);
    if (csrValC     ) free(csrValC);

    if (handle      ) cusparseDestroy(handle);
    if (stream      ) cudaStreamDestroy(stream);
    if (descrA      ) cusparseDestroyMatDescr(descrA);
    if (descrC      ) cusparseDestroyMatDescr(descrC);
    if (info        ) cusparseDestroyPruneInfo(info);

    cudaDeviceReset();

    return 0;
}
```

Chapter 17. Appendix C: Examples of csrsm2

17.1. Forward Triangular Solver

This section provides a simple example in the C programming language of csrsm2.

The example solves a lower triangular system with 2 right hand side vectors.

```
/*
 * How to compile (assume cuda is installed at /usr/local/cuda/)
 *   nvcc -c -I/usr/local/cuda/include csrms2.cpp
 *   g++ -o csrm2 csrsm2.o -L/usr/local/cuda/lib64 -lcusparse -lcudart
 */
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <cuda_runtime.h>
#include <cusparse.h>

/* compute | b - A*x|_inf */
void residaul_eval(
    int n,
    const cusparseMatDescr_t descrA,
    const float *csrVal,
    const int *csrRowPtr,
    const int *csrColInd,
    const float *b,
    const float *x,
    float *r_nrminf_ptr)
{
    const int base = (cusparseGetMatIndexBase(descrA) != CUSPARSE_INDEX_BASE_ONE)?
 0:1 ;
    const int lower = (CUSPARSE_FILL_MODE_LOWER == cusparseGetMatFillMode(descrA))?
 1:0;
    const int unit  = (CUSPARSE_DIAG_TYPE_UNIT == cusparseGetMatDiagType(descrA))?
 1:0;

    float r_nrminf = 0;
    for(int row = 0 ; row < n ; row++){
        const int start = csrRowPtr[row]   - base;
        const int end   = csrRowPtr[row+1] - base;
        float dot = 0;
        for(int colidx = start ; colidx < end; colidx++){
            const int col = csrColInd[colidx] - base;
            float Aij = csrVal[colidx];
            float xj  = x[col];
```

```
            if ( (row == col) && unit ){
                Aij = 1.0;
            }
            int valid = (row >= col) && lower ||
                        (row <= col) && !lower ;
            if ( valid ){
                dot += Aij*xj;
            }
        }
        float ri = b[row] - dot;
        r_nrminf = (r_nrminf > fabs(ri))? r_nrminf : fabs(ri);
    }
    *r_nrminf_ptr = r_nrminf;
}

int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrA = NULL;
    csrsm2Info_t info = NULL;
```

```
    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    const int nrhs = 2;
    const int n = 4;
    const int nnzA = 9;
    const cusparseSolvePolicy_t policy = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
    const float h_one = 1.0;
/*
 *      |    1     0     2     -3  |
 *      |    0     4     0     0   |
 *  A = |    5     0     6     7   |
 *      |    0     8     0     9   |
 *
 *  Regard A as a lower triangle matrix L with non-unit diagonal.
 *             | 1   5 |              | 1          5      |
 *  Given  B = | 2   6 |, X = L \ B = | 0.5       1.5     |
 *             | 3   7 |              | -0.3333   -3      |
 *             | 4   8 |              |  0        -0.4444 |
 */
    const int csrRowPtrA[n+1] = { 1, 4, 5, 8, 10};
    const int csrColIndA[nnzA] = { 1, 3, 4, 2, 1, 3, 4, 2, 4};
    const float csrValA[nnzA] = {1, 2, -3, 4, 5, 6, 7, 8, 9};
    const float B[n*nrhs] = {1,2,3,4,5,6,7,8};
    float X[n*nrhs];

    int *d_csrRowPtrA = NULL;
    int *d_csrColIndA = NULL;
    float *d_csrValA = NULL;
    float *d_B = NULL;

    size_t lworkInBytes = 0;
    char *d_work = NULL;

    const int algo = 0; /* non-block version */

    printf("example of csrsm2 \n");

/* step 1: create cusparse handle, bind a stream */
    cudaStat1 = cudaStreamCreateWithFlags(&stream, cudaStreamNonBlocking);
    assert(cudaSuccess == cudaStat1);

    status = cusparseCreate(&handle);
```

```
    assert(CUSPARSE_STATUS_SUCCESS == status);

    status = cusparseSetStream(handle, stream);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    status = cusparseCreateCsrsm2Info(&info);
    assert(CUSPARSE_STATUS_SUCCESS == status);

/* step 2: configuration of matrix A */
    status = cusparseCreateMatDescr(&descrA);
    assert(CUSPARSE_STATUS_SUCCESS == status);
/* A is base-1*/
    cusparseSetMatIndexBase(descrA,CUSPARSE_INDEX_BASE_ONE);

    cusparseSetMatType(descrA, CUSPARSE_MATRIX_TYPE_GENERAL);
/* A is lower triangle */
    cusparseSetMatFillMode(descrA, CUSPARSE_FILL_MODE_LOWER);
/* A has non unit diagonal */
    cusparseSetMatDiagType(descrA, CUSPARSE_DIAG_TYPE_NON_UNIT);
```

```
    cudaStat1 = cudaMalloc ((void**)&d_csrRowPtrA, sizeof(int)*(n+1) );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrColIndA, sizeof(int)*nnzA );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrValA   , sizeof(float)*nnzA );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_B         , sizeof(float)*n*nrhs );
    assert(cudaSuccess == cudaStat1);

    cudaStat1 = cudaMemcpy(d_csrRowPtrA, csrRowPtrA, sizeof(int)*(n+1),
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(d_csrColIndA, csrColIndA, sizeof(int)*nnzA,
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(d_csrValA   , csrValA   , sizeof(float)*nnzA,
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(d_B         , B         , sizeof(float)*n*nrhs,
 cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);

/* step 3: query workspace */
    status = cusparseScsrsm2_bufferSizeExt(
        handle,
        algo,
        CUSPARSE_OPERATION_NON_TRANSPOSE, /* transA */
        CUSPARSE_OPERATION_NON_TRANSPOSE, /* transB */
        n,
        nrhs,
        nnzA,
        &h_one,
        descrA,
        d_csrValA,
        d_csrRowPtrA,
        d_csrColIndA,
        d_B,
        n,   /* ldb */
        info,
        policy,
        &lworkInBytes);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    printf("lworkInBytes  = %lld \n", (long long)lworkInBytes);
```

```
    if (NULL != d_work) { cudaFree(d_work); }
    cudaStat1 = cudaMalloc((void**)&d_work, lworkInBytes);
    assert(cudaSuccess == cudaStat1);

/* step 4: analysis */
    status = cusparseScsrsm2_analysis(
        handle,
        algo,
        CUSPARSE_OPERATION_NON_TRANSPOSE, /* transA */
        CUSPARSE_OPERATION_NON_TRANSPOSE, /* transB */
        n,
        nrhs,
        nnzA,
        &h_one,
        descrA,
        d_csrValA,
        d_csrRowPtrA,
        d_csrColIndA,
        d_B,
        n,   /* ldb */
        info,
        policy,
        d_work);
    assert(CUSPARSE_STATUS_SUCCESS == status);
```

```
/* step 5: solve L * X = B */
    status = cusparseScsrsm2_solve(
        handle,
        algo,
        CUSPARSE_OPERATION_NON_TRANSPOSE, /* transA */
        CUSPARSE_OPERATION_NON_TRANSPOSE, /* transB */
        n,
        nrhs,
        nnzA,
        &h_one,
        descrA,
        d_csrValA,
        d_csrRowPtrA,
        d_csrColIndA,
        d_B,
        n,   /* ldb */
        info,
        policy,
        d_work);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    cudaStat1 = cudaDeviceSynchronize();
    assert(cudaSuccess == cudaStat1);

/* step 6:measure residual B - A*X */
    cudaStat1 = cudaMemcpy(X, d_B, sizeof(float)*n*nrhs, cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    cudaDeviceSynchronize();

    printf("==== x1 = inv(A)*b1 \n");
    for(int j = 0 ; j < n; j++){
        printf("x1[%d] = %f\n", j, X[j]);
    }
    float r1_nrminf;
    residaul_eval(
        n,
        descrA,
        csrValA,
        csrRowPtrA,
        csrColIndA,
        B,
```

```
        X,
        &r1_nrminf
    );
    printf("|b1 - A*x1| = %E\n", r1_nrminf);

    printf("==== x2 = inv(A)*b2 \n");
    for(int j = 0 ; j < n; j++){
        printf("x2[%d] = %f\n", j, X[n+j]);
    }
    float r2_nrminf;
    residaul_eval(
        n,
        descrA,
        csrValA,
        csrRowPtrA,
        csrColIndA,
        B+n,
        X+n,
        &r2_nrminf
    );
    printf("|b2 - A*x2| = %E\n", r2_nrminf);
```

```
/* free resources */
    if (d_csrRowPtrA  ) cudaFree(d_csrRowPtrA);
    if (d_csrColIndA  ) cudaFree(d_csrColIndA);
    if (d_csrValA     ) cudaFree(d_csrValA);
    if (d_B           ) cudaFree(d_B);

    if (handle        ) cusparseDestroy(handle);
    if (stream        ) cudaStreamDestroy(stream);
    if (descrA        ) cusparseDestroyMatDescr(descrA);
    if (info          ) cusparseDestroyCsrsm2Info(info);

    cudaDeviceReset();

    return 0;
}
```

Chapter 18. Appendix D: Acknowledgements

NVIDIA would like to thank the following individuals and institutions for their contributions:

- The cusparse<t>gtsv implementation is derived from a version developed by Li-Wen Chang from the University of Illinois.
- The cusparse<t>gtsvInterleavedBatch adopts cuThomasBatch developed by Pedro Valero-Lara and Ivan Martínez-Pérez from Barcelona Supercomputing Center and BSC/UPC NVIDIA GPU Center of Excellence.
- This product includes {fmt} - A modern formatting library https://fmt.dev Copyright (c) 2012 - present, Victor Zverovich.

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