CUSPARSE LIBRARY

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**TABLE OF CONTENTS**

Chapter 1. Introduction........................................................................................................... 1
  1.1. Naming Conventions................................................................................................... 1
  1.2. Asynchronous Execution.......................................................................................... 2
  1.3. Static Library support.............................................................................................. 2

Chapter 2. Using the cuSPARSE API..................................................................................... 4
  2.1. Thread Safety............................................................................................................. 4
  2.2. Scalar Parameters..................................................................................................... 4
  2.3. Parallelism with Streams........................................................................................ 4
  2.4. Compatibility and Versioning................................................................................. 5
  2.5. Optimization Notes............................................................................................... 5

Chapter 3. cuSPARSE Indexing and Data Formats................................................................. 6
  3.1. Index Base Format.................................................................................................... 6
    3.1.1. Vector Formats..................................................................................................... 6
      3.1.1.1. Dense Format............................................................................................... 6
      3.1.1.2. Sparse Format............................................................................................. 6
    3.2. Matrix Formats...................................................................................................... 7
      3.2.1. Dense Format.................................................................................................... 7
      3.2.2. Coordinate Format (COO)................................................................................. 7
      3.2.3. Compressed Sparse Row Format (CSR)............................................................ 8
      3.2.4. Compressed Sparse Column Format (CSC)......................................................... 9
      3.2.5. Block Compressed Sparse Row Format (BSR)..................................................... 10
      3.2.6. Extended BSR Format (BSRX)........................................................................ 12

Chapter 4. cuSPARSE Types Reference............................................................................... 14
  4.1. Data types................................................................................................................ 14
  4.2. cusparseStatus_t..................................................................................................... 14
  4.3. cusparseHandle_t..................................................................................................... 15
  4.4. cusparsePointerMode_t.......................................................................................... 15
  4.5. cusparseOperation_t.............................................................................................. 16
  4.6. cusparseAction_t..................................................................................................... 16
  4.7. cusparseDirection_t.............................................................................................. 16
  4.8. cusparseMatDescr_t............................................................................................... 17
    4.8.1. cusparseDiagType_t.......................................................................................... 17
    4.8.2. cusparseFillMode_t.......................................................................................... 17
    4.8.3. cusparseIndexBase_t...................................................................................... 17
    4.8.4. cusparseMatrixType_t..................................................................................... 17
  4.9. cusparseAlgMode_t................................................................................................. 18
  4.10. cusparseColorInfo_t............................................................................................. 18
  4.11. cusparseSolvePolicy_t......................................................................................... 18
  4.12. bsric02Info_t....................................................................................................... 19
  4.13. bsrilu02Info_t...................................................................................................... 19
4.14. bsrsm2Info_t................................................................. 19
4.15. bsrsv2Info_t............................................................... 19
4.16. csrgemm2Info_t.......................................................... 19
4.17. csric02Info_t............................................................ 19
4.18. csrilu02Info_t........................................................... 19
4.19. csrsm2Info_t............................................................. 20
4.20. csrsv2Info_t............................................................. 20

Chapter 5. cuSPARSE Management Function Reference........................................ 21
5.1. cusparseCreate()....................................................... 21
5.2. cusparseDestroy()..................................................... 21
5.3. cusparseGetErrorName()............................................ 22
5.4. cusparseGetErrorString().......................................... 22
5.5. cusparseGetProperty()............................................... 22
5.6. cusparseGetVersion()............................................... 23
5.7. cusparseGetPointerMode()........................................ 23
5.8. cusparseSetPointerMode()........................................ 23
5.9. cusparseGetStream()................................................ 24
5.10. cusparseSetStream()............................................... 24

Chapter 6. cuSPARSE Helper Function Reference................................................ 25
6.1. cusparseCreateColorInfo()........................................ 25
6.2. cusparseCreateMatDescr()......................................... 25
6.3. cusparseDestroyColorInfo()...................................... 26
6.4. cusparseDestroyMatDescr()....................................... 26
6.5. cusparseGetMatDiagType()....................................... 26
6.6. cusparseGetMatFillMode()........................................ 26
6.7. cusparseGetMatIndexBase()...................................... 27
6.8. cusparseGetMatType()................................................ 27
6.9. cusparseSetMatDiagType()........................................ 27
6.10. cusparseSetMatFillMode()....................................... 28
6.11. cusparseSetMatIndexBase()..................................... 28
6.12. cusparseSetMatType()............................................. 28
6.13. cusparseCreateCsrsv2Info()..................................... 29
6.14. cusparseDestroyCsrsv2Info()................................... 29
6.15. cusparseCreateCsrsm2Info().................................... 29
6.16. cusparseDestroyCsrsm2Info().................................. 29
6.17. cusparseCreateCsric02Info().................................. 30
6.18. cusparseDestroyCsric02Info()................................. 30
6.19. cusparseCreateCsrilu02Info()................................ 30
6.20. cusparseDestroyCsrilu02Info()................................. 31
6.21. cusparseCreateBsrsv2Info().................................... 31
6.22. cusparseDestroyBsrsv2Info().................................. 31
6.23. cusparseCreateBsrsm2Info().................................... 32
6.24. cusparseDestroyBsrsm2Info()................................. 32
<table>
<thead>
<tr>
<th>Function</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>cusparseCreateBsrc02Info()</td>
<td>32</td>
</tr>
<tr>
<td>cusparseDestroyBsrc02Info()</td>
<td>32</td>
</tr>
<tr>
<td>cusparseCreateBsrilu02Info()</td>
<td>33</td>
</tr>
<tr>
<td>cusparseDestroyBsrilu02Info()</td>
<td>33</td>
</tr>
<tr>
<td>cusparseCreateCsrgemm2Info()</td>
<td>33</td>
</tr>
<tr>
<td>cusparseDestroyCsrgemm2Info()</td>
<td>34</td>
</tr>
<tr>
<td>cusparseCreatePruneInfo()</td>
<td>34</td>
</tr>
<tr>
<td>cusparseDestroyPruneInfo()</td>
<td>34</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;axpyi() [DEPRECATED]</td>
<td>36</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;gthr() [DEPRECATED]</td>
<td>37</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;gthrz() [DEPRECATED]</td>
<td>39</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;roti() [DEPRECATED]</td>
<td>40</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;sctr() [DEPRECATED]</td>
<td>42</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;bsrmv()</td>
<td>45</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;bsrxmv()</td>
<td>48</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;bsrsv2_bufferSize()</td>
<td>52</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;bsrsv2_analysis()</td>
<td>55</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;bsrsv2_solve()</td>
<td>58</td>
</tr>
<tr>
<td>cusparseXbsrsv2_zeroPivot()</td>
<td>62</td>
</tr>
<tr>
<td>cusparseCsrsv2Ex()</td>
<td>64</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;csrsv2_bufferSize()</td>
<td>66</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;csrsv2_analysis()</td>
<td>68</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;csrsv2_solve()</td>
<td>71</td>
</tr>
<tr>
<td>cusparseXcsrsv2_zeroPivot()</td>
<td>75</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;gemvi() [DEPRECATED]</td>
<td>76</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;bsrmm()</td>
<td>80</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;bsrsm2_bufferSize()</td>
<td>84</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;bsrsm2_analysis()</td>
<td>87</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;bsrsm2_solve()</td>
<td>90</td>
</tr>
<tr>
<td>cusparseXbsrsm2_zeroPivot()</td>
<td>93</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;csrsm2_bufferSizeExt()</td>
<td>94</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;csrsm2_analysis()</td>
<td>97</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;csrsm2_solve()</td>
<td>100</td>
</tr>
<tr>
<td>cusparseXcsrsm2_zeroPivot()</td>
<td>103</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;gemmi() [DEPRECATED]</td>
<td>104</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;csrgeam2()</td>
<td>107</td>
</tr>
<tr>
<td>cusparse&lt;t&gt;csrgemm2() [DEPRECATED]</td>
<td>111</td>
</tr>
<tr>
<td>Incomplete Cholesky Factorization: level 0</td>
<td>116</td>
</tr>
</tbody>
</table>
13.11. cusparse<t>csr2dense()............................................................................. 207
13.12. cusparse<t>csr2csr_compress()................................................................... 209
13.13. cusparse<t>dense2csc()............................................................................ 213
13.14. cusparse<t>dense2csr()............................................................................. 215
13.15. cusparse<t>nnz().................................................................................... 217
13.16. cusparseCreateIdentityPermutation()......................................................... 218
13.17. cusparseXcoosort().................................................................................. 219
13.18. cusparseXcsrsort()................................................................................... 221
13.19. cusparseXcscsort()................................................................................... 223
13.20. cusparseXcsru2csr()................................................................................. 226
13.21. cusparseXpruneDense2csr()........................................................................ 230
13.22. cusparseXpruneCsr2csr().......................................................................... 233
13.23. cusparseXpruneDense2csrPercentage()......................................................... 236
13.24. cusparseXpruneCsr2csrByPercentage()......................................................... 239
13.25. cusparse<t>nnz_compress()........................................................................ 242

Chapter 14. cuSPARSE Generic API Reference.......................................................... 244
14.1. Generic Types Reference............................................................................ 244
    14.1.1. cudaDataType_t.................................................................................. 244
    14.1.2. cusparseFormat_t............................................................................... 245
    14.1.3. cusparseOrder_t............................................................................... 245
    14.1.4. cusparseIndexType_t............................................................................ 246
14.2. Sparse Vector APIs................................................................................... 246
    14.2.1. cusparseCreateSpVec()......................................................................... 246
    14.2.2. cusparseDestroySpVec()........................................................................ 246
    14.2.3. cusparseSpVecGet()............................................................................. 247
    14.2.4. cusparseSpVecGetIndexBase()............................................................... 247
    14.2.5. cusparseSpVecGetValues().................................................................... 248
    14.2.6. cusparseSpVecSetValues()...................................................................... 248
14.3. Sparse Matrix APIs.................................................................................... 248
    14.3.1. cusparseCreateCoo()............................................................................ 248
    14.3.2. cusparseCreateCooAoS()....................................................................... 249
    14.3.3. cusparseCreateCsr()............................................................................. 250
    14.3.4. cusparseDestroySpMat()........................................................................ 250
    14.3.5. cusparseCooGet()............................................................................... 251
    14.3.6. cusparseCooAosGet().......................................................................... 251
    14.3.7. cusparseCsrGet()................................................................................ 252
    14.3.8. cusparseCsrSetPointers()..................................................................... 253
    14.3.9. cusparseSpMatGetSize()........................................................................ 253
    14.3.10. cusparseSpMatGetFormat().................................................................. 253
    14.3.11. cusparseSpMatGetIndexBase()............................................................. 254
    14.3.12. cusparseSpMatGetValues().................................................................. 254
    14.3.13. cusparseSpMatSetValues().................................................................. 254
    14.3.14. cusparseSpMatGetStridedBatch()........................................................ 255
Chapter 14. Dense Vector APIs

14.4.1. cusparseCreateDnVec() ................................................................. 256
14.4.2. cusparseDestroyDnVec() ................................................................. 257
14.4.3. cusparseDnVecGet() .................................................................... 257
14.4.4. cusparseDnVecGetValues() ............................................................. 257
14.4.5. cusparseDnVecSetValues() ............................................................... 258

Chapter 14. Dense Matrix APIs

14.5.1. cusparseCreateDnMat() ................................................................. 258
14.5.2. cusparseDestroyDnMat() ................................................................. 259
14.5.3. cusparseDnMatGet() ..................................................................... 259
14.5.4. cusparseDnMatGetValues() ............................................................. 259
14.5.5. cusparseDnSetValues() ................................................................. 260
14.5.6. cusparseDnMatGetStridedBatch() ..................................................... 260
14.5.7. cusparseDnMatSetStridedBatch() ..................................................... 260

Chapter 14. Generic API Functions

14.6.1. cusparseAxpby() ......................................................................... 261
14.6.2. cusparseGather() ........................................................................ 262
14.6.3. cusparseScatter() ....................................................................... 263
14.6.4. cusparseRot() ............................................................................ 265
14.6.5. cusparseSpVV() .......................................................................... 266
14.6.6. cusparseSpMV() ........................................................................ 268
14.6.7. cusparseSpMM() ........................................................................ 271
14.6.8. cusparseConstrainedGeMM() ......................................................... 275
14.6.9. cusparseSpGEMM() .................................................................... 278

Chapter 15. Appendix B: cuSPARSE Fortran Bindings .................................................... 281

15.1. Fortran Application ........................................................................ 283

Chapter 16. Appendix B: Examples of sorting ........................................................ 284

16.1. COO Sort ....................................................................................... 284

Chapter 17. Appendix C: Examples of prune ......................................................... 286

17.1. Prune Dense to Sparse .................................................................. 286
17.2. Prune Sparse to Sparse .................................................................. 288
17.3. Prune Dense to Sparse by Percentage ............................................. 290
17.4. Prune Sparse to Sparse by Percentage ............................................ 292

Chapter 18. Appendix D: Examples of gpsv .......................................................... 294

18.1. Batched Penta-diagonal Solver ......................................................... 294

Chapter 19. Appendix E: Examples of csrsm2 ...................................................... 296

19.1. Forward Triangular Solver .............................................................. 296

Chapter 20. Appendix F: Acknowledgements ...................................................... 298

Chapter 21. Bibliography ........................................................................ 299
Chapter 1.  
INTRODUCTION

The cuSPARSE library contains a set of basic linear algebra subroutines used for handling sparse matrices. The library targets matrices with a number of (structural) zero elements which represent > 95% of the total entries.

It is implemented on top of the NVIDIA® CUDA™ runtime (which is part of the CUDA Toolkit) and is designed to be called from C and C++.

The library routines can be classified into four categories:

- Level 1: operations between a vector in sparse format and a vector in dense format
- Level 2: operations between a matrix in sparse format and a vector in dense format
- Level 3: operations between a matrix in sparse format and a set of vectors in dense format (which can also usually be viewed as a dense tall matrix)
- Conversion: operations that allow conversion between different matrix formats, and compression of csr matrices.

The cuSPARSE library allows developers to access the computational resources of the NVIDIA graphics processing unit (GPU), although it does not auto-parallelize across multiple GPUs. The cuSPARSE API assumes that input and output data reside in GPU (device) memory, unless it is explicitly indicated otherwise by the string `DevHostPtr` in a function parameter’s name.

It is the responsibility of the developer to allocate memory and to copy data between GPU memory and CPU memory using standard CUDA runtime API routines, such as `cudaMalloc()`, `cudaFree()`, `cudaMemcpy()`, and `cudaMemcpyAsync()`.

1.1. Naming Conventions

The cuSPARSE library functions are available for data types `float`, `double`, `cuComplex`, and `cuDoubleComplex`. The sparse Level 1, Level 2, and Level 3 functions follow this naming convention:

`cusparse<t>[<matrix data format>]<operation>[<output matrix data format>]`
where \(<t>\) can be \(S, D, C, Z,\) or \(X,\) corresponding to the data types \(\text{float, double,}\)
\(\text{cuComplex, cuDoubleComplex,}\) and the generic type, respectively.

The \(<\text{matrix data format}>\) can be \(\text{dense, coo, csr, or csc,}\) corresponding to the
dense, coordinate, compressed sparse row, and compressed sparse column formats,
respectively.

Finally, the \(<\text{operation}>\) can be \(\text{axpyi, gthr, gthrz, roti, or sctr,}\) corresponding to
the Level 1 functions; it also can be \(\text{mv or sv,}\) corresponding to the Level 2 functions, as
well as \(\text{mm or sm,}\) corresponding to the Level 3 functions.

All of the functions have the return type \text{cusparseStatus\_t}\) and are explained in more
detail in the chapters that follow.

1.2. Asynchronous Execution

The cuSPARSE library functions are executed asynchronously with respect to the
host and may return control to the application on the host before the result is ready.
Developers can use the \text{cudaDeviceSynchronize()} function to ensure that the
execution of a particular cuSPARSE library routine has completed.

A developer can also use the \text{cudaMemcpy()} routine to copy data from the
device to the host and vice versa, using the \text{cudaMemcpyDeviceToHost} and
\text{cudaMemcpyHostToDevice} parameters, respectively. In this case there is no need to add
a call to \text{cudaDeviceSynchronize()} because the call to \text{cudaMemcpy()} with the above
parameters is blocking and completes only when the results are ready on the host.

1.3. Static Library support

Starting with release 6.5, the cuSPARSE Library is also delivered in a static form as
\text{libcusparse\_static.a} on Linux and Mac OSes. The static cuSPARSE library and all others
static maths libraries depend on a common thread abstraction layer library called
\text{libculibos.a} on Linux and Mac and culibos.lib on Windows.

For example, on linux, to compile a small application using cuSPARSE against the
dynamic library, the following command can be used:

\[
\text{nvcc myCusparseApp.c -lcusparse -o myCusparseApp}
\]

Whereas to compile against the static cuSPARSE library, the following command has to be
used:

\[
\text{nvcc myCusparseApp.c -lcusparse\_static -lculibos -o myCusparseApp}
\]

It is also possible to use the native Host C++ compiler. Depending on the Host Operating
system, some additional libraries like \text{pthread} or \text{dl} might be needed on the linking
line. The following command on Linux is suggested:

\[
\text{g++ myCusparseApp.c -lcusparse\_static -lculibos -lcudart\_static -lpthread -ldl -I <cuda-toolkit-path>/include -L <cuda-toolkit-path>/lib64 -o myCusparseApp}
\]
Note that in the latter case, the library **cuda** is not needed. The CUDA Runtime will try to open explicitly the **cuda** library if needed. In the case of a system which does not have the CUDA driver installed, this allows the application to gracefully manage this issue and potentially run if a CPU-only path is available.
This chapter describes how to use the cuSPARSE library API. It is not a reference for the cuSPARSE API data types and functions; that is provided in subsequent chapters.

2.1. Thread Safety

The library is thread safe and its functions can be called from multiple host threads. However, simultaneous read/writes of the same objects (or of the same handle) are not safe. Hence the handle must be private per thread, i.e., only one handle per thread is safe.

2.2. Scalar Parameters

In the cuSPARSE API, the scalar parameters $\alpha$ and $\beta$ can be passed by reference on the host or the device.

The few functions that return a scalar result, such as `nnz()`, return the resulting value by reference on the host or the device. Even though these functions return immediately, similarly to those that return matrix and vector results, the scalar result is not ready until execution of the routine on the GPU completes. This requires proper synchronization be used when reading the result from the host.

This feature allows the cuSPARSE library functions to execute completely asynchronously using streams, even when $\alpha$ and $\beta$ are generated by a previous kernel. This situation arises, for example, when the library is used to implement iterative methods for the solution of linear systems and eigenvalue problems [3].

2.3. Parallelism with Streams

If the application performs several small independent computations, or if it makes data transfers in parallel with the computation, CUDA streams can be used to overlap these tasks.
The application can conceptually associate a stream with each task. To achieve the overlap of computation between the tasks, the developer should create CUDA streams using the function `cudaStreamCreate()` and set the stream to be used by each individual cuSPARSE library routine by calling `cusparseSetStream()` just before calling the actual cuSPARSE routine. Then, computations performed in separate streams would be overlapped automatically on the GPU, when possible. This approach is especially useful when the computation performed by a single task is relatively small and is not enough to fill the GPU with work, or when there is a data transfer that can be performed in parallel with the computation.

When streams are used, we recommend using the new cuSPARSE API with scalar parameters and results passed by reference in the device memory to achieve maximum computational overlap.

Although a developer can create many streams, in practice it is not possible to have more than 16 concurrent kernels executing at the same time.

### 2.4. Compatibility and Versioning

The cuSPARSE APIs are intended to be backward compatible at the source level with future releases (unless stated otherwise in the release notes of a specific future release). In other words, if a program uses cuSPARSE, it should continue to compile and work correctly with newer versions of cuSPARSE without source code changes. cuSPARSE is not guaranteed to be backward compatible at the binary level. Using different versions of the `cusparse.h` header file and the shared library is not supported. Using different versions of cuSPARSE and the CUDA runtime is not supported. The APIs should be backward compatible at the source level for public functions in most cases.

### 2.5. Optimization Notes

Most of the cuSPARSE routines can be optimized by exploiting CUDA Graphs capture and Hardware Memory Compression features.

More in details, a single cuSPARSE call or a sequence of calls can be captured by a CUDA Graph and executed in a second moment. This minimizes kernels launch overhead and allows the CUDA runtime to optimize the whole workflow. A full example of CUDA graphs capture applied to a cuSPARSE routine can be found in cuSPARSE Library Samples - CUDA Graph.

Secondly, the data types and functionalities involved in cuSPARSE are suitable for Hardware Memory Compression available in Ampere GPU devices (compute capability 8.0) or above. The feature allows memory compression for data with enough zero bytes without no loss of information. The device memory must be allocation with the CUDA driver APIs. A full example of Hardware Memory Compression applied to a cuSPARSE routine can be found in cuSPARSE Library Samples - Memory Compression.
Chapter 3.
CUSPARSE INDEXING AND DATA FORMATS

The cuSPARSE library supports dense and sparse vector, and dense and sparse matrix formats.

3.1. Index Base Format

The library supports zero- and one-based indexing. The index base is selected through the `cusparseIndexBase_t` type, which is passed as a standalone parameter or as a field in the matrix descriptor `cusparseMatDescr_t` type.

3.1.1. Vector Formats

This section describes dense and sparse vector formats.

3.1.1.1. Dense Format

Dense vectors are represented with a single data array that is stored linearly in memory, such as the following $7 \times 1$ dense vector.

\[
\begin{bmatrix}
1.0 & 0.0 & 0.0 & 2.0 & 3.0 & 0.0 & 4.0
\end{bmatrix}
\]

(This vector is referenced again in the next section.)

3.1.1.2. Sparse Format

Sparse vectors are represented with two arrays.

- The data array has the nonzero values from the equivalent array in dense format.
- The integer index array has the positions of the corresponding nonzero values in the equivalent array in dense format.

For example, the dense vector in section 3.2.1 can be stored as a sparse vector with one-based indexing.

\[
\begin{bmatrix}
1.0 & 2.0 & 3.0 & 4.0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 4 & 5 & 7
\end{bmatrix}
\]
It can also be stored as a sparse vector with zero-based indexing.

\[
\begin{bmatrix}
1.0 & 2.0 & 3.0 & 4.0 \\
0 & 3 & 4 & 6
\end{bmatrix}
\]

In each example, the top row is the data array and the bottom row is the index array, and it is assumed that the indices are provided in increasing order and that each index appears only once.

3.2. Matrix Formats

Dense and several sparse formats for matrices are discussed in this section.

3.2.1. Dense Format

The dense matrix \( X \) is assumed to be stored in column-major format in memory and is represented by the following parameters.

<table>
<thead>
<tr>
<th>m</th>
<th>(integer)</th>
<th>The number of rows in the matrix.</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>(integer)</td>
<td>The number of columns in the matrix.</td>
</tr>
<tr>
<td>( \text{ldX} )</td>
<td>(integer)</td>
<td>The leading dimension of ( X ), which must be greater than or equal to ( m ). If ( \text{ldX} ) is greater than ( m ), then ( X ) represents a sub-matrix of a larger matrix stored in memory.</td>
</tr>
<tr>
<td>( X )</td>
<td>(pointer)</td>
<td>Points to the data array containing the matrix elements. It is assumed that enough storage is allocated for ( X ) to hold all of the matrix elements and that cuSPARSE library functions may access values outside of the sub-matrix, but will never overwrite them.</td>
</tr>
</tbody>
</table>

For example, \( m \times n \) dense matrix \( X \) with leading dimension \( \text{ldX} \) can be stored with one-based indexing as shown.

\[
\begin{bmatrix}
X_{1,1} & X_{1,2} & \cdots & X_{1,n} \\
X_{2,1} & X_{2,2} & \cdots & X_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
X_{m,1} & X_{m,2} & \cdots & X_{m,n} \\
X_{\text{ldX},1} & X_{\text{ldX},2} & \cdots & X_{\text{ldX},n}
\end{bmatrix}
\]

Its elements are arranged linearly in memory in the order below.

\[
\begin{bmatrix}
X_{1,1} & X_{2,1} & \cdots & X_{m,1} & X_{\text{ldX},1} & X_{1,n} & X_{2,n} & \cdots & X_{m,n}
\end{bmatrix}
\]

This format and notation are similar to those used in the NVIDIA CUDA cuBLAS library.

3.2.2. Coordinate Format (COO)

The \( m \times n \) sparse matrix \( A \) is represented in COO format by the following parameters.
### nnz
(integer)
The number of nonzero elements in the matrix.

<table>
<thead>
<tr>
<th>cooValA</th>
<th>(pointer)</th>
<th>Points to the data array of length ( \text{nnz} ) that holds all nonzero values of ( A ) in row-major format.</th>
</tr>
</thead>
<tbody>
<tr>
<td>cooRowIndA</td>
<td>(pointer)</td>
<td>Points to the integer array of length ( \text{nnz} ) that contains the row indices of the corresponding elements in array ( \text{cooValA} ).</td>
</tr>
<tr>
<td>cooColIndA</td>
<td>(pointer)</td>
<td>Points to the integer array of length ( \text{nnz} ) that contains the column indices of the corresponding elements in array ( \text{cooValA} ).</td>
</tr>
</tbody>
</table>

A sparse matrix in COO format is assumed to be stored in row-major format: the index arrays are first sorted by row indices and then within the same row by compressed column indices. It is assumed that each pair of row and column indices appears only once.

For example, consider the following \( 4 \times 5 \) matrix \( A \).

\[
\begin{bmatrix}
1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\
5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\
0.0 & 0.0 & 9.0 & 0.0 & 6.0 \\
\end{bmatrix}
\]

It is stored in COO format with zero-based indexing this way.

\[
\begin{align*}
\text{cooValA} &= [1.0, 4.0, 2.0, 3.0, 5.0, 7.0, 8.0, 9.0, 6.0] \\
\text{cooRowIndA} &= [0, 0, 1, 1, 2, 2, 2, 3, 3] \\
\text{cooColIndA} &= [0, 1, 1, 2, 0, 3, 4, 2, 4]
\end{align*}
\]

In the COO format with one-based indexing, it is stored as shown.

\[
\begin{align*}
\text{cooValA} &= [1.0, 4.0, 2.0, 3.0, 5.0, 7.0, 8.0, 9.0, 6.0] \\
\text{cooRowIndA} &= [1, 1, 2, 2, 3, 3, 3, 4, 4] \\
\text{cooColIndA} &= [1, 2, 2, 3, 1, 4, 5, 3, 5]
\end{align*}
\]

#### 3.2.3. Compressed Sparse Row Format (CSR)

The only way the CSR differs from the COO format is that the array containing the row indices is compressed in CSR format. The \( m \times n \) sparse matrix \( A \) is represented in CSR format by the following parameters.

<table>
<thead>
<tr>
<th>nnz</th>
<th>(integer)</th>
<th>The number of nonzero elements in the matrix.</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrValA</td>
<td>(pointer)</td>
<td>Points to the data array of length ( \text{nnz} ) that holds all nonzero values of ( A ) in row-major format.</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>(pointer)</td>
<td>Points to the integer array of length ( m+1 ) that holds indices into the arrays ( \text{csrColIndA} ) and ( \text{csrValA} ). The first ( m ) entries of this array contain the indices of the first nonzero element in the ( i )th row for ( i=1, \ldots, m ), while the last entry contains ( \text{nnz} + \text{csrRowPtrA}(0) ). In general, ( \text{csrRowPtrA}(0) ) is 0 or 1 for zero- and one-based indexing, respectively.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>(pointer)</td>
<td>Points to the integer array of length ( \text{nnz} ) that contains the column indices of the corresponding elements in array ( \text{csrValA} ).</td>
</tr>
</tbody>
</table>
Sparse matrices in CSR format are assumed to be stored in row-major CSR format, in other words, the index arrays are first sorted by row indices and then within the same row by column indices. It is assumed that each pair of row and column indices appears only once.

Consider again the $4 \times 5$ matrix $A$.

$$
\begin{bmatrix}
1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\
5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\
0.0 & 0.0 & 9.0 & 0.0 & 6.0 \\
\end{bmatrix}
$$

It is stored in CSR format with zero-based indexing as shown.

```plaintext
csrValA = [1.0 4.0 2.0 3.0 5.0 7.0 8.0 9.0 6.0]
csrRowPtrA = [0 2 4 7 9 ]
csrColIndA = [0 1 1 2 0 3 4 2 4 ]
```

This is how it is stored in CSR format with one-based indexing.

```plaintext
csrValA = [1.0 4.0 2.0 3.0 5.0 7.0 8.0 9.0 6.0]
csrRowPtrA = [1 3 5 8 10 ]
csrColIndA = [1 2 2 3 1 4 5 3 5 ]
```

### 3.2.4. Compressed Sparse Column Format (CSC)

The CSC format is different from the COO format in two ways: the matrix is stored in column-major format, and the array containing the column indices is compressed in CSC format. The $m \times n$ matrix $A$ is represented in CSC format by the following parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>nnz</td>
<td>(integer) The number of nonzero elements in the matrix.</td>
</tr>
<tr>
<td>cscValA</td>
<td>(pointer) Points to the data array of length $\text{nnz}$ that holds all nonzero values of $A$ in column-major format.</td>
</tr>
<tr>
<td>cscRowIndA</td>
<td>(pointer) Points to the integer array of length $\text{nnz}$ that contains the row indices of the corresponding elements in array $\text{cscValA}$.</td>
</tr>
<tr>
<td>cscColPtrA</td>
<td>(pointer) Points to the integer array of length $n+1$ that holds indices into the arrays $\text{cscRowIndA}$ and $\text{cscValA}$. The first $n$ entries of this array contain the indices of the first nonzero element in the $i$th row for $i=i,\ldots,n$, while the last entry contains $\text{nnz}+\text{cscColPtrA}(0)$. In general, $\text{cscColPtrA}(0)$ is 0 or 1 for zero- and one-based indexing, respectively.</td>
</tr>
</tbody>
</table>

The matrix $A$ in CSR format has exactly the same memory layout as its transpose in CSC format (and vice versa).

For example, consider once again the $4 \times 5$ matrix $A$.

$$
\begin{bmatrix}
1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\
5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\
0.0 & 0.0 & 9.0 & 0.0 & 6.0 \\
\end{bmatrix}
$$

It is stored in CSC format with zero-based indexing this way.
In CSC format with one-based indexing, this is how it is stored.

\[
\begin{array}{cccccccc}
1 & 2 & 0 & 1 & 1 & 3 & 2 & 2 \\
3 & 2 & 4 & 6 & 7 & 9 & & \\
\end{array}
\]

Each pair of row and column indices appears only once.

### 3.2.5. Block Compressed Sparse Row Format (BSR)

The only difference between the CSR and BSR formats is the format of the storage element. The former stores primitive data types (single, double, cuComplex, and cuDoubleComplex) whereas the latter stores a two-dimensional square block of primitive data types. The dimension of the square block is `blockDim`. The \(m \times n\) sparse matrix \(A\) is equivalent to a block sparse matrix \(A_b\) with \(mb = \frac{m + \text{blockDim} - 1}{\text{blockDim}}\) block rows and \(nb = \frac{n + \text{blockDim} - 1}{\text{blockDim}}\) block columns. If \(m\) or \(n\) is not multiple of `blockDim`, then zeros are filled into \(A_b\).

\(A\) is represented in BSR format by the following parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>blockDim</td>
<td>(integer) Block dimension of matrix (A).</td>
</tr>
<tr>
<td>mb</td>
<td>(integer) The number of block rows of (A).</td>
</tr>
<tr>
<td>nb</td>
<td>(integer) The number of block columns of (A).</td>
</tr>
<tr>
<td>nnzb</td>
<td>(integer) The number of nonzero blocks in the matrix.</td>
</tr>
<tr>
<td>bsrValA</td>
<td>(pointer) Points to the data array of length (nnzb \times \text{blockDim}^2) that holds all elements of nonzero blocks of (A). The block elements are stored in either column-major order or row-major order.</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>(pointer) Points to the integer array of length (mb+1) that holds indices into the arrays bsrColIndA and bsrValA. The first (mb) entries of this array contain the indices of the first nonzero block in the (i)th block row for (i=1,\ldots,mb), while the last entry contains (nnzb+\text{bsrRowPtrA}(0)). In general, (\text{bsrRowPtrA}(0)) is 0 or 1 for zero- and one-based indexing, respectively.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>(pointer) Points to the integer array of length (nnzb) that contains the column indices of the corresponding blocks in array bsrValA.</td>
</tr>
</tbody>
</table>

As with CSR format, (row, column) indices of BSR are stored in row-major order. The index arrays are first sorted by row indices and then within the same row by column indices.

For example, consider again the \(4 \times 5\) matrix \(A\).
If \( \text{blockDim} \) is equal to 2, then \( mb \) is 2, \( nb \) is 3, and matrix \( A \) is split into \( 2 \times 3 \) block matrix \( A_p \). The dimension of \( A_p \) is \( 4 \times 6 \), slightly bigger than matrix \( A \), so zeros are filled in the last column of \( A_p \). The element-wise view of \( A_p \) is this.

\[
\begin{bmatrix}
1.0 & 4.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & 3.0 & 0.0 & 0.0 & 0.0 \\
5.0 & 0.0 & 0.0 & 7.0 & 8.0 & 0.0 \\
0.0 & 0.0 & 9.0 & 0.0 & 0.0 & 6.0
\end{bmatrix}
\]

Based on zero-based indexing, the block-wise view of \( A_B \) can be represented as follows.

\[
A_B = \begin{bmatrix}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12}
\end{bmatrix}
\]

The basic element of BSR is a nonzero \( A_{ij} \) block, one that contains at least one nonzero element of \( A \). Five of six blocks are nonzero in \( A_p \).

\[
A_{00} = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}, \quad A_{01} = \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}, \quad A_{10} = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} 9 & 7 \\ 0 & 0 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 8 & 0 \\ 6 & 0 \end{bmatrix}
\]

BSR format only stores the information of nonzero blocks, including block indices \((i, j)\) and values \( A_{ij} \). Also row indices are compressed in CSR format.

\[
\begin{align*}
\text{bsrValA} &= [A_{00} \ A_{01} \ A_{10} \ A_{11} \ A_{12}]
\text{bsrRowPtrA} &= [0 \ 2 \ 5]
\text{bsrColIndA} &= [0 \ 1 \ 0 \ 1 \ 2]
\end{align*}
\]

There are two ways to arrange the data element of block \( A_{ij} \): row-major order and column-major order. Under column-major order, the physical storage of \( \text{bsrValA} \) is this.

\[
\text{bsrValA} = [1 \ 0 \ 4 \ 2 \ 0 \ 3 \ 0 \ 0 \ 5 \ 0 \ 0 \ 0 \ 0 \ 9 \ 7 \ 0 \ 8 \ 6 \ 0 \ 0]
\]

Under row-major order, the physical storage of \( \text{bsrValA} \) is this.

\[
\text{bsrValA} = [1 \ 4 \ 0 \ 2 \ 0 \ 0 \ 3 \ 0 \ 5 \ 0 \ 0 \ 0 \ 0 \ 7 \ 9 \ 0 \ 8 \ 0 \ 6 \ 0]
\]

Similarly, in BSR format with one-based indexing and column-major order, \( A \) can be represented by the following.

\[
A_B = \begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23}
\end{bmatrix}
\]

\[
\text{bsrValA} = [1 \ 0 \ 4 \ 2 \ 0 \ 3 \ 0 \ 0 \ 5 \ 0 \ 0 \ 0 \ 0 \ 9 \ 7 \ 0 \ 8 \ 6 \ 0 \ 0]
\]
The general BSR format has two parameters, \( rowBlockDim \) and \( colBlockDim \). \( rowBlockDim \) is number of rows within a block and \( colBlockDim \) is number of columns within a block. If \( rowBlockDim = colBlockDim \), general BSR format is the same as BSR format. If \( rowBlockDim = colBlockDim = 1 \), general BSR format is the same as CSR format. The conversion routine \texttt{gebsr2gebsr} is used to do conversion among CSR, BSR and general BSR.

In the cuSPARSE Library, the storage format of blocks in BSR format can be column-major or row-major, independently of the base index. However, if the developer uses BSR format from the Math Kernel Library (MKL) and wants to directly interface with the cuSPARSE Library, then \texttt{cusparseDirection_t CUSPARSE_DIRECTION_COLUMN} should be used if the base index is one; otherwise, \texttt{cusparseDirection_t CUSPARSE_DIRECTION_ROW} should be used.

### 3.2.6. Extended BSR Format (BSRX)

BSRX is the same as the BSR format, but the array \( \text{bsrRowPtrA} \) is separated into two parts. The first nonzero block of each row is still specified by the array \( \text{bsrRowPtrA} \), which is the same as in BSR, but the position next to the last nonzero block of each row is specified by the array \( \text{bsrEndPtrA} \). Briefly, BSRX format is simply like a 4-vector variant of BSR format.

Matrix \( \mathbf{A} \) is represented in BSRX format by the following parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>blockDim</td>
<td>(integer) Block dimension of matrix ( \mathbf{A} ).</td>
</tr>
<tr>
<td>( mb )</td>
<td>(integer) The number of block rows of ( \mathbf{A} ).</td>
</tr>
<tr>
<td>( nb )</td>
<td>(integer) The number of block columns of ( \mathbf{A} ).</td>
</tr>
<tr>
<td>( nnzb )</td>
<td>(integer) Number of nonzero blocks in ( \mathbf{A} ).</td>
</tr>
<tr>
<td>( \text{bsrValA} )</td>
<td>(pointer) Points to the data array of length ( nnzb \times \text{blockDim}^2 ) that holds all the elements of the nonzero blocks of ( \mathbf{A} ). The block elements are stored in either column-major order or row-major order.</td>
</tr>
<tr>
<td>( \text{bsrRowPtrA} )</td>
<td>(pointer) Points to the integer array of length ( mb ) that holds indices into the arrays ( \text{bsrColIndA} ) and ( \text{bsrValA} ); ( \text{bsrRowPtrA}(i) ) is the position of the first nonzero block of the ( i )th block row in ( \text{bsrColIndA} ) and ( \text{bsrValA} ).</td>
</tr>
<tr>
<td>( \text{bsrEndPtrA} )</td>
<td>(pointer) Points to the integer array of length ( mb ) that holds indices into the arrays ( \text{bsrColIndA} ) and ( \text{bsrValA} ); ( \text{bsrRowPtrA}(i) ) is the position next to the last nonzero block of the ( i )th block row in ( \text{bsrColIndA} ) and ( \text{bsrValA} ).</td>
</tr>
<tr>
<td>( \text{bsrColIndA} )</td>
<td>(pointer) Points to the integer array of length ( nnzb ) that contains the column indices of the corresponding blocks in array ( \text{bsrValA} ).</td>
</tr>
</tbody>
</table>

A simple conversion between BSR and BSRX can be done as follows. Suppose the developer has a \( 2 \times 3 \) block sparse matrix \( \mathbf{A}_b \) represented as shown.

\[
\mathbf{A}_b = \begin{bmatrix}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12}
\end{bmatrix}
\]
Assume it has this BSR format.

\[
\begin{align*}
\text{bsrValA of BSR} &= \begin{bmatrix} A_{00} & A_{01} & A_{10} & A_{11} & A_{12} \end{bmatrix} \\
\text{bsrRowPtrA of BSR} &= [0 \ 2 \ 5] \\
\text{bsrColIndA of BSR} &= [0 \ 1 \ 0 \ 1 \ 2]
\end{align*}
\]

The `bsrRowPtrA` of the BSRX format is simply the first two elements of the `bsrRowPtrA` BSR format. The `bsrEndPtrA` of BSRX format is the last two elements of the `bsrRowPtrA` of BSR format.

\[
\begin{align*}
\text{bsrRowPtrA of BSRX} &= [0 \ 2] \\
\text{bsrEndPtrA of BSRX} &= [2 \ 5]
\end{align*}
\]

The advantage of the BSRX format is that the developer can specify a submatrix in the original BSR format by modifying `bsrRowPtrA` and `bsrEndPtrA` while keeping `bsrColIndA` and `bsrValA` unchanged.

For example, to create another block matrix \( \hat{A} = \begin{bmatrix} 0 & O & O \\ O & A_{11} & O \end{bmatrix} \) that is slightly different from \( A \), the developer can keep `bsrColIndA` and `bsrValA`, but reconstruct \( \hat{A} \) by properly setting of `bsrRowPtrA` and `bsrEndPtrA`. The following 4-vector characterizes \( \hat{A} \).

\[
\begin{align*}
\text{bsrValA of } \hat{A} &= \begin{bmatrix} A_{00} & A_{01} & A_{10} & A_{11} & A_{12} \end{bmatrix} \\
\text{bsrColIndA of } \hat{A} &= [0 \ 1 \ 0 \ 1 \ 2] \\
\text{bsrRowPtrA of } \hat{A} &= [0 \ 3] \\
\text{bsrEndPtrA of } \hat{A} &= [0 \ 4]
\end{align*}
\]
4.1. Data types

The `float`, `double`, `cuComplex`, and `cuDoubleComplex` data types are supported. The first two are standard C data types, while the last two are exported from `cuComplex.h`.

4.2. `cusparseStatus_t`

This data type represents the status returned by the library functions and it can have the following values:

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_STATUS_SUCCESS</td>
<td>The operation completed successfully</td>
</tr>
<tr>
<td></td>
<td>The cuSPARSE library was not initialized. This is usually caused by the</td>
</tr>
<tr>
<td></td>
<td>lack of a prior call, an error in the CUDA Runtime API called by the</td>
</tr>
<tr>
<td></td>
<td>cuSPARSE routine, or an error in the hardware setup</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_NOT_INITIAl</td>
<td>To correct: call <code>cusparseCreate()</code> prior to the function call; and</td>
</tr>
<tr>
<td></td>
<td>check that the hardware, an appropriate version of the driver, and the</td>
</tr>
<tr>
<td></td>
<td>cuSPARSE library are correctly installed</td>
</tr>
<tr>
<td></td>
<td>The error also applies to generic APIs (Generic APIs reference) for</td>
</tr>
<tr>
<td></td>
<td>indicating a matrix/vector descriptor not initialized</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_ALLOC_FAIL</td>
<td>Resource allocation failed inside the cuSPARSE library. This is usually</td>
</tr>
<tr>
<td></td>
<td>caused by a device memory allocation (<code>cudaMalloc()</code>) or by a host memory</td>
</tr>
<tr>
<td></td>
<td>allocation failure</td>
</tr>
<tr>
<td></td>
<td>To correct: prior to the function call, deallocate previously allocated</td>
</tr>
<tr>
<td></td>
<td>memory as much as possible</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_INVALID_VAL</td>
<td>An unsupported value or parameter was passed to the function (a negative</td>
</tr>
<tr>
<td></td>
<td>vector size, for example)</td>
</tr>
<tr>
<td>Value</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_ARCH_MISMATCH</td>
<td>The function requires a feature absent from the device architecture. <strong>To correct:</strong> compile and run the application on a device with appropriate compute capability</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_EXECUTION_FAILED</td>
<td>The GPU program failed to execute. This is often caused by a launch failure of the kernel on the GPU, which can be caused by multiple reasons. <strong>To correct:</strong> check that the hardware, an appropriate version of the driver, and the cuSPARSE library are correctly installed</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_INTERNAL_ERROR</td>
<td>An internal cuSPARSE operation failed. <strong>To correct:</strong> check that the hardware, an appropriate version of the driver, and the cuSPARSE library are correctly installed. Also, check that the memory passed as a parameter to the routine is not being deallocated prior to the routine completion</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED</td>
<td>The matrix type is not supported by this function. This is usually caused by passing an invalid matrix descriptor to the function. <strong>To correct:</strong> check that the fields in cusparseMatDescr_t descrA were set correctly</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_NOT_SUPPORTED</td>
<td>The operation or data type combination is currently not supported by the function. <strong>To correct:</strong> check that the fields in cusparseMatDescr_t descrA were set correctly</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_INSUFFICIENT_RESOURCES</td>
<td>The resources for the computation, such as GPU global or shared memory, are not sufficient to complete the operation. The error can also indicate that the current computation mode (e.g. bit size of sparse matrix indices) does not allow to handle the given input</td>
</tr>
</tbody>
</table>

### 4.3. cusparseHandle_t

This is a pointer type to an opaque cuSPARSE context, which the user must initialize by calling prior to calling `cusparseCreate()` any other library function. The handle created and returned by `cusparseCreate()` must be passed to every cuSPARSE function.

### 4.4. cusparsePointerMode_t

This type indicates whether the scalar values are passed by reference on the host or device. It is important to point out that if several scalar values are passed by reference in the function call, all of them will conform to the same single pointer mode. The
pointer mode can be set and retrieved using `cusparseSetPointerMode()` and `cusparseGetPointerMode()` routines, respectively.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_POINTER_MODE_HOST</td>
<td>the scalars are passed by reference on the host.</td>
</tr>
<tr>
<td>CUSPARSE.PointerMode_DEVICE</td>
<td>the scalars are passed by reference on the device.</td>
</tr>
</tbody>
</table>

### 4.5. cusparseOperation_t

This type indicates which operations need to be performed with the sparse matrix.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_OPERATION_NON_TRANSPOSE</td>
<td>the non-transpose operation is selected.</td>
</tr>
<tr>
<td>CUSPARSE_OPERATION_TRANSPOSE</td>
<td>the transpose operation is selected.</td>
</tr>
<tr>
<td>CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE</td>
<td>the conjugate transpose operation is selected.</td>
</tr>
</tbody>
</table>

### 4.6. cusparseAction_t

This type indicates whether the operation is performed only on indices or on data and indices.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_ACTION_SYMBOLIC</td>
<td>the operation is performed only on indices.</td>
</tr>
<tr>
<td>CUSPARSE_ACTION_NUMERIC</td>
<td>the operation is performed on data and indices.</td>
</tr>
</tbody>
</table>

### 4.7. cusparseDirection_t

This type indicates whether the elements of a dense matrix should be parsed by rows or by columns (assuming column-major storage in memory of the dense matrix) in function cusparse[S|D|C|Z]nnz. Besides storage format of blocks in BSR format is also controlled by this type.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_DIRECTION_ROW</td>
<td>the matrix should be parsed by rows.</td>
</tr>
<tr>
<td>CUSPARSE_DIRECTION_COLUMN</td>
<td>the matrix should be parsed by columns.</td>
</tr>
</tbody>
</table>
4.8. cusparseMatDescr_t

This structure is used to describe the shape and properties of a matrix.

```c
typedef struct {
    cusparseMatrixType_t MatrixType;
    cusparseFillMode_t FillMode;
    cusparseDiagType_t DiagType;
    cusparseIndexBase_t IndexBase;
} cusparseMatDescr_t;
```

4.8.1. cusparseDiagType_t

This type indicates if the matrix diagonal entries are unity. The diagonal elements are always assumed to be present, but if `CUSPARSE_DIAG_TYPE_UNIT` is passed to an API routine, then the routine assumes that all diagonal entries are unity and will not read or modify those entries. Note that in this case the routine assumes the diagonal entries are equal to one, regardless of what those entries are actually set to in memory.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_DIAG_TYPE_NON_UNIT</td>
<td>the matrix diagonal has non-unit elements.</td>
</tr>
<tr>
<td>CUSPARSE_DIAG_TYPE_UNIT</td>
<td>the matrix diagonal has unit elements.</td>
</tr>
</tbody>
</table>

4.8.2. cusparseFillMode_t

This type indicates if the lower or upper part of a matrix is stored in sparse storage.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_FILL_MODE_LOWER</td>
<td>the lower triangular part is stored.</td>
</tr>
<tr>
<td>CUSPARSE_FILL_MODE_UPPER</td>
<td>the upper triangular part is stored.</td>
</tr>
</tbody>
</table>

4.8.3. cusparseIndexBase_t

This type indicates if the base of the matrix indices is zero or one.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_INDEX_BASE_ZERO</td>
<td>the base index is zero.</td>
</tr>
<tr>
<td>CUSPARSE_INDEX_BASE_ONE</td>
<td>the base index is one.</td>
</tr>
</tbody>
</table>

4.8.4. cusparseMatrixType_t

This type indicates the type of matrix stored in sparse storage. Notice that for symmetric, Hermitian and triangular matrices only their lower or upper part is assumed to be stored.

The whole idea of matrix type and fill mode is to keep minimum storage for symmetric/Hermitian matrix, and also to take advantage of symmetric property on SpMV.
Sparse Matrix Vector multiplication). To compute \( y = A \times x \) when \( A \) is symmetric and only lower triangular part is stored, two steps are needed. First step is to compute \( y = (L + D) \times x \) and second step is to compute \( y = L^T \times x + y \). Given the fact that the transpose operation \( y = L^T \times x \) is 10x slower than non-transpose version \( y = L \times x \), the symmetric property does not show up any performance gain. It is better for the user to extend the symmetric matrix to a general matrix and apply \( y = A \times x \) with matrix type CUSPARSE_MATRIX_TYPE_GENERAL.

In general, SpMV, preconditioners (incomplete Cholesky or incomplete LU) and triangular solver are combined together in iterative solvers, for example PCG and GMRES. If the user always uses general matrix (instead of symmetric matrix), there is no need to support other than general matrix in preconditioners. Therefore the new routines, \([bsr|csr]sv2\) (triangular solver), \([bsr|csr]ilu02\) (incomplete LU) and \([bsr|csr]ic02\) (incomplete Cholesky), only support matrix type CUSPARSE_MATRIX_TYPE_GENERAL.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_MATRIX_TYPE_GENERAL</td>
<td>the matrix is general.</td>
</tr>
<tr>
<td>CUSPARSE_MATRIX_TYPE_SYMMETRIC</td>
<td>the matrix is symmetric.</td>
</tr>
<tr>
<td>CUSPARSE_MATRIX_TYPE_HERMITIAN</td>
<td>the matrix is Hermitian.</td>
</tr>
<tr>
<td>CUSPARSE_MATRIX_TYPE_TRIANGULAR</td>
<td>the matrix is triangular.</td>
</tr>
</tbody>
</table>

4.9. cusparseAlgMode_t

This is type for algorithm parameter to cusparseCsrmvEx() and cusparseCsrmvEx_bufferSize() functions.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_ALG_MERGE_PATH</td>
<td>Use load-balancing algorithm that suits better for irregular nonzero-patterns.</td>
</tr>
</tbody>
</table>

4.10. cusparseColorInfo_t

This is a pointer type to an opaque structure holding the information used in csrcolor().

4.11. cusparseSolvePolicy_t

This type indicates whether level information is generated and used in csrsv2, csric02, csrilu02, bsrsv2, bsric02 and bsrilu02.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_SOLVE_POLICY_NO_LEVEL</td>
<td>no level information is generated and used.</td>
</tr>
<tr>
<td>Value</td>
<td>Meaning</td>
</tr>
<tr>
<td>--------------------------------------------</td>
<td>----------------------------------------------</td>
</tr>
<tr>
<td>CUSPARSE_SOLVE_POLICY_USE_LEVEL</td>
<td>generate and use level information.</td>
</tr>
</tbody>
</table>

4.12. bsric02Info_t
This is a pointer type to an opaque structure holding the information used in bsric02_bufferSize(), bsric02_analysis(), and bsric02().

4.13. bsrлу02Info_t
This is a pointer type to an opaque structure holding the information used in bsrлу02_bufferSize(), bsrлу02_analysis(), and bsrлу02().

4.14. bsrsm2Info_t
This is a pointer type to an opaque structure holding the information used in bsrsm2_bufferSize(), bsrsm2_analysis(), and bsrsm2_solve().

4.15. bsrsv2Info_t
This is a pointer type to an opaque structure holding the information used in bsrsv2_bufferSize(), bsrsv2_analysis(), and bsrsv2_solve().

4.16. csrgemm2Info_t
This is a pointer type to an opaque structure holding the information used in csrgemm2_bufferSizeExt(), and csrgemm2().

4.17. csric02Info_t
This is a pointer type to an opaque structure holding the information used in csric02_bufferSize(), csric02_analysis(), and csric02().

4.18. csrilu02Info_t
This is a pointer type to an opaque structure holding the information used in csrilu02_bufferSize(), csrilu02_analysis(), and csrilu02().
4.19. csrsm2Info_t
This is a pointer type to an opaque structure holding the information used in csrsm2_bufferSize(), csrsm2_analysis(), and csrsm2_solve().

4.20. csrsv2Info_t
This is a pointer type to an opaque structure holding the information used in csrsv2_bufferSize(), csrsv2_analysis(), and csrsv2_solve().
The cuSPARSE functions for managing the library are described in this section.

5.1. cusparseCreate()

```
cusparseStatus_t
cusparseCreate(cusparseHandle_t *handle)
```

This function initializes the cuSPARSE library and creates a handle on the cuSPARSE context. It must be called before any other cuSPARSE API function is invoked. It allocates hardware resources necessary for accessing the GPU.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>IN</td>
<td>The pointer to the handle to the cuSPARSE context</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.

5.2. cusparseDestroy()

```
cusparseStatus_t
cusparseDestroy(cusparseHandle_t handle)
```

This function releases CPU-side resources used by the cuSPARSE library. The release of GPU-side resources may be deferred until the application shuts down.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>IN</td>
<td>The handle to the cuSPARSE context</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
5.3. cusparseGetErrorName()

const char*
cusparseGetErrorString(cusparseStatus_t status)

The function returns the string representation of an error code enum name. If the error code is not recognized, "unrecognized error code" is returned.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>status</td>
<td>IN</td>
<td>Error code to convert to string</td>
</tr>
<tr>
<td>const char*</td>
<td>OUT</td>
<td>Pointer to a NULL-terminated string</td>
</tr>
</tbody>
</table>

5.4. cusparseGetErrorString()

const char*
cusparseGetErrorString(cusparseStatus_t status)

Returns the description string for an error code. If the error code is not recognized, "unrecognized error code" is returned.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>status</td>
<td>IN</td>
<td>Error code to convert to string</td>
</tr>
<tr>
<td>const char*</td>
<td>OUT</td>
<td>Pointer to a NULL-terminated string</td>
</tr>
</tbody>
</table>

5.5. cusparseGetProperty()

cusparseStatus_t
cusparseGetProperty(libraryPropertyType type,
int* value)

The function returns the value of the requested property. Refer to libraryPropertyType for supported types.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>type</td>
<td>IN</td>
<td>Requested property</td>
</tr>
<tr>
<td>value</td>
<td>OUT</td>
<td>Value of the requested property</td>
</tr>
</tbody>
</table>

libraryPropertyType (defined in library_types.h):

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAJOR_VERSION</td>
<td>Enumerator to query the major version</td>
</tr>
<tr>
<td>MINOR_VERSION</td>
<td>Enumerator to query the minor version</td>
</tr>
<tr>
<td>PATCH_LEVEL</td>
<td>Number to identify the patch level</td>
</tr>
</tbody>
</table>
See `cusparseStatus_t` for the description of the return status

### 5.6. `cusparseGetVersion()`

```c
cusparseStatus_t
 cusparseGetVersion(cusparseHandle_t handle,
                    int* version)
```

This function returns the version number of the cuSPARSE library.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>IN</td>
<td>cuSPARSE handle</td>
</tr>
<tr>
<td>version</td>
<td>OUT</td>
<td>The version number of the library</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

### 5.7. `cusparseGetPointerMode()`

```c
cusparseStatus_t
 cusparseGetPointerMode(cusparseHandle_t handle,
                        cusparsePointerMode_t *mode)
```

This function obtains the pointer mode used by the cuSPARSE library. Please see the section on the `cusparsePointerMode_t` type for more details.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>IN</td>
<td>The handle to the cuSPARSE context</td>
</tr>
<tr>
<td>mode</td>
<td>OUT</td>
<td>One of the enumerated pointer mode types</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

### 5.8. `cusparseSetPointerMode()`

```c
cusparseStatus_t
 cusparseSetPointerMode(cusparseHandle_t handle,
                        cusparsePointerMode_t mode)
```

This function sets the pointer mode used by the cuSPARSE library. The `default` is for the values to be passed by reference on the host. Please see the section on the `cublasPointerMode_t` type for more details.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>IN</td>
<td>The handle to the cuSPARSE context</td>
</tr>
<tr>
<td>mode</td>
<td>IN</td>
<td>One of the enumerated pointer mode types</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status
5.9. cusparseGetStream()

cusparseStatus_t

cusparseGetStream(cusparseHandle_t handle, cudaStream_t *streamId)

This function gets the cuSPARSE library stream, which is being used to to execute all calls to the cuSPARSE library functions. If the cuSPARSE library stream is not set, all kernels use the default NULL stream.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>IN</td>
<td>The handle to the cuSPARSE context</td>
</tr>
<tr>
<td>streamId</td>
<td>OUT</td>
<td>The stream used by the library</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

5.10. cusparseSetStream()

cusparseStatus_t

cusparseSetStream(cusparseHandle_t handle, cudaStream_t streamId)

This function sets the stream to be used by the cuSPARSE library to execute its routines.

<table>
<thead>
<tr>
<th>Param.</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>IN</td>
<td>The handle to the cuSPARSE context</td>
</tr>
<tr>
<td>streamId</td>
<td>IN</td>
<td>The stream to be used by the library</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status
Chapter 6.
CUSPARSE HELPER FUNCTION REFERENCE

The cuSPARSE helper functions are described in this section.

6.1. cusparseCreateColorInfo()

cusparseStatus_t
cusparseCreateColorInfo(cusparseColorInfo_t* info)

This function creates and initializes the cusparseColorInfo_t structure to default values.
Input

| info         | the pointer to the cusparseColorInfo_t structure |

See cusparseStatus_t for the description of the return status.

6.2. cusparseCreateMatDescr()

cusparseStatus_t
cusparseCreateMatDescr(cusparseMatDescr_t *descrA)

This function initializes the matrix descriptor. It sets the fields MatrixType and IndexBase to the default values CUSPARSE_MATRIX_TYPE_GENERAL and CUSPARSE_INDEX_BASE_ZERO, respectively, while leaving other fields uninitialized.
Input

| descrA | the pointer to the matrix descriptor. |

See cusparseStatus_t for the description of the return status.
6.3. cusparseDestroyColorInfo()

```c
cusparseStatus_t
    cusparseDestroyColorInfo(cusparseColorInfo_t info)
```

This function destroys and releases any memory required by the structure.

**Input**

| info | the pointer to the structure of `csrColor()` |

See `cusparseStatus_t` for the description of the return status.

6.4. cusparseDestroyMatDescr()

```c
cusparseStatus_t
    cusparseDestroyMatDescr(cusparseMatDescr_t descrA)
```

This function releases the memory allocated for the matrix descriptor.

**Input**

| descrA | the matrix descriptor. |

See `cusparseStatus_t` for the description of the return status.

6.5. cusparseGetMatDiagType()

```c
cusparseDiagType_t
    cusparseGetMatDiagType(const cusparseMatDescr_t descrA)
```

This function returns the **DiagType** field of the matrix descriptor `descrA`.

**Input**

| descrA | the matrix descriptor. |

**Returned**

| | One of the enumerated diagType types. |

6.6. cusparseGetMatFillMode()

```c
cusparseFillMode_t
    cusparseGetMatFillMode(const cusparseMatDescr_t descrA)
```

This function returns the **FillMode** field of the matrix descriptor `descrA`.

**Input**

| descrA | the matrix descriptor. |
6.7. `cusparseGetMatIndexBase()`

```c
CUSPARSE_INDEX_BASE_TYPE
cusparseGetMatIndexBase(const cusparseMatDescr_t descrA)
```

This function returns the `IndexBase` field of the matrix descriptor `descrA`.

**Input**
- `descrA`: the matrix descriptor.

**Returned**
- One of the enumerated `IndexBase` types.

6.8. `cusparseGetMatType()`

```c
CUSPARSE_MATRIX_TYPE
cusparseGetMatType(const cusparseMatDescr_t descrA)
```

This function returns the `MatrixType` field of the matrix descriptor `descrA`.

**Input**
- `descrA`: the matrix descriptor.

**Returned**
- One of the enumerated `MatrixType` types.

6.9. `cusparseSetMatDiagType()`

```c
CUSPARSE_STATUS_T
cusparseSetMatDiagType(cusparseMatDescr_t descrA,
                        cusparseDiagType_t diagType)
```

This function sets the `DiagType` field of the matrix descriptor `descrA`.

**Input**
- `diagType`: One of the enumerated `DiagType` types.
- `descrA`: the matrix descriptor.

**Output**
- `descrA`: the matrix descriptor.

See `cusparseStatus_t` for the description of the return status.
6.10. `cusparseSetMatFillMode()`

```c
 cusparseStatus_t
cusparseSetMatFillMode(cusparseMatDescr_t descrA,
            cusparseFillMode_t fillMode)
```

This function sets the `FillMode` field of the matrix descriptor `descrA`.

**Input**

| fillMode | One of the enumerated fillMode types. |

**Output**

| descrA | the matrix descriptor. |

See `cusparseStatus_t` for the description of the return status.

6.11. `cusparseSetMatIndexBase()`

```c
 cusparseStatus_t
cusparseSetMatIndexBase(cusparseMatDescr_t descrA,
               cusparseIndexBase_t base)
```

This function sets the `IndexBase` field of the matrix descriptor `descrA`.

**Input**

| base | One of the enumerated indexBase types. |

**Output**

| descrA | the matrix descriptor. |

See `cusparseStatus_t` for the description of the return status.

6.12. `cusparseSetMatType()`

```c
 cusparseStatus_t
cusparseSetMatType(cusparseMatDescr_t descrA,
               cusparseMatrixType_t type)
```

This function sets the `MatrixType` field of the matrix descriptor `descrA`.

**Input**

| type | One of the enumerated matrix types. |

**Output**

| descrA | the matrix descriptor. |

See `cusparseStatus_t` for the description of the return status.
6.13. cusparseCreateCsrsv2Info()

cusparseStatus_t
cusparseCreateCsrsv2Info(csrsv2Info_t *info);

This function creates and initializes the solve and analysis structure of csrsv2 to default values.

Input

| info          | the pointer to the solve and analysis structure of csrsv2. |

See cusparseStatus_t for the description of the return status

6.14. cusparseDestroyCsrsv2Info()

cusparseStatus_t
cusparseDestroyCsrsv2Info(csrsv2Info_t info);

This function destroys and releases any memory required by the structure.

Input

| info          | the solve (csrsv2_solve) and analysis (csrsv2_analysis) structure. |

See cusparseStatus_t for the description of the return status

6.15. cusparseCreateCsrsm2Info()

cusparseStatus_t
cusparseCreateCsrsm2Info(csrsm2Info_t *info);

This function creates and initializes the solve and analysis structure of csrsm2 to default values.

Input

| info          | the pointer to the solve and analysis structure of csrsm2. |

See cusparseStatus_t for the description of the return status

6.16. cusparseDestroyCsrsm2Info()

cusparseStatus_t
cusparseDestroyCsrsm2Info(csrsm2Info_t info);
This function destroys and releases any memory required by the structure.

**Input**

<table>
<thead>
<tr>
<th>info</th>
</tr>
</thead>
<tbody>
<tr>
<td>the solve (csrsm2_solve) and analysis (csrsm2_analysis) structure.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

### 6.17. cusparseCreateCsric02Info()

```c
 cusparseStatus_t
cusparseCreateCsric02Info(csric02Info_t *info);
```

This function creates and initializes the solve and analysis structure of incomplete Cholesky to *default* values.

**Input**

<table>
<thead>
<tr>
<th>info</th>
</tr>
</thead>
<tbody>
<tr>
<td>the pointer to the solve and analysis structure of incomplete Cholesky.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

### 6.18. cusparseDestroyCsric02Info()

```c
 cusparseStatus_t
cusparseDestroyCsric02Info(csric02Info_t info);
```

This function destroys and releases any memory required by the structure.

**Input**

<table>
<thead>
<tr>
<th>info</th>
</tr>
</thead>
<tbody>
<tr>
<td>the solve (csric02_solve) and analysis (csric02_analysis) structure.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

### 6.19. cusparseCreateCsrilu02Info()

```c
 cusparseStatus_t
cusparseCreateCsrilu02Info(csrilu02Info_t *info);
```

This function creates and initializes the solve and analysis structure of incomplete LU to *default* values.

**Input**

<table>
<thead>
<tr>
<th>info</th>
</tr>
</thead>
<tbody>
<tr>
<td>the pointer to the solve and analysis structure of incomplete LU.</td>
</tr>
</tbody>
</table>
6.20. cusparseDestroyCsrilu02Info()

```
cusparseStatus_t
cusparseDestroyCsrilu02Info(csrilu02Info_t info);
```

This function destroys and releases any memory required by the structure.

**Input**

| info | the solve (csrilu02_solve) and analysis (csrilu02_analysis) structure. |

See `cusparseStatus_t` for the description of the return status

6.21. cusparseCreateBsrsv2Info()

```
cusparseStatus_t
cusparseCreateBsrsv2Info(bsrsv2Info_t *info);
```

This function creates and initializes the solve and analysis structure of bsrsv2 to `default` values.

**Input**

| info | the pointer to the solve and analysis structure of bsrsv2. |

See `cusparseStatus_t` for the description of the return status

6.22. cusparseDestroyBsrsv2Info()

```
cusparseStatus_t
cusparseDestroyBsrsv2Info(bsrsv2Info_t info);
```

This function destroys and releases any memory required by the structure.

**Input**

| info | the solve (bsrsv2_solve) and analysis (bsrsv2_analysis) structure. |

See `cusparseStatus_t` for the description of the return status
6.23. cusparseCreateBsrsm2Info()

cusparseStatus_t
cusparseCreateBsrsm2Info(bsrsm2Info_t *info);

This function creates and initializes the solve and analysis structure of bsrsm2 to default values.

Input

| info               | the pointer to the solve and analysis structure of bsrsm2. |

See cusparseStatus_t for the description of the return status

6.24. cusparseDestroyBsrsm2Info()

cusparseStatus_t
cusparseDestroyBsrsm2Info(bsrsm2Info_t info);

This function destroys and releases any memory required by the structure.

Input

| info               | the solve (bsrsm2_solve) and analysis (bsrsm2_analysis) structure. |

See cusparseStatus_t for the description of the return status

6.25. cusparseCreateBsric02Info()

cusparseStatus_t
cusparseCreateBsric02Info(bsric02Info_t *info);

This function creates and initializes the solve and analysis structure of block incomplete Cholesky to default values.

Input

| info               | the pointer to the solve and analysis structure of block incomplete Cholesky. |

See cusparseStatus_t for the description of the return status

6.26. cusparseDestroyBsric02Info()

cusparseStatus_t
cusparseDestroyBsric02Info(bsric02Info_t info);
This function destroys and releases any memory required by the structure.

**Input**

| info                  | the solve (bsric02_solve) and analysis (bsric02_analysis) structure. |

See [cusparseStatus_t](#) for the description of the return status.

### 6.27. cusparseCreateBsrilu02Info()

```c
 cusparseStatus_t cusparseCreateBsrilu02Info(bsrilu02Info_t *info);
```

This function creates and initializes the solve and analysis structure of block incomplete LU to *default* values.

**Input**

| info                  | the pointer to the solve and analysis structure of block incomplete LU. |

See [cusparseStatus_t](#) for the description of the return status.

### 6.28. cusparseDestroyBsrilu02Info()

```c
 cusparseStatus_t cusparseDestroyBsrilu02Info(bsrilu02Info_t info);
```

This function destroys and releases any memory required by the structure.

**Input**

| info                  | the solve (bsrilu02_solve) and analysis (bsrilu02_analysis) structure. |

See [cusparseStatus_t](#) for the description of the return status.

### 6.29. cusparseCreateCsrgemm2Info()

```c
 cusparseStatus_t cusparseCreateCsrgemm2Info(csrgemm2Info_t *info);
```

This function creates and initializes analysis structure of general sparse matrix-matrix multiplication.

**Input**

| info                  | the pointer to the analysis structure of general sparse matrix-matrix multiplication. |
See `cusparseStatus_t` for the description of the return status

### 6.30. `cusparseDestroyCsrGemm2Info()`

```c
 cusparseStatus_t
cusparseDestroyCsrGemm2Info(csrGemm2Info_t info);
```

This function destroys and releases any memory required by the structure.

**Input**

| info | opaque structure of csrGemm2. |

See `cusparseStatus_t` for the description of the return status

### 6.31. `cusparseCreatePruneInfo()`

```c
 cusparseStatus_t
cusparseCreatePruneInfo(pruneInfo_t *info);
```

This function creates and initializes structure of `prune` to default values.

**Input**

| info | the pointer to the structure of prune. |

See `cusparseStatus_t` for the description of the return status

### 6.32. `cusparseDestroyPruneInfo()`

```c
 cusparseStatus_t
cusparseDestroyPruneInfo(pruneInfo_t info);
```

This function destroys and releases any memory required by the structure.

**Input**

| info | the structure of prune. |

See `cusparseStatus_t` for the description of the return status
This chapter describes sparse linear algebra functions that perform operations between dense and sparse vectors.
7.1. cusparse<t>axpyi() [DEPRECATED]

[DEPRECATED] use cusparseAxpby() instead. The routine will be removed in the next major release.

This function multiplies the vector \( x \) in sparse format by the constant \( \alpha \) and adds the result to the vector \( y \) in dense format. This operation can be written as

\[
y = y + \alpha x
\]

In other words,

\[
y[xInd[i]-idxBase] = y[xInd[i]-idxBase] + alpha*xVal[i]
\]

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input
### 7.2. `cusparse<t>gthr()` [DEPRECATED]

**[[DEPRECATED]] use `cusparseGather()` instead. The routine will be removed in the next major release**

```c
cusparseStatus_t
cusparseSgthr(cusparseHandle_t handle,  
              int nnz,  
              const float* y,  
              float* xVal,  
              const int* xInd,  
              cusparseIndexBase_t idxBase)

cusparseStatus_t
cusparseDgthr(cusparseHandle_t handle,  
              int nnz,  
              const double* y,  
              double* xVal,  
              const int* xInd,  
              cusparseIndexBase_t idxBase)

cusparseStatus_t
cusparseCgthr(cusparseHandle_t handle,  
              int nnz,  
              const cuComplex* y,  
              cuComplex* xVal,  
              const int* xInd,  
              cusparseIndexBase_t idxBase)

cusparseStatus_t
cusparseZgthr(cusparseHandle_t handle,  
              int nnz,  
              const cuDoubleComplex* y,  
              cuDoubleComplex* xVal,  
              const int* xInd,  
              cusparseIndexBase_t idxBase)
```
This function gathers the elements of the vector \( y \) listed in the index array \( x\text{Ind} \) into the data array \( x\text{Val} \).

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>nnz</td>
<td>number of elements in vector ( x ).</td>
</tr>
<tr>
<td>( y )</td>
<td>(&lt;\text{type}&gt; ) vector in dense format (of size ( \geq \max(x\text{Ind})-\text{idxBase}+1 )).</td>
</tr>
<tr>
<td>( x\text{Ind} )</td>
<td>integer vector with ( nnz ) indices of the nonzero values of vector ( x ).</td>
</tr>
<tr>
<td>idxBase</td>
<td>CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x\text{Val} )</td>
<td>(&lt;\text{type}&gt; ) vector with ( nnz ) nonzero values that were gathered from vector ( y ) (that is unchanged if ( nnz == 0 )).</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
7.3. cusparse<t>gthrz() [DEPRECATED]

[[DEPRECATED]] use cusparseGather() instead. The routine will be removed in the next major release.

The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

This function gathers the elements of the vector \( y \) listed in the index array \( x\text{Ind} \) into the data array \( x\text{Val} \). Also, it zeros out the gathered elements in the vector \( y \).

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>nnz</td>
<td>number of elements in vector ( x ).</td>
</tr>
<tr>
<td>( y )</td>
<td>(&lt;\text{type}&gt; ) vector in dense format (of (&lt;\text{type}&gt; ) ( \times ) max(( x\text{Ind} ) - ( \text{idxBase} ) + 1)).</td>
</tr>
<tr>
<td>( x\text{Ind} )</td>
<td>integer vector with ( nnz ) indices of the nonzero values of vector ( x ).</td>
</tr>
</tbody>
</table>
idxBase | CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.

**Output**

<table>
<thead>
<tr>
<th>xVal</th>
<th>&lt;type&gt; vector with nnz nonzero values that were gathered from vector y (that is unchanged if nnz == 0).</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>&lt;type&gt; vector in dense format with elements indexed by xInd set to zero (it is unchanged if nnz == 0).</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status.

### 7.4. cusparse<t>roti() [DEPRECATED]

[[DEPRECATED]] use cusparseRot() instead. The routine will be removed in the next major release.

```c
cusparseStatus_t cusparseSroti(cusparseHandle_t handle,
                               int nnz,
                               float* xVal,
                               const int* xInd,
                               float* y,
                               const float* c,
                               const float* s,
                               cusparseIndexBase_t idxBase)

cusparseStatus_t cusparseDroti(cusparseHandle_t handle,
                               int nnz,
                               double* xVal,
                               const int* xInd,
                               double* y,
                               const double* c,
                               const double* s,
                               cusparseIndexBase_t idxBase)
```

This function applies the Givens rotation matrix

\[
G = \begin{pmatrix}
    c & s \\
    -s & c
\end{pmatrix}
\]

to sparse \( x \) and dense \( y \) vectors. In other words,

for \( i=0 \) to \( \text{nnz}-1 \)

- \( y[xInd[i]-idxBase] = c * y[xInd[i]-idxBase] - s*xVal[i] \)
- \( x[i] = c * xVal[i] + s * y[xInd[i]-idxBase] \)

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>nnz</td>
<td>number of elements in vector $x$.</td>
</tr>
<tr>
<td>xVal</td>
<td>&lt;type&gt; vector with $nnz$ nonzero values of vector $x$.</td>
</tr>
<tr>
<td>xInd</td>
<td>integer vector with $nnz$ indices of the nonzero values of vector $x$.</td>
</tr>
<tr>
<td>y</td>
<td>&lt;type&gt; vector in dense format.</td>
</tr>
<tr>
<td>c</td>
<td>cosine element of the rotation matrix.</td>
</tr>
<tr>
<td>s</td>
<td>sine element of the rotation matrix.</td>
</tr>
<tr>
<td>idxBase</td>
<td>CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
</tbody>
</table>

Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>xVal</td>
<td>&lt;type&gt; updated vector in sparse format (that is unchanged if $nnz == 0$).</td>
</tr>
<tr>
<td>y</td>
<td>&lt;type&gt; updated vector in dense format (that is unchanged if $nnz == 0$).</td>
</tr>
</tbody>
</table>

See **cusparseStatus_t** for the description of the return status.
7.5. cusparse<t>sctr() [DEPRECATED]

[[DEPRECATED]] use cusparseScatter() instead. The routine will be removed in the next major release

```
cusparseStatus_t
cusparseSsctr(cusparseHandle_t handle,
              int nnz,
              const float* xVal,
              const int* xInd,
              float* y,
              cusparseIndexBase_t idxBase)

cusparseStatus_t
cusparseDsctr(cusparseHandle_t handle,
              int nnz,
              const double* xVal,
              const int* xInd,
              double* y,
              cusparseIndexBase_t idxBase)

cusparseStatus_t
cusparseCsctr(cusparseHandle_t handle,
              int nnz,
              const cuComplex* xVal,
              const int* xInd,
              cuComplex* y,
              cusparseIndexBase_t idxBase)

cusparseStatus_t
cusparseZsctr(cusparseHandle_t handle,
              int nnz,
              const cuDoubleComplex* xVal,
              const int* xInd,
              cuDoubleComplex* y,
              cusparseIndexBase_t idxBase)
```

This function scatters the elements of the vector \( x \) in sparse format into the vector \( y \) in dense format. It modifies only the elements of \( y \) whose indices are listed in the array \( xInd \).

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>nnz</code></td>
<td>number of elements in vector ( x ).</td>
</tr>
<tr>
<td><code>xVal</code></td>
<td>&lt;type&gt; vector with ( nnz ) nonzero values of vector ( x ).</td>
</tr>
<tr>
<td><code>xInd</code></td>
<td>integer vector with ( nnz ) indices of the nonzero values of vector ( x ).</td>
</tr>
<tr>
<td><strong>y</strong></td>
<td><code>&lt;type&gt;</code> dense vector (of size $\geq \max(x\text{Ind}) - idx\text{Base} + 1$).</td>
</tr>
<tr>
<td>-------</td>
<td>------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>idxBase</strong></td>
<td>CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
</tbody>
</table>

**Output**

| **y** | `<type>` vector with `nnz` nonzero values that were scattered from vector `x` (that is unchanged if `nnz == 0`). |

See `cusparseStatus_t` for the description of the return status.
Chapter 8.
CUSPARSE LEVEL 2 FUNCTION REFERENCE

This chapter describes the sparse linear algebra functions that perform operations between sparse matrices and dense vectors.

In particular, the solution of sparse triangular linear systems is implemented in two phases. First, during the analysis phase, the sparse triangular matrix is analyzed to determine the dependencies between its elements by calling the appropriate `csrsrv2_analysis()` function. The analysis is specific to the sparsity pattern of the given matrix and to the selected `cusparseOperation_t` type. The information from the analysis phase is stored in the parameter of type `csrsrv2Info_t` that has been initialized previously with a call to `cusparseCreateCsrsv2Info()`.

Second, during the solve phase, the given sparse triangular linear system is solved using the information stored in the `csrsrv2Info_t` parameter by calling the appropriate `csrsrv2_solve()` function. The solve phase may be performed multiple times with different right-hand sides, while the analysis phase needs to be performed only once. This is especially useful when a sparse triangular linear system must be solved for a set of different right-hand sides one at a time, while its coefficient matrix remains the same.

Finally, once all the solves have completed, the opaque data structure pointed to by the `csrsrv2Info_t` parameter can be released by calling `cusparseDestroyCsrsv2Info()`.
8.1. cusparse<t>bsrmv()

cusparseStatus_t cusparseSbsrmv(cusparseHandle_t handle,
cusparseDirection_t dir,
cusparseOperation_t trans,
int mb,
int nb,
int nnzb,
const float* alpha,
const cusparseMatDescr_t descr,
const float* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int blockDim,
const float* x,
const float* beta,
float* y)

cusparseStatus_t cusparseDbsrmv(cusparseHandle_t handle,
cusparseDirection_t dir,
cusparseOperation_t trans,
int mb,
int nb,
int nnzb,
const double* alpha,
const cusparseMatDescr_t descr,
const double* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int blockDim,
const double* x,
const double* beta,
double* y)

cusparseStatus_t cusparseCbsrmv(cusparseHandle_t handle,
cusparseDirection_t dir,
cusparseOperation_t trans,
int mb,
int nb,
int nnzb,
const cuComplex* alpha,
const cusparseMatDescr_t descr,
const cuComplex* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int blockDim,
const cuComplex* x,
const cuComplex* beta,
cuComplex* y)

cusparseStatus_t cusparseZbsrmv(cusparseHandle_t handle,
cusparseDirection_t dir,
cusparseOperation_t trans,
int mb,
int nb,
int nnzb,
const cuDoubleComplex* alpha,
const cusparseMatDescr_t descr,
const cuDoubleComplex* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
This function performs the matrix-vector operation

\[ y = \alpha \cdot \text{op}(A) \cdot x + \beta \cdot y \]

where \( A \) is an \( (mb \times blockDim) \times (nb \times blockDim) \) sparse matrix that is defined in BSR storage format by the three arrays \( \text{bsrVal}, \text{bsrRowPtr}, \text{and bsrColInd} \); \( x \) and \( y \) are vectors; \( \alpha \) and \( \beta \) are scalars; and

\[
\text{op}(A) = \begin{cases} 
A & \text{if trans} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if trans} = \text{CUSPARSE_OPERATION_TRANSPOSE} \\
A^H & \text{if trans} = \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE}
\end{cases}
\]

\text{bsrmv()} has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Several comments on \text{bsrmv()}:

- Only \texttt{blockDim} > 1 is supported
- Only \texttt{CUSPARSE_OPERATION_NON_TRANSPOSE} is supported, that is

\[
y = \alpha \cdot A \cdot x + \beta \cdot y
\]

- Only \texttt{CUSPARSE_MATRIX_TYPE_GENERAL} is supported.
- The size of vector \( x \) should be \( (nb \times blockDim) \) at least, and the size of vector \( y \) should be \( (mb \times blockDim) \) at least; otherwise, the kernel may return \texttt{CUSPARSE_STATUS_EXECUTION_FAILED} because of an out-of-bounds array.

For example, suppose the user has a CSR format and wants to try \text{bsrmv()}, the following code demonstrates how to use \text{csr2bsr()} conversion and \text{bsrmv()} multiplication in single precision.

```c
// Suppose that A is m x n sparse matrix represented by CSR format,
// hx is a host vector of size n, and hy is also a host vector of size m.
// m and n are not multiple of blockDim.
// step 1: transform CSR to BSR with column-major order
int base, nnz;
int nnzb;
cusparseDirection_t dirA = CUSPARSE_DIRECTION_COLUMN;
int mb = (m + blockDim-1)/blockDim;
int nb = (n + blockDim-1)/blockDim;
cudaMalloc((void**)&bsrRowPtrC, sizeof(int) *(mb+1));
cusparseXcsr2bsrNnz(handle, dirA, m, n,
    descrA, csrRowPtrA, csrColIndA, blockDim,
    descrC, bsrRowPtrC, &nnzb);
cudaMalloc((void**)&bsrColIndC, sizeof(int)*(nnzb));
cudaMalloc((void**)&bsrValC, sizeof(float)*(blockDim*blockDim)*nnzb);
cusparseScsr2bsr(handle, dirA, m, n,
    descrA, csrRowPtrA, csrColIndA, blockDim,
    descrC, bsrRowPtrC, bsrColIndC, bsrValC);
// step 2: allocate vector x and vector y large enough for bsrmv
cudaMalloc((void**)&x, sizeof(float)*(nb*blockDim));
cudaMalloc((void**)&y, sizeof(float)*(mb*blockDim));
cudaMemcpy(x, hx, sizeof(float)*n, cudaMemcpyHostToDevice);
cudaMemcpy(y, hy, sizeof(float)*m, cudaMemcpyHostToDevice);
// step 3: perform bsrmv
cusparseSbsrmv(handle, dirA, transA, mb, nb, nnzb, &alpha,
    descrC, bsrValC, bsrRowPtrC, bsrColIndC, blockDim, x, &beta, y);
```
### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>dir</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>trans</td>
<td>the operation ( op(A) ). Only CUSPARSE_OPERATION_NON_TRANSPOSE is supported.</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows of matrix ( A ).</td>
</tr>
<tr>
<td>nb</td>
<td>number of block columns of matrix ( A ).</td>
</tr>
<tr>
<td>nnzb</td>
<td>number of nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td>alpha</td>
<td>(&lt;\text{type}&gt;) scalar used for multiplication.</td>
</tr>
<tr>
<td>descr</td>
<td>the descriptor of matrix ( A ). The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>bsrVal</td>
<td>(&lt;\text{type}&gt;) array of ( \text{nnz} = \text{csrRowPtrA}(mb) - \text{csrRowPtrA}(0) ) nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td>bsrRowPtr</td>
<td>integer array of ( mb + 1 ) elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>bsrColInd</td>
<td>integer array of ( \text{nnz} = \text{csrRowPtrA}(mb) - \text{csrRowPtrA}(0) ) column indices of the nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix ( A ), larger than zero.</td>
</tr>
<tr>
<td>x</td>
<td>(&lt;\text{type}&gt;) vector of ( nb \times blockDim ) elements.</td>
</tr>
<tr>
<td>beta</td>
<td>(&lt;\text{type}&gt;) scalar used for multiplication. If ( \beta ) is zero, ( y ) does not have to be a valid input.</td>
</tr>
<tr>
<td>y</td>
<td>(&lt;\text{type}&gt;) vector of ( mb + blockDim ) elements.</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>(&lt;\text{type}&gt;) updated vector.</td>
</tr>
</tbody>
</table>

See \texttt{cusparseStatus_t} for the description of the return status.
8.2. cusparse<t>bsrvmv()

cusparseStatus_t
cusparseSbsrvmv(cusparseHandle_t handle,
cusparseDirection_t dir,
cusparseOperation_t trans,
int sizeOfMask,
int mb,
int nb,
int nnzb,
const float* alpha,
const cusparseMatDescr_t descr,
const float* bsrVal,
const int* bsrMaskPtr,
const int* bsrRowPtr,
const int* bsrEndPtr,
const int* bsrColInd,
int blockDim,
const float* x,
const float* beta,
float* y)

cusparseStatus_t
cusparseDbsrvmv(cusparseHandle_t handle,
cusparseDirection_t dir,
cusparseOperation_t trans,
int sizeOfMask,
int mb,
int nb,
int nnzb,
const double* alpha,
const cusparseMatDescr_t descr,
const double* bsrVal,
const int* bsrMaskPtr,
const int* bsrRowPtr,
const int* bsrEndPtr,
const int* bsrColInd,
int blockDim,
const double* x,
const double* beta,
double* y)

cusparseStatus_t
cusparseCbsrvmv(cusparseHandle_t handle,
cusparseDirection_t dir,
cusparseOperation_t trans,
int sizeOfMask,
int mb,
int nb,
int nnzb,
const cuComplex* alpha,
const cusparseMatDescr_t descr,
const cuComplex* bsrVal,
const int* bsrMaskPtr,
const int* bsrRowPtr,
const int* bsrEndPtr,
const int* bsrColInd,
int blockDim,
const cuComplex* x,
const cuComplex* beta,
cuComplex* y)

cusparseStatus_t
cusparseZbsrvmv(cusparseHandle_t handle,
cusparseDirection_t dir,
cusparseOperation_t trans,
int sizeOfMask,
int mb,
int nb,
int nnzb,
const cuDoubleComplex* alpha,
const cusparseMatDescr_t descr,
const cuDoubleComplex* bsrVal,
const int* bsrMaskPtr,
const int* bsrRowPtr,
const int* bsrEndPtr,
const int* bsrColInd,
int blockDim,
const cuDoubleComplex* x,
const cuDoubleComplex* beta,
cuDoubleComplex* y)
This function performs a \texttt{bsrmv} and a mask operation

\[ y(\text{mask}) = (\alpha \ast \text{op}(A) \ast x + \beta \ast y)(\text{mask}) \]

where \( A \) is an \((mb \times \text{ blockDim}) \times (nb \times \text{ blockDim})\) sparse matrix that is defined in BSRX storage format by the four arrays \texttt{bsrVal}, \texttt{bsrRowPtr}, \texttt{bsrEndPtr}, and \texttt{bsrColInd}); \( x \) and \( y \) are vectors; \( \alpha \) and \( \beta \) are scalars; and

\[
\text{op}(A) = \begin{cases} 
A & \text{if trans} = \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\
A^T & \text{if trans} = \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\
A^H & \text{if trans} = \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE}
\end{cases}
\]

The mask operation is defined by array \texttt{bsrMaskPtr} which contains updated block row indices of \( y \). If row \( i \) is not specified in \texttt{bsrMaskPtr}, then \texttt{bsrxmv()} does not touch row block \( i \) of \( A \) and \( y \).

For example, consider the 2\( \times \)3 block matrix \( A \):

\[
A = \begin{bmatrix} 
A_{11} & A_{12} & O \\
A_{21} & A_{22} & A_{23}
\end{bmatrix}
\]

and its one-based BSR format (three vector form) is

\[
\begin{align*}
\text{bsrVal} &= [A_{11} \ A_{12} \ A_{21} \ A_{22} \ A_{23}] \\
\text{bsrRowPtr} &= [1 \ 3 \ 6] \\
\text{bsrColInd} &= [1 \ 2 \ 1 \ 2 \ 3]
\end{align*}
\]

Suppose we want to do the following \texttt{bsrmv} operation on a matrix \( \bar{A} \) which is slightly different from \( A \).

\[
\begin{bmatrix} 
Y_1 \\
Y_2
\end{bmatrix} := \alpha \ast \left( \bar{A} \ast \begin{bmatrix} O & O & O \\
O & A_{22} & O
\end{bmatrix} \ast \begin{bmatrix} x_1 \\
x_2 \\
x_3
\end{bmatrix} + \beta \ast y_2 \right)
\]

We don’t need to create another BSR format for the new matrix \( \bar{A} \), all that we should do is to keep \texttt{bsrVal} and \texttt{bsrColInd} unchanged, but modify \texttt{bsrRowPtr} and add an additional array \texttt{bsrEndPtr} which points to the last nonzero elements per row of \( \bar{A} \) plus 1.

For example, the following \texttt{bsrRowPtr} and \texttt{bsrEndPtr} can represent matrix \( \bar{A} \):

\[
\begin{align*}
\text{bsrRowPtr} &= [1 \ 4] \\
\text{bsrEndPtr} &= [1 \ 5]
\end{align*}
\]

Further we can use a mask operator (specified by array \texttt{bsrMaskPtr}) to update particular block row indices of \( y \) only because \( y_1 \) is never changed. In this case, \texttt{bsrMaskPtr} = [2] and \texttt{sizeOfMask}=1.

The mask operator is equivalent to the following operation:

\[
\begin{bmatrix} 
? \\
Y_2
\end{bmatrix} := \alpha \ast \left( \begin{bmatrix} O & ? & ? \\
O & A_{22} & O
\end{bmatrix} \ast \begin{bmatrix} x_1 \\
x_2 \\
x_3
\end{bmatrix} + \beta \ast Y_2 \right)
\]
If a block row is not present in the `bsrMaskPtr`, then no calculation is performed on that row, and the corresponding value in $y$ is unmodified. The question mark "?" is used to indicate row blocks not in `bsrMaskPtr`.

In this case, first row block is not present in `bsrMaskPtr`, so `bsrRowPtr[0]` and `bsrEndPtr[0]` are not touched also.

```
bsrRowPtr = [? 4]
bsrEndPtr = [? 5]
```

`bsrXmv()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

A couple of comments on `bsrXmv()`:

- Only `blockDim > 1` is supported
- Only `CUSPARSE_OPERATION_NON_TRANSPOSE` and `CUSPARSE_MATRIX_TYPE_GENERAL` are supported.
- Parameters `bsrMaskPtr`, `bsrRowPtr`, `bsrEndPtr` and `bsrColInd` are consistent with base index, either one-based or zero-based. The above example is one-based.

## Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>dir</code></td>
<td>storage format of blocks, either <code>CUSPARSE_DIRECTION_ROW</code> or <code>CUSPARSE_DIRECTION_COLUMN</code>.</td>
</tr>
<tr>
<td><code>trans</code></td>
<td>the operation $\text{op}(A)$. Only <code>CUSPARSE_OPERATION_NON_TRANSPOSE</code> is supported.</td>
</tr>
<tr>
<td><code>sizeOfMask</code></td>
<td>number of updated block rows of $y$.</td>
</tr>
<tr>
<td><code>mb</code></td>
<td>number of block rows of matrix $A$.</td>
</tr>
<tr>
<td><code>nb</code></td>
<td>number of block columns of matrix $A$.</td>
</tr>
<tr>
<td><code>nnzb</code></td>
<td>number of nonzero blocks of matrix $A$.</td>
</tr>
<tr>
<td><code>alpha</code></td>
<td><code>&lt;type&gt;</code> scalar used for multiplication.</td>
</tr>
<tr>
<td><code>descr</code></td>
<td>the descriptor of matrix $A$. The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code>. Also, the supported index bases are <code>CUSPARSE_INDEX_BASE_ZERO</code> and <code>CUSPARSE_INDEX_BASE_ONE</code>.</td>
</tr>
<tr>
<td><code>bsrVal</code></td>
<td><code>&lt;type&gt;</code> array of <code>nnz</code> nonzero blocks of matrix $A$.</td>
</tr>
<tr>
<td><code>bsrMaskPtr</code></td>
<td>integer array of <code>sizeOfMask</code> elements that contains the indices corresponding to updated block rows.</td>
</tr>
<tr>
<td><code>bsrRowPtr</code></td>
<td>integer array of <code>mb</code> elements that contains the start of every block row.</td>
</tr>
<tr>
<td>bsrEndPtr</td>
<td>integer array of mb elements that contains the end of the every block row plus one.</td>
</tr>
<tr>
<td>---------------------</td>
<td>----------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>bsrColInd</td>
<td>integer array of nnzb column indices of the nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix A, larger than zero.</td>
</tr>
<tr>
<td>x</td>
<td>&lt;type&gt; vector of nb * blockDim elements.</td>
</tr>
<tr>
<td>beta</td>
<td>&lt;type&gt; scalar used for multiplication. If beta is zero, y does not have to be a valid input.</td>
</tr>
<tr>
<td>y</td>
<td>&lt;type&gt; vector of mb * blockDim elements.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
## 8.3. cusparse<\text{t}>bsrsv2_bufferSize()

<table>
<thead>
<tr>
<th>Type</th>
<th>Function Name</th>
<th>Description</th>
</tr>
</thead>
</table>
| cusparseStatus_t | cusparseSbsrsv2_bufferSize          | Computes the buffer size for single-precision CSR
|            | handle,                            |                                                 |
|            | dirA,                              |                                                 |
|            | transA,                            |                                                 |
|            | mb,                                |                                                 |
|            | nnzb,                              |                                                 |
|            | descrA,                            |                                                 |
|            | bsrValA,                           |                                                 |
|            | bsrRowPtrA,                        |                                                 |
|            | bsrColIndA,                        |                                                 |
|            | blockDim,                          |                                                 |
|            | info,                              |                                                 |
|            | pBufferSizeInBytes                 |                                                 |

|            | cusparseDbsrsv2_bufferSize          | Computes the buffer size for double-precision CSR
|            | handle,                            |                                                 |
|            | dirA,                              |                                                 |
|            | transA,                            |                                                 |
|            | mb,                                |                                                 |
|            | nnzb,                              |                                                 |
|            | descrA,                            |                                                 |
|            | bsrValA,                           |                                                 |
|            | bsrRowPtrA,                        |                                                 |
|            | bsrColIndA,                        |                                                 |
|            | blockDim,                          |                                                 |
|            | info,                              |                                                 |
|            | pBufferSizeInBytes                 |                                                 |

|            | cusparseCbsrsv2_bufferSize          | Computes the buffer size for single-precision CSR
|            | handle,                            |                                                 |
|            | dirA,                              |                                                 |
|            | transA,                            |                                                 |
|            | mb,                                |                                                 |
|            | nnzb,                              |                                                 |
|            | descrA,                            |                                                 |
|            | bsrValA,                           |                                                 |
|            | bsrRowPtrA,                        |                                                 |
|            | bsrColIndA,                        |                                                 |
|            | blockDim,                          |                                                 |
|            | info,                              |                                                 |
|            | pBufferSizeInBytes                 |                                                 |

|            | cusparseZbsrsv2_bufferSize          | Computes the buffer size for double-precision CSR
|            | handle,                            |                                                 |
|            | dirA,                              |                                                 |
|            | transA,                            |                                                 |
|            | mb,                                |                                                 |
|            | nnzb,                              |                                                 |
|            | descrA,                            |                                                 |
|            | bsrValA,                           |                                                 |
|            | bsrRowPtrA,                        |                                                 |
|            | bsrColIndA,                        |                                                 |
|            | blockDim,                          |                                                 |
|            | info,                              |                                                 |
|            | pBufferSizeInBytes                 |                                                 |
This function returns size of the buffer used in `bsrsv2`, a new sparse triangular linear system \( \text{op}(A) \ast y = \alpha x \).

\( A \) is an \((mb \ast \text{blockDim}) \times (mb \ast \text{blockDim})\) sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`; \( x \) and \( y \) are the right-hand-side and the solution vectors; \( \alpha \) is a scalar; and

\[
\text{op}(A) = \begin{cases} 
A & \text{if } \text{trans} == \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if } \text{trans} == \text{CUSPARSE_OPERATION_TRANSPOSE} \\
A^H & \text{if } \text{trans} == \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE}
\end{cases}
\]

Although there are six combinations in terms of parameter `trans` and the upper (lower) triangular part of \( A \), `bsrsv2_bufferSize()` returns the maximum size buffer among these combinations. The buffer size depends on the dimensions `mb`, `blockDim`, and the number of nonzero blocks of the matrix `nnzb`. If the user changes the matrix, it is necessary to call `bsrsv2_bufferSize()` again to have the correct buffer size; otherwise a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>dirA</code></td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td><code>transA</code></td>
<td>the operation ( \text{op}(A) ).</td>
</tr>
<tr>
<td><code>mb</code></td>
<td>number of block rows of matrix ( A ).</td>
</tr>
<tr>
<td><code>nnzb</code></td>
<td>number of nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td><code>descrA</code></td>
<td>the descriptor of matrix ( A ). The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.</td>
</tr>
<tr>
<td><code>bsrValA</code></td>
<td>(&lt;\text{type}&gt;) array of <code>nnzb</code> (= <code>bsrRowPtrA(mb) - bsrRowPtrA(0)</code>) nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td><code>bsrRowPtrA</code></td>
<td>integer array of <code>mb</code> + 1 elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td><code>bsrColIndA</code></td>
<td>integer array of <code>nnzb</code> (= <code>bsrRowPtrA(mb) - bsrRowPtrA(0)</code>) column indices of the nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td><code>blockDim</code></td>
<td>block dimension of sparse matrix ( A ); must be larger than zero.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>info</code></td>
<td>record of internal states based on different algorithms.</td>
</tr>
<tr>
<td>pBufferSizeInBytes</td>
<td>number of bytes of the buffer used in the bsrsv2_analysis() and bsrsv2_solve().</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
8.4. `cusparse<t>bsrsv2_analysis()`

```c
cusparseStatus_t cusparseSbsrsv2_analysis(cusparseHandle_t handle,
                          cusparseDirection_t dirA,
                          cusparseOperation_t transA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr_t descrA,
                          const float* bsrValA,
                          const int* bsrRowPtrA,
                          const int* bsrColIndA,
                          int blockDim,
                          bsrsv2Info_t info,
                          cusparseSolvePolicy_t policy,
                          void* pBuffer)

cusparseStatus_t cusparseDbsrsv2_analysis(cusparseHandle_t handle,
                          cusparseDirection_t dirA,
                          cusparseOperation_t transA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr_t descrA,
                          const double* bsrValA,
                          const int* bsrRowPtrA,
                          const int* bsrColIndA,
                          int blockDim,
                          bsrsv2Info_t info,
                          cusparseSolvePolicy_t policy,
                          void* pBuffer)

cusparseStatus_t cusparseZbsrsv2_analysis(cusparseHandle_t handle,
                           cusparseDirection_t dirA,
                           cusparseOperation_t transA,
                           int mb,
                           int nnzb,
                           const cusparseMatDescr_t descrA,
                           const cuComplex* bsrValA,
                           const int* bsrRowPtrA,
                           const int* bsrColIndA,
                           int blockDim,
                           bsrsv2Info_t info,
                           cusparseSolvePolicy_t policy,
                           void* pBuffer)

cusparseStatus_t cusparseCbsrsv2_analysis(cusparseHandle_t handle,
                           cusparseDirection_t dirA,
                           cusparseOperation_t transA,
                           int mb,
                           int nnzb,
                           const cusparseMatDescr_t descrA,
                           const cuDoubleComplex* bsrValA,
                           const int* bsrRowPtrA,
                           const int* bsrColIndA,
                           int blockDim,
                           bsrsv2Info_t info,
                           cusparseSolvePolicy_t policy,
                           void* pBuffer)
```

This function performs the analysis phase of `bsrsv2`, a new sparse triangular linear system $\text{op}(A) \cdot y = \alpha \cdot x$.

$A$ is an $(mb \cdot \text{blockDim}) \times (mb \cdot \text{blockDim})$ sparse matrix that is defined in BSR storage format by the three arrays $\text{bsrValA}$, $\text{bsrRowPtrA}$, and $\text{bsrColIndA}$; $x$ and $y$ are the right-hand side and the solution vectors; $\alpha$ is a scalar; and

$$\text{op}(A) = \begin{cases} A & \text{if trans} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\ A^T & \text{if trans} = \text{CUSPARSE_OPERATION_TRANSPOSE} \\ A^H & \text{if trans} = \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} \end{cases}$$

The block of BSR format is of size $\text{blockDim} \times \text{blockDim}$, stored as column-major or row-major as determined by parameter $\text{dirA}$, which is either `CUSPARSE_DIRECTION_COLUMN` or `CUSPARSE_DIRECTION_ROW`. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, and the fill mode and diagonal type are ignored.

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires a buffer size returned by `bsrsv2_bufferSize()`. The address of $\text{pBuffer}$ must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `bsrsv2_analysis()` reports a structural zero and computes level information, which stored in the opaque structure $\text{info}$. The level information can extract more parallelism for a triangular solver. However `bsrsv2_solve()` can be done without level information. To disable level information, the user needs to specify the policy of the triangular solver as `CUSPARSE_SOLVE_POLICY_NO_LEVEL`.

Function `bsrsv2_analysis()` always reports the first structural zero, even when parameter $\text{policy}$ is `CUSPARSE_SOLVE_POLICY_NO_LEVEL`. No structural zero is reported if `CUSPARSE_DIAG_TYPE_UNIT` is specified, even if block $A(j,j)$ is missing for some $j$. The user needs to call `cusparseXbsrsv2_zeroPivot()` to know where the structural zero is.

It is the user's choice whether to call `bsrsv2_solve()` if `bsrsv2_analysis()` reports a structural zero. In this case, the user can still call `bsrsv2_solve()`, which will return a numerical zero at the same position as a structural zero. However the result $x$ is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine does **not** support asynchronous execution
- The routine does **not** support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>dirA</code></td>
<td>storage format of blocks, either <code>CUSPARSE_DIRECTION_ROW</code> or <code>CUSPARSE_DIRECTION_COLUMN</code>.</td>
</tr>
<tr>
<td><code>transA</code></td>
<td>the operation $\text{op}(A)$.</td>
</tr>
<tr>
<td><code>mb</code></td>
<td>number of block rows of matrix $A$.</td>
</tr>
<tr>
<td><code>nnzb</code></td>
<td>number of nonzero blocks of matrix $A$.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix $A$. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.</td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>bsrValA</td>
<td>&lt;type&gt; array of $nnzb = bsrRowPtrA(mb) - bsrRowPtrA(0)$ nonzero blocks of matrix $A$.</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of $mb + 1$ elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of $nnzb = bsrRowPtrA(mb) - bsrRowPtrA(0)$ column indices of the nonzero blocks of matrix $A$.</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix $A$, larger than zero.</td>
</tr>
<tr>
<td>info</td>
<td>structure initialized using cusparseCreateBsrsv2Info().</td>
</tr>
<tr>
<td>policy</td>
<td>the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user, the size is return by bsrsv2_bufferSize().</td>
</tr>
</tbody>
</table>

**Output**

| info                   | structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged). |

See cusparseStatus_t for the description of the return status.
8.5. cusparse<t>bsrsv2_solve()

cusparseStatus_t
cusparseSbsrsv2_solve(cusparseHandle_t handle,
cusparseDirection_t dirA,
cusparseOperation_t transA,
int mb,
int nnzb,
const float* alpha,
const cusparseMatDescr_t descrA,
const float* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
bsrsv2Info_t info,
const float* x,
float* y,
cusparseSolvePolicy_t policy,
void* pBuffer)

cusparseStatus_t
cusparseDbsrsv2_solve(cusparseHandle_t handle,
cusparseDirection_t dirA,
cusparseOperation_t transA,
int mb,
int nnzb,
const double* alpha,
const cusparseMatDescr_t descrA,
const double* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
bsrsv2Info_t info,
const double* x,
double* y,
cusparseSolvePolicy_t policy,
void* pBuffer)

cusparseStatus_t
cusparseCbsrsv2_solve(cusparseHandle_t handle,
cusparseDirection_t dirA,
cusparseOperation_t transA,
int mb,
int nnzb,
cusparseMatDescr_t descrA,
cusparseMatDescr_t descrA,
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cusparseMatDescr_t descrA,
This function performs the solve phase of \texttt{bsrsv2}, a new sparse triangular linear system \( \text{op}(A) \, y = \alpha \, x \).

A is an \((\text{mb} \times \text{blockDim}) \times (\text{mb} \times \text{blockDim})\) sparse matrix that is defined in BSR storage format by the three arrays \texttt{bsrValA}, \texttt{bsrRowPtrA}, and \texttt{bsrColIndA}; \( x \) and \( y \) are the right-hand-side and the solution vectors; \( \alpha \) is a scalar; and

\[
\text{op}(A) = \begin{cases} 
A & \text{if trans} = = \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\
A^T & \text{if trans} = = \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\
A^H & \text{if trans} = = \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE}
\end{cases}
\]

The block in BSR format is of size \( \text{blockDim} \times \text{blockDim} \), stored as column-major or row-major as determined by parameter \texttt{dirA}, which is either \texttt{CUSPARSE\_DIRECTION\_COLUMN} or \texttt{CUSPARSE\_DIRECTION\_ROW}. The matrix type must be \texttt{CUSPARSE\_MATRIX\_TYPE\_GENERAL}, and the fill mode and diagonal type are ignored.

Function \texttt{bsrsv02\_solve()} can support an arbitrary \texttt{blockDim}.

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires a buffer size returned by \texttt{bsrsv2\_bufferSize()}. The address of \texttt{pBuffer} must be multiple of 128 bytes. If it is not, \texttt{CUSPARSE\_STATUS\_INVALID\_VALUE} is returned.

Although \texttt{bsrsv2\_solve()} can be done without level information, the user still needs to be aware of consistency. If \texttt{bsrsv2\_analysis()} is called with policy \texttt{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL}, \texttt{bsrsv2\_solve()} can be run with or without levels. On the other hand, if \texttt{bsrsv2\_analysis()} is called with \texttt{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL}, \texttt{bsrsv2\_solve()} can only accept \texttt{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL}; otherwise, \texttt{CUSPARSE\_STATUS\_INVALID\_VALUE} is returned.

The level information may not improve the performance, but may spend extra time doing analysis. For example, a tridiagonal matrix has no parallelism.

In this case, \texttt{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL} performs better than \texttt{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL}. If the user has an iterative solver, the best approach is to do \texttt{bsrsv2\_analysis()} with \texttt{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL} once. Then do \texttt{bsrsv2\_solve()} with \texttt{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL} in the first run, and with \texttt{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL} in the second run, and pick the fastest one to perform the remaining iterations.

Function \texttt{bsrsv02\_solve()} has the same behavior as \texttt{csrsv02\_solve()}. That is, \( \text{bsr2csr(} \texttt{bsrsv02\,(} \texttt{A}) \text{)} = \text{csrsv02\,(} \texttt{bsr2csr\,(} \texttt{A}) \text{)} \). The numerical zero of \texttt{csrsv02\_solve()} means there exists some zero \( A(j,j) \). The numerical zero of \texttt{bsrsv02\_solve()} means there exists some block \( A(j,j) \) that is not invertible.

Function \texttt{bsrsv2\_solve()} reports the first numerical zero, including a structural zero. No numerical zero is reported if \texttt{CUSPARSE\_DIAG\_TYPE\_UNIT} is specified, even if \( A(j,j) \) is not invertible for some \( j \). The user needs to call \texttt{cusparseXbsrsv2\_zeroPivot()} to know where the numerical zero is.

The function supports the following properties if \texttt{pBuffer \(!= NULL}\):

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture
For example, suppose \( L \) is a lower triangular matrix with unit diagonal, then the following code solves \( L \cdot y = x \) by level information.

```c
// Suppose that L is m x m sparse matrix represented by BSR format,
// The number of block rows/columns is mb, and
// the number of nonzero blocks is nnzb.
// L is lower triangular with unit diagonal.
// Assumption:
// - dimension of matrix L is m(=mb*blockDim),
// - matrix L has nnz(=nnzb*blockDim*blockDim) nonzero elements,
// - handle is already created by cusparseCreate(),
// - (d_bsrRowPtr, d_bsrColInd, d_bsrVal) is BSR of L on device memory,
// - d_y is right hand side vector on device memory.
// - d_x and d_y are of size m.
cusparseMatDescr_t descr = 0;
bsrsv2Info_t info = 0;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;

// step 1: create a descriptor which contains
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has unit diagonal, specified by parameter CUSPARSE_DIAG_TYPE_UNIT
// (L may not have all diagonal elements.)
cusparseCreateMatDescr(&descr);
cusparseSetMatIndexBase(descr, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatFillMode(descr, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr, CUSPARSE_DIAG_TYPE_UNIT);

// step 2: create a empty info structure
cusparseCreateBsrsv2Info(&info);

// step 3: query how much memory used in bsrsv2, and allocate the buffer
cusparseBsrsv2_bufferSize(handle, dir, trans, mb, nnzb, descr,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, pBufferSize);

// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis
cusparseBsrsv2_analysis(handle, dir, trans, mb, nnzb, descr,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info, policy, pBuffer);

// L has unit diagonal, so no structural zero is reported.
status = cusparseXbsrsv2_zeroPivot(handle, info, &structural_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
    printf("L(%d,%d) is missing\n", structural_zero, structural_zero);
}

// step 5: solve L \cdot y = x
cusparseBsrsv2_solve(handle, dir, trans, mb, nnzb, &alpha, descr,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info,
    d_x, d_y, policy, pBuffer);

// L has unit diagonal, so no numerical zero is reported.
status = cusparseXbsrsv2_zeroPivot(handle, info, &numerical_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
    printf("L(%d,%d) is zero\n", numerical_zero, numerical_zero);
}

// step 6: free resources
cudaFree(pBuffer);
cusparseDestroyBsrsv2Info(info);
cusparseDestroyMatDescr(descr);
cusparseDestroy(handle);
```
# cuSPARSE Level 2 Function Reference

## Input

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>dirA</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>transA</td>
<td>the operation op(A).</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows and block columns of matrix A.</td>
</tr>
<tr>
<td>alpha</td>
<td>&lt;type&gt; scalar used for multiplication.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.</td>
</tr>
<tr>
<td>bsrValA</td>
<td>&lt;type&gt; array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0)) nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of mb + 1 elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0)) column indices of the nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix A, larger than zero.</td>
</tr>
<tr>
<td>info</td>
<td>structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).</td>
</tr>
<tr>
<td>x</td>
<td>&lt;type&gt; right-hand-side vector of size m.</td>
</tr>
<tr>
<td>policy</td>
<td>the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user, the size is returned by bsrsv2_bufferSize().</td>
</tr>
</tbody>
</table>

## Output

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>&lt;type&gt; solution vector of size m.</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status

## 8.6. cusparseXbsrsv2_zeroPivot()

```c
#define CUSPARSE_VERSION_MAJOR 7
#define CUSPARSE_VERSION_MINOR 3
#define CUSPARSE_VERSION_PATCH 4

int cusparseXbsrsv2_zeroPivot(cusparseHandle_t handle, 
                      bsrsv2Info_t info, 
                      int* position)
```

www.nvidia.com
cuSPARSE Library

DU-06709-001_v11.0 | 62
If the returned error code is `CUSPARSE_STATUS_ZERO_PIVOT`, `position=j` means $A(j,j)$ is either structural zero or numerical zero (singular block). Otherwise `position=-1`.

The `position` can be 0-based or 1-based, the same as the matrix.

Function `cusparseXbsrsv2_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The `position` can be in the host memory or device memory. The user can set the proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>info</td>
<td>info contains a structural zero or numerical zero if the user already called <code>bsrsv2_analysis()</code> or <code>bsrsv2_solve()</code>.</td>
</tr>
</tbody>
</table>

### Output

| position     | if no structural or numerical zero, `position` is -1; otherwise if $A(j,j)$ is missing or $U(j,j)$ is zero, `position=j`. |

See `cusparseStatus_t` for the description of the return status.
8.7. cusparseCsrMVEx()

```c
 cusparseStatus_t
cusparseCsrMVEx_bufferSize(cusparseHandle_t handle,
cusparseAlgMode_t alg,
cusparseOperation_t transA,
int m,
int n,
int nnz,
const void* alpha,
cudaDataType alphatype,
const cusparseMatDescr_t descrA,
const void* csrValA,
cudaDataType csrValAtype,
const int* csrRowPtrA,
const int* csrColIndA,
const void* x,
cudaDataType xtype,
const void* beta,
cudaDataType betatype,
void* y,
cudaDataType ytype,
cudaDataType executiontype,
size_t* bufferSizeInBytes)
```

```c
 cusparseStatus_t
cusparseCsrMVEx(cusparseHandle_t handle,
cusparseAlgMode_t alg,
cusparseOperation_t transA,
int m,
int n,
int nnz,
const void* alpha,
cudaDataType alphatype,
const cusparseMatDescr_t descrA,
const void* csrValA,
cudaDataType csrValAtype,
const int* csrRowPtrA,
const int* csrColIndA,
const void* x,
cudaDataType xtype,
const void* beta,
cudaDataType betatype,
void* y,
cudaDataType ytype,
cudaDataType executiontype,
void* buffer)
```

This function performs the matrix-vector operation

\[ y = \alpha \cdot \text{op}(A) \cdot x + \beta \cdot y \]

\( A \) is an \( m \times n \) sparse matrix that is defined in CSR storage format by the three arrays \( \text{csrValA}, \text{csrRowPtrA}, \) and \( \text{csrColIndA} \); \( x \) and \( y \) are vectors;

The function \texttt{cusparseCsrMVEx_bufferSize} returns the size of the workspace needed by \texttt{cusparseCsrMVEx}. 
The function has the following limitations:

- All pointers should be aligned with 128 bytes
- Only `CUSPARSE_OPERATION_NON_TRANSPOSE` operation is supported
- Only `CUSPARSE_MATRIX_TYPE_GENERAL` matrix type is supported
- Only `CUSPARSE_INDEX_BASE_ZERO` indexing is supported
- Half-precision is not supported
- The minimum GPU architecture supported is SM_53

The function has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input specifically required by `cusparseCsrmvEx`**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>alg</code></td>
<td>Algorithm implementation for csrmv, see <code>cusparseAlgMode_t</code> for possible values.</td>
</tr>
<tr>
<td><code>alphatype</code></td>
<td>Data type of <code>alpha</code>.</td>
</tr>
<tr>
<td><code>csrValAtype</code></td>
<td>Data type of <code>csrValA</code>.</td>
</tr>
<tr>
<td><code>xtype</code></td>
<td>Data type of <code>x</code>.</td>
</tr>
<tr>
<td><code>betatype</code></td>
<td>Data type of <code>beta</code>.</td>
</tr>
<tr>
<td><code>ytype</code></td>
<td>Data type of <code>y</code>.</td>
</tr>
<tr>
<td><code>executiontype</code></td>
<td>Data type used for computation.</td>
</tr>
<tr>
<td><code>bufferSizeInBytes</code></td>
<td>Pointer to a <code>size_t</code> variable, which will be assigned with the size of workspace needed by <code>cusparseCsrmvEx</code>.</td>
</tr>
<tr>
<td><code>buffer</code></td>
<td>Pointer to workspace buffer</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
8.8. cusparse<t>csrsv2_bufferSize()

This function returns the size of the buffer used in \texttt{csrsv2}, a new sparse triangular linear system \( \text{op}(A) \cdot y = \alpha x \).

\( A \) is an \( m \times m \) sparse matrix that is defined in CSR storage format by the three arrays \texttt{csrValA}, \texttt{csrRowPtrA}, and \texttt{csrColIndA}; \( x \) and \( y \) are the right-hand-side and the solution vectors; \( \alpha \) is a scalar; and
Although there are six combinations in terms of the parameter `trans` and the upper (lower) triangular part of \( A \), `csrsv2_bufferSize()` returns the maximum size buffer of these combinations. The buffer size depends on the dimension and the number of nonzero elements of the matrix. If the user changes the matrix, it is necessary to call `csrsv2_bufferSize()` again to have the correct buffer size; otherwise, a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>transA</code></td>
<td>the operation ( \text{op}(A) ).</td>
</tr>
<tr>
<td><code>m</code></td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td><code>nnz</code></td>
<td>number of nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td><code>descrA</code></td>
<td>the descriptor of matrix ( A ). The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code>, while the supported diagonal types are <code>CUSPARSE_DIAG_TYPE_UNIT</code> and <code>CUSPARSE_DIAG_TYPE_NON_UNIT</code>.</td>
</tr>
<tr>
<td><code>csrValA</code></td>
<td><code>&lt;type&gt;</code> array of ( nnz ) nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td><code>csrRowPtrA</code></td>
<td>integer array of ( m + 1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td><code>csrColIndA</code></td>
<td>integer array of ( nnz ) column indices of the nonzero elements of matrix ( A ).</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>info</code></td>
<td>record of internal states based on different algorithms.</td>
</tr>
<tr>
<td><code>pBufferSizeInBytes</code></td>
<td>number of bytes of the buffer used in the <code>csrsv2_analysis</code> and <code>csrsv2_solve</code>.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
8.9. cusparse<t>csrsv2_analysis()

This function performs the analysis phase of \(\text{csrsv2}\), a new sparse triangular linear system \(\text{op}(A) \cdot y = \alpha \cdot x\).
A is an \( m \times m \) sparse matrix that is defined in CSR storage format by the three arrays \( \text{csrValA} \), \( \text{csrRowPtrA} \), and \( \text{csrColIndA} \); \( \mathbf{x} \) and \( \mathbf{y} \) are the right-hand-side and the solution vectors; \( \alpha \) is a scalar; and

\[
\text{op}(A) = \begin{cases} 
A & \text{if } \text{trans} == \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if } \text{trans} == \text{CUSPARSE_OPERATION_TRANSPOSE} \\
A^H & \text{if } \text{trans} == \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} 
\end{cases}
\]

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires a buffer size returned by \( \text{csrsv2_bufferSize()} \). The address of \( \text{pBuffer} \) must be multiple of 128 bytes. If it is not, \( \text{CUSPARSE_STATUS_INVALID_VALUE} \) is returned.

Function \( \text{csrsv2_analysis()} \) reports a structural zero and computes level information that is stored in opaque structure \( \text{info} \). The level information can extract more parallelism for a triangular solver. However \( \text{csrsv2_solve()} \) can be done without level information. To disable level information, the user needs to specify the policy of the triangular solver as \( \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL} \).

Function \( \text{csrsv2_analysis()} \) always reports the first structural zero, even if the policy is \( \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL} \). No structural zero is reported if \( \text{CUSPARSE_DIAG_TYPE_UNIT} \) is specified, even if \( A(j,j) \) is missing for some \( j \). The user needs to call \( \text{cusparseXcsrsv2_zeroPivot()} \) to know where the structural zero is.

It is the user’s choice whether to call \( \text{csrsv2_solve()} \) if \( \text{csrsv2_analysis()} \) reports a structural zero. In this case, the user can still call \( \text{csrsv2_solve()} \) which will return a numerical zero in the same position as the structural zero. However the result \( \mathbf{x} \) is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine does \textit{not} support asynchronous execution
- The routine does \textit{not} support CUDA graph capture

\[\text{Input}\]

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>transA</td>
<td>the operation ( \text{op}(A) ).</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>nnz</td>
<td>number of nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix ( A ). The supported matrix type is ( \text{CUSPARSE_MATRIX_TYPE_GENERAL} ), while the supported diagonal types are ( \text{CUSPARSE_DIAG_TYPE_UNIT} ) and ( \text{CUSPARSE_DIAG_TYPE_NON_UNIT} ).</td>
</tr>
<tr>
<td>csrValA</td>
<td>(&lt;\text{type}&gt;) array of ( \text{nnz} (= \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0)) ) nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of ( m + 1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
</tbody>
</table>
### `csrColIndA`<br>integer array of `nnz (= csrRowPtrA(m) - csrRowPtrA(0))` column indices of the nonzero elements of matrix `A`.

### `info`<br>structure initialized using `cusparseCreateCsrsv2Info()`.

### `policy`<br>The supported policies are `CUSPARSE_SOLVE_POLICY_NO_LEVEL` and `CUSPARSE_SOLVE_POLICY_USE_LEVEL`.

### `pBuffer`<br>buffer allocated by the user, the size is returned by `csrsv2_bufferSize()`.

#### Output

| `info` | structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged). |

See `cusparseStatus_t` for the description of the return status.
8.10. cuSparse<t>csrsv2_solve()

cusparseStatus_t cusparseScsrsv2_solve(cusparseHandle_t handle, cusparseOperation_t transA, int m, int nnz, const float* alpha, const cusparseMatDescr_t descra, const float* csrValA, const int* csrRowPtrA, const int* csrColIndA, csrsv2Info_t info, const float* x, float* y, cusparseSolvePolicy_t policy, void* pBuffer)

cusparseStatus_t cusparseDcsrsv2_solve(cusparseHandle_t handle, cusparseOperation_t transA, int m, int nnz, const double* alpha, const cusparseMatDescr_t descra, const double* csrValA, const int* csrRowPtrA, const int* csrColIndA, csrsv2Info_t info, const double* x, double* y, cusparseSolvePolicy_t policy, void* pBuffer)

cusparseStatus_t cusparseCcsrsv2_solve(cusparseHandle_t handle, cusparseOperation_t transA, int m, int nnz, const cuComplex* alpha, const cusparseMatDescr_t descra, const cuComplex* csrValA, const int* csrRowPtrA, const int* csrColIndA, csrsv2Info_t info, const cuComplex* x, cuComplex* y, cusparseSolvePolicy_t policy, void* pBuffer)

cusparseStatus_t cusparseZcsrsv2_solve(cusparseHandle_t handle, cusparseOperation_t transA, int m, int nnz, const cuDoubleComplex* alpha, const cusparseMatDescr_t descra, const cuDoubleComplex* csrValA, const int* csrRowPtrA, const int* csrColIndA, csrsv2Info_t info, const cuDoubleComplex* x, cuDoubleComplex* y, cusparseSolvePolicy_t policy, void* pBuffer)
This function performs the solve phase of `csrsv2`, a new sparse triangular linear system
\[ \text{op}(A) * y = \alpha x. \]

A is an \( m \times m \) sparse matrix that is defined in CSR storage format by the three arrays \( \text{csrValA}, \text{csrRowPtrA}, \) and \( \text{csrColIndA} \); \( x \) and \( y \) are the right-hand-side and the solution vectors; \( \alpha \) is a scalar; and

\[
\text{op}(A) = \begin{cases} 
A & \text{if } \text{trans} == \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if } \text{trans} == \text{CUSPARSE_OPERATION_TRANSPOSE} \\
A^H & \text{if } \text{trans} == \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} 
\end{cases}
\]

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires the buffer size returned by `csrsv2_bufferSize()`. The address of \( \text{pBuffer} \) must be multiple of 128 bytes. If it is not, \text{CUSPARSE_STATUS_INVALID_VALUE} is returned.

Although `csrsv2_solve()` can be done without level information, the user still needs to be aware of consistency. If `csrsv2_analysis()` is called with policy \text{CUSPARSE_SOLVE_POLICY_USE_LEVEL}, `csrsv2_solve()` can be run with or without levels. On the contrary, if `csrsv2_analysis()` is called with \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL}, `csrsv2_solve()` can only accept \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL}; otherwise, \text{CUSPARSE_STATUS_INVALID_VALUE} is returned.

The level information may not improve the performance but spend extra time doing analysis. For example, a tridiagonal matrix has no parallelism. In this case, \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL} performs better than \text{CUSPARSE_SOLVE_POLICY_USE_LEVEL}. If the user has an iterative solver, the best approach is to do `csrsv2_analysis()` with \text{CUSPARSE_SOLVE_POLICY_USE_LEVEL} once. Then do `csrsv2_solve()` with \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL} in the first run and with \text{CUSPARSE_SOLVE_POLICY_USE_LEVEL} in the second run, picking faster one to perform the remaining iterations.

Function `csrsv2_solve()` reports the first numerical zero, including a structural zero. If \text{status} is 0, no numerical zero was found. Furthermore, no numerical zero is reported if \text{CUSPARSE_DIAG_TYPE_UNIT} is specified, even if \( A(j,j) \) is zero for some \( j \). The user needs to call `cusparseXcsrsv2_zeroPivot()` to know where the numerical zero is.
For example, suppose $L$ is a lower triangular matrix with unit diagonal, the following code solves $L\mathbf{y} = \mathbf{x}$ by level information.

```c
// Suppose that $L$ is m x m sparse matrix represented by CSR format,
// $L$ is lower triangular with unit diagonal.
// Assumption:
// - dimension of matrix $L$ is m,
// - matrix $L$ has nnz number zero elements,
// - handle is already created by cusparseCreate(),
// - (d_csrRowPtr, d_csrColInd, d_csrVal) is CSR of $L$ on device memory,
// - d_x is right hand side vector on device memory,
// - d_y is solution vector on device memory.

cusparseMatDescr_t descr = 0;
csrsv2Info_t info = 0;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans = CUSPARSE_OPERATION_NON_TRANSPOSE;

// step 1: create a descriptor which contains
// - matrix $L$ is base-1
// - matrix $L$ is lower triangular
// - matrix $L$ has unit diagonal, specified by parameter CUSPARSE_DIAG_TYPE_UNIT
// (L may not have all diagonal elements.)
cusparseCreateMatDescr(&descr);
cusparseSetMatIndexBase(descr, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatFillMode(descr, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr, CUSPARSE_DIAG_TYPE_UNIT);

// step 2: create an empty info structure
cusparseCreateCsrsv2Info(&info);

// step 3: query how much memory used in csrsv2, and allocate the buffer
cusparseDcsrsv2_bufferSize(handle, trans, m, nnz, descr,
    d_csrVal, d_csrRowPtr, d_csrColInd, &pBufferSize);
// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis
cusparseDcsrsv2_analysis(handle, trans, m, nnz, descr,
    d_csrVal, d_csrRowPtr, d_csrColInd, info, policy, pBuffer);
// L has unit diagonal, so no structural zero is reported.
status = cusparseXcsrsv2_zeroPivot(handle, info, &structural_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
    printf("L(%d,%d) is missing\n", structural_zero, structural_zero);
}

// step 5: solve L\mathbf{y} = \mathbf{x}
cusparseDcsrsv2_solve(handle, trans, m, nnz, &alpha, descr,
    d_csrVal, d_csrRowPtr, d_csrColInd, info,
    d_x, d_y, policy, pBuffer);
// L has unit diagonal, so no numerical zero is reported.
status = cusparseXcsrsv2_zeroPivot(handle, info, &numerical_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
    printf("L(%d,%d) is zero\n", numerical_zero, numerical_zero);
}

// step 6: free resources
cudaFree(pBuffer);
cusparseDestroyCsrsv2Info(info);
cusparseDestroyMatDescr(descr);
cusparseDestroy(handle);
```
Remark: \texttt{csrsv2\_solve()} needs more nonzeros per row to achieve good performance. It would perform better if more than 16 nonzeros per row in average.

The function supports the following properties if \texttt{pBuffer} != \texttt{NULL}:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>transA</td>
<td>the operation \text{op}(A).</td>
</tr>
<tr>
<td>m</td>
<td>number of rows and columns of matrix A.</td>
</tr>
<tr>
<td>alpha</td>
<td>(&lt;\text{type}&gt;) scalar used for multiplication.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is \text{CUSPARSE_MATRIX_TYPE_GENERAL}, while the supported diagonal types are \text{CUSPARSE_DIAG_TYPE_UNIT} and \text{CUSPARSE_DIAG_TYPE_NON_UNIT}.</td>
</tr>
<tr>
<td>csrValA</td>
<td>(&lt;\text{type}&gt;) array of \text{nnz} (= \text{csrRowPtrA}(m) – \text{csrRowPtrA}(0)) nonzero elements of matrix A.</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of (m + 1) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of (\text{nnz} (= \text{csrRowPtrA}(m) – \text{csrRowPtrA}(0))) column indices of the nonzero elements of matrix A.</td>
</tr>
<tr>
<td>info</td>
<td>structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).</td>
</tr>
<tr>
<td>x</td>
<td>(&lt;\text{type}&gt;) right-hand-side vector of size (m).</td>
</tr>
<tr>
<td>policy</td>
<td>The supported policies are \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL} and \text{CUSPARSE_SOLVE_POLICY_USE_LEVEL}.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user, the size is return by \text{csrsv2_bufferSize}.</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>(&lt;\text{type}&gt;) solution vector of size (m).</td>
</tr>
</tbody>
</table>

See \texttt{cusparseStatus\_t} for the description of the return status.
8.11. cusparseXcsrsv2_zeroPivot()

cusparseStatus_t
cusparseXcsrsv2_zeroPivot(cusparseHandle_t handle,
                          csrsv2Info_t     info,
                          int*             position)

If the returned error code is CUSPARSE_STATUS_ZERO_PIVOT, position=j means
A(j,j) has either a structural zero or a numerical zero. Otherwise position=-1.

The position can be 0-based or 1-based, the same as the matrix.

Function cusparseXcsrsv2_zeroPivot() is a blocking call. It calls
cudaDeviceSynchronize() to make sure all previous kernels are done.

The position can be in the host memory or device memory. The user can set the proper
mode with cusparseSetPointerMode().

- The routine requires no extra storage
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

Input

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
</table>
| info   | info contains structural zero or numerical zero if
|        | the user already called csrsv2_analysis() or csrsv2_solve(). |

Output

| position | if no structural or numerical zero, position is -1; otherwise, if A(j,j) is missing or U(j,j) is zero, position=j. |

See cusparseStatus_t for the description of the return status.
8.12. cusparse<t>gemvi()

cusparseStatus_t
cusparseSgemvi_bufferSize( cusparseHandle_t    handle,
cusparseOperation_t    transA,
int                 m,
int                 n,
int                 nnz,
int*                pBufferSize)

cusparseStatus_t
cusparseDgemvi_bufferSize( cusparseHandle_t    handle,
cusparseOperation_t    transA,
int                 m,
int                 n,
int                 nnz,
int*                pBufferSize)

cusparseStatus_t
cusparseCgemvi_bufferSize( cusparseHandle_t    handle,
cusparseOperation_t    transA,
int                 m,
int                 n,
int                 nnz,
int*                pBufferSize)

cusparseStatus_t
cusparseZgemvi_bufferSize( cusparseHandle_t    handle,
cusparseOperation_t    transA,
int                 m,
int                 n,
int                 nnz,
int*                pBufferSize)

cusparseStatus_t
cusparseSgemvi(    handle,
cusparseOperation_t    transA,
int                 m,
int                 n,
const float*         alpha,
const float*         A,
int                 lda,
int                  nnz,
const float*         x,
const int*           xInd,
const float*         beta,
float*               y,
cusparseIndexBase_t  idxBase,
void*                pBuffer)

cusparseStatus_t
cusparseDgemvi(    handle,
cusparseOperation_t    transA,
int                 m,
int                 n,
const double*        alpha,
const double*        A,
int                 lda,
int                  nnz,
const double*        x,
const int*           xInd,
const float*         beta,
double*              y,
cusparseIndexBase_t  idxBase,
void*                pBuffer)

cusparseStatus_t
cusparseCgemvi(    handle,
cusparseOperation_t    transA,
int                 m,
int                 n,
const cuComplex*     alpha,
const cuComplex*     A,
int                 lda,
int                  nnz,
const cuComplex*     x,
const int*           xInd,
const float*         beta,
cuComplex*           y,
cusparseIndexBase_t  idxBase,
void*                pBuffer)

cusparseStatus_t
cusparseZgemvi(    handle,
cusparseOperation_t    transA,
int                 m,
int                 n,
const cuDoubleComplex* alpha,
const cuDoubleComplex* A,
int                 lda,
int                  nnz,
const cuDoubleComplex* x,
const int*           xInd,
const float*         beta,
cuDoubleComplex*     y,
cusparseIndexBase_t  idxBase,
void*                pBuffer)
This function performs the matrix-vector operation

\[ y = \alpha \times \text{op}(A) \times x + \beta \times y \]

\( A \) is an \( m \times n \) dense matrix and a sparse vector \( x \) that is defined in a sparse storage format by the two arrays \( xVal, xInd \) of length \( \text{nnz} \), and \( y \) is a dense vector; \( \alpha \) and \( \beta \) are scalars; and

\[
\text{op}(A) = \begin{cases} 
A & \text{if trans} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if trans} = \text{CUSPARSE_OPERATION_TRANSPOSE} \\
A^H & \text{if trans} = \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE}
\end{cases}
\]

To simplify the implementation, we have not (yet) optimized the transpose multiple case. We recommend the following for users interested in this case.

1. Convert the matrix from CSR to CSC format using one of the \texttt{csr2csc()} functions. Notice that by interchanging the rows and columns of the result you are implicitly transposing the matrix.

2. Call the \texttt{gemvi()} function with the \texttt{cusparseOperation_t} parameter set to \texttt{CUSPARSE_OPERATION_NON_TRANSPOSE} and with the interchanged rows and columns of the matrix stored in CSC format. This (implicitly) multiplies the vector by the transpose of the matrix in the original CSR format.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

The function \texttt{cusparse\textless t\textgreater gemvi\_bufferSize()} returns size of buffer used in \texttt{cusparse\textless t\textgreater gemvi()}.

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>trans</td>
<td>the operation ( \text{op}(A) ).</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>n</td>
<td>number of columns of matrix ( A ).</td>
</tr>
<tr>
<td>alpha</td>
<td>&lt;type&gt; scalar used for multiplication.</td>
</tr>
<tr>
<td>A</td>
<td>the pointer to dense matrix ( A ).</td>
</tr>
<tr>
<td>lda</td>
<td>size of the leading dimension of ( A ).</td>
</tr>
<tr>
<td>nnz</td>
<td>number of nonzero elements of vector ( x ).</td>
</tr>
<tr>
<td>( x )</td>
<td>&lt;type&gt; sparse vector of ( \text{nnz} ) elements of size ( n ) if ( \text{op}(A) = A ), and size ( m ) if ( \text{op}(A) = A^T ) or ( \text{op}(A) = A^H ).</td>
</tr>
<tr>
<td>xInd</td>
<td>Indices of non-zero values in ( x ).</td>
</tr>
<tr>
<td>beta</td>
<td>&lt;type&gt; scalar used for multiplication. If ( \text{beta} ) is zero, ( y ) does not have to be a valid input.</td>
</tr>
<tr>
<td>y</td>
<td>&lt;type&gt; dense vector of ( m ) elements if ( \text{op}(A) = A ), and ( n ) elements if ( \text{op}(A) = A^T ) or ( \text{op}(A) = A^H ).</td>
</tr>
</tbody>
</table>
idxBase | 0 or 1, for 0 based or 1 based indexing, respectively
---|---
pBufferSize | number of elements needed the buffer used in cusparse<t>gemvi()
 pBuffer | working space buffer

**Output**

| y | <type> updated dense vector.

See `cusparseStatus_t` for the description of the return status
This chapter describes sparse linear algebra functions that perform operations between sparse and (usually tall) dense matrices.

In particular, the solution of sparse triangular linear systems with multiple right-hand sides is implemented in two phases. First, during the analysis phase, the sparse triangular matrix is analyzed to determine the dependencies between its elements by calling the appropriate `csrsm2_analysis()` function. The analysis is specific to the sparsity pattern of the given matrix and to the selected `cusparseOperation_t` type. The information from the analysis phase is stored in the parameter of type `csrsm2Info_t` that has been initialized previously with a call to `cusparseCreateCsrsm2Info()`.

Second, during the solve phase, the given sparse triangular linear system is solved using the information stored in the `csrsm2Info_t` parameter by calling the appropriate `csrsm2_solve()` function. The solve phase may be performed multiple times with different multiple right-hand sides, while the analysis phase needs to be performed only once. This is especially useful when a sparse triangular linear system must be solved for different sets of multiple right-hand sides one at a time, while its coefficient matrix remains the same.

Finally, once all the solves have completed, the opaque data structure pointed to by the `csrsm2Info_t` parameter can be released by calling `cusparseDestroyCsrsm2Info()`.
9.1. cusparse<t>bsrmm()

cusparseStatus_t cusparseSbsrmm(cusparseHandle_t handle,
cusparseDirection_t dirA,
cusparseOperation_t transA,
cusparseOperation_t transB,
int mb,
int n,
int kb,
int nnzb,
const float* alpha,
const cusparseMatDescr_t descrA,
const float* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
const float* B,
int ldb,
const float* beta,
float* C,
int ldc)

cusparseStatus_t cusparseDbsrmm(cusparseHandle_t handle,
cusparseDirection_t dirA,
cusparseOperation_t transA,
cusparseOperation_t transB,
int mb,
int n,
int kb,
int nnzb,
const double* alpha,
const cusparseMatDescr_t descrA,
const double* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
const double* B,
int ldb,
const double* beta,
double* C,
int ldc)

cusparseStatus_t cusparseCbsrmm(cusparseHandle_t handle,
cusparseDirection_t dirA,
cusparseOperation_t transA,
cusparseOperation_t transB,
int mb,
int n,
int kb,
int nnzb,
const cuComplex* alpha,
const cusparseMatDescr_t descrA,
const cuComplex* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
const cuComplex* B,
int ldb,
const cuComplex* beta,
cuComplex* C,
int ldc)

cusparseStatus_t cusparseZbsrmm(cusparseHandle_t handle,
cusparseDirection_t dirA,
cusparseOperation_t transA,
cusparseOperation_t transB,
int mb,
int n,
int kb,
int nnzb,
const cuDoubleComplex* alpha,
const cusparseMatDescr_t descrA,
const cuDoubleComplex* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
const cuDoubleComplex* B,
int ldb,
const cuDoubleComplex* beta,
cuDoubleComplex* C,
int ldc)
This function performs one of the following matrix-matrix operations:

\[ C = \alpha \cdot \text{op}(A) \cdot \text{op}(B) + \beta \cdot C \]

\( A \) is an \( mb \times kb \) sparse matrix that is defined in BSR storage format by the three arrays \( bsrValA, bsrRowPtrA, \) and \( bsrColIndA; \) \( B \) and \( C \) are dense matrices; \( \alpha \) and \( \beta \) are scalars; and

\[
\text{op}(A) = \begin{cases} 
A & \text{if } \text{transA} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if } \text{transA} = \text{CUSPARSE_OPERATION_TRANSPOSE} \text{ (not supported)} \\
A^H & \text{if } \text{transA} = \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} \text{ (not supported)} 
\end{cases}
\]

and

\[
\text{op}(B) = \begin{cases} 
B & \text{if } \text{transB} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
B^T & \text{if } \text{transB} = \text{CUSPARSE_OPERATION_TRANSPOSE} \\
B^H & \text{if } \text{transB} = \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} \text{ (not supported)} 
\end{cases}
\]

The function has the following limitations:

- Only \textbf{CUSPARSE\_MATRIX\_TYPE\_GENERAL} matrix type is supported
- Only \textbf{blockDim} > 1 is supported

The motivation of \textbf{transpose(B)} is to improve memory access of matrix \( B \). The computational pattern of \( A \cdot \text{transpose}(B) \) with matrix \( B \) in column-major order is equivalent to \( A \cdot B \) with matrix \( B \) in row-major order.

In practice, no operation in an iterative solver or eigenvalue solver uses \( A \cdot \text{transpose}(B) \). However, we can perform \( A \cdot \text{transpose}(\text{transpose}(B)) \) which is the same as \( A \cdot B \). For example, suppose \( A \) is \( mb \times kb \), \( B \) is \( k \times n \) and \( C \) is \( m \times n \), the following code shows usage of \textbf{cusparseDbsrmm}().

```cpp
// A is mb*kb, B is k*n and C is m*n
const int m = mb*blockSize;
const int k = kb*blockSize;
const int ldb_B = k; // leading dimension of B
const int ldc   = m; // leading dimension of C
// perform C:=alpha*A*B + beta*C
cusparseSetMatType(descrA, CUSPARSE\_MATRIX\_TYPE\_GENERAL );
cusparseDbsrmm(cusparse_handle,
    CUSPARSE\_DIRECTION\_COLUMN,
    CUSPARSE\_OPERATION\_NON\_TRANSPOSE,
    CUSPARSE\_OPERATION\_NON\_TRANSPOSE,
    mb, n, kb, nnzb, alpha,
    descrA, bsrValA, bsrRowPtrA, bsrColIndA, blockSize,
    B, ldb_B,
    beta, C, ldc);
```
Instead of using $A*B$, our proposal is to transpose $B$ to $B^t$ by first calling `cublas<t>geam()`, and then to perform $A^t B^t$.

```c
// step 1: $B^t := \text{transpose}(B)$
const int m = mb*blockSize;
const int k = kb*blockSize;
double *Bt;
const int ldb_Bt = n; // leading dimension of $B$
cudaMalloc((void**)&Bt, sizeof(double)*ldb_Bt*k);
double one  = 1.0;
double zero = 0.0;
cublasSetPointerMode(cublas_handle, CUBLAS_POINTER_MODE_HOST);
cublasDgeam(cublas_handle, CUBLAS_OP_T, CUBLAS_OP_T,
n, k, &one, B, int ldb_B, &zero, B, int ldb_B, Bt, ldb_Bt);

// step 2: perform $C := \alpha A^t B^t + \beta C$
cusparseDbsrmm(cusparse_handle,
CUSPARSE_DIRECTION_COLUMN,
CUSPARSE_OPERATION_NON_TRANSPOSE,
CUSPARSE_OPERATION_TRANSPOSE,
mb, n, kb, nnzb, alpha,
descrA, bsrValA, bsrRowPtrA, bsrColIndA, blockSize,
Bt, ldb_Bt,
beta, C, ldc);
```

`bsrmm()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>dir</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>transA</td>
<td>the operation op(A).</td>
</tr>
<tr>
<td>transB</td>
<td>the operation op(B).</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows of sparse matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>number of columns of dense matrix op(B) and A.</td>
</tr>
<tr>
<td>kb</td>
<td>number of block columns of sparse matrix A.</td>
</tr>
<tr>
<td>nnzb</td>
<td>number of non-zero blocks of sparse matrix A.</td>
</tr>
<tr>
<td>alpha</td>
<td>&lt;type&gt; scalar used for multiplication.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>bsrValA</td>
<td>&lt;type&gt; array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0)) nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of ( mb + 1 ) elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of ( nnzb = bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix ( A ), larger than zero.</td>
</tr>
<tr>
<td>B</td>
<td>array of dimensions ((ldb, n)) if ( \text{op}(B) = B ) and ((ldb, k)) otherwise.</td>
</tr>
<tr>
<td>ldb</td>
<td>leading dimension of ( B ). If ( \text{op}(B) = B ), it must be at least ( \max(1, k) ) if ( \text{op}(B) \neq B ), it must be at least ( \max(1, n) ).</td>
</tr>
<tr>
<td>beta</td>
<td>(&lt;\text{type}&gt;) scalar used for multiplication. If ( \beta ) is zero, ( C ) does not have to be a valid input.</td>
</tr>
<tr>
<td>C</td>
<td>array of dimensions ((ldc, n)).</td>
</tr>
<tr>
<td>ldc</td>
<td>leading dimension of ( C ). It must be at least ( \max(1, m) ) if ( \text{op}(A) = A ) and at least ( \max(1, k) ) otherwise.</td>
</tr>
</tbody>
</table>

Output

\[ C \quad \text{<type> updated array of dimensions \((ldc, n)\).} \]

See \texttt{cusparseStatus_t} for the description of the return status.
9.2. cusparse\textless t\textgreater bsrsm2\_bufferSize()

\begin{verbatim}
cusparseStatus_t cusparseSbsrsm2_bufferSize(cusparseHandle_t handle, cusparseDirection_t dirA, cusparseOperation_t transA, cusparseOperation_t transX, int mb, int n, int nnzb, const cusparseMatDescr_t descrA, float* bsrSortedValA, const int* bsrSortedRowPtrA, const int* bsrSortedColIndA, int blockDim, bsrsm2Info_t info, int* pBufferSizeInBytes)
cusparseStatus_t cusparseDbsrsm2_bufferSize(cusparseHandle_t handle, cusparseDirection_t dirA, cusparseOperation_t transA, cusparseOperation_t transX, int mb, int n, int nnzb, const cusparseMatDescr_t descrA, double* bsrSortedValA, const int* bsrSortedRowPtrA, const int* bsrSortedColIndA, int blockDim, bsrsm2Info_t info, int* pBufferSizeInBytes)
cusparseStatus_t cusparseCbsrsm2_bufferSize(cusparseHandle_t handle, cusparseDirection_t dirA, cusparseOperation_t transA, cusparseOperation_t transX, int mb, int n, int nnzb, const cusparseMatDescr_t descrA, cuComplex* bsrSortedValA, const int* bsrSortedRowPtrA, const int* bsrSortedColIndA, int blockDim, bsrsm2Info_t info, int* pBufferSizeInBytes)
cusparseStatus_t cusparseZbsrsm2_bufferSize(cusparseHandle_t handle, cusparseDirection_t dirA, cusparseOperation_t transA, cusparseOperation_t transX, int mb, int n, int nnzb, const cusparseMatDescr_t descrA, cuDoubleComplex* bsrSortedValA, const int* bsrSortedRowPtrA, const int* bsrSortedColIndA, int blockDim, bsrsm2Info_t info, int* pBufferSizeInBytes)
\end{verbatim}
This function returns size of buffer used in `bsrsm2()`, a new sparse triangular linear system \( \text{op}(A) \ast \text{op}(X) = \alpha \text{op}(B) \).

\( A \) is an \((\text{mb} \times \text{blockDim}) \times (\text{mb} \times \text{blockDim})\) sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`; \( B \) and \( X \) are the right-hand-side and the solution matrices; \( \alpha \) is a scalar; and

\[
\text{op}(A) = \begin{cases} 
A & \text{if } \text{trans} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if } \text{trans} = \text{CUSPARSE_OPERATION_TRANSPOSE} \\
A^H & \text{if } \text{trans} = \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE}
\end{cases}
\]

Although there are six combinations in terms of parameter `trans` and the upper (and lower) triangular part of \( A \), \( \text{bsrsm2\_bufferSize()} \) returns the maximum size of the buffer among these combinations. The buffer size depends on dimension `mb`, `blockDim` and the number of nonzeros of the matrix, `nnzb`. If the user changes the matrix, it is necessary to call `bsrsm2\_bufferSize()` again to get the correct buffer size, otherwise a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>dirA</code></td>
<td>storage format of blocks, either <code>CUSPARSE_DIRECTION_ROW</code> or <code>CUSPARSE_DIRECTION_COLUMN</code>.</td>
</tr>
<tr>
<td><code>transA</code></td>
<td>the operation ( \text{op}(A) ).</td>
</tr>
<tr>
<td><code>transX</code></td>
<td>the operation ( \text{op}(X) ).</td>
</tr>
<tr>
<td><code>mb</code></td>
<td>number of block rows of matrix ( A ).</td>
</tr>
<tr>
<td><code>n</code></td>
<td>number of columns of matrix ( \text{op}(B) ) and ( \text{op}(X) ).</td>
</tr>
<tr>
<td><code>nnzb</code></td>
<td>number of nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td><code>descrA</code></td>
<td>the descriptor of matrix ( A ). The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code>, while the supported diagonal types are <code>CUSPARSE_DIAG_TYPE_UNIT</code> and <code>CUSPARSE_DIAG_TYPE_NON_UNIT</code>.</td>
</tr>
<tr>
<td><code>bsrValA</code></td>
<td>(&lt;\text{type}&gt;) array of ( \text{nnzb} (= \text{bsrRowPtrA}(\text{mb}) - \text{bsrRowPtrA}(0)) ) nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td><code>bsrRowPtrA</code></td>
<td>integer array of ( \text{mb} + 1 ) elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td><code>bsrColIndA</code></td>
<td>integer array of ( \text{nnzb} (= \text{bsrRowPtrA}(\text{mb}) - \text{bsrRowPtrA}(0)) ) column indices of the nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td><code>blockDim</code></td>
<td>block dimension of sparse matrix ( A ); larger than zero.</td>
</tr>
</tbody>
</table>

### Output
<table>
<thead>
<tr>
<th>info</th>
<th>record internal states based on different algorithms.</th>
</tr>
</thead>
<tbody>
<tr>
<td>pBufferSizeInBytes</td>
<td>number of bytes of the buffer used in bsrsm2_analysis() and bsrsm2_solve().</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
9.3. cusparse<t>bsrsm2_analysis()

cusparseStatus_t
cusparseSbsrsm2_analysis(cusparseHandle_t         handle,
cusparseDirection_t      dirA,
cusparseOperation_t      transA,
cusparseOperation_t      transX,
int                      mb,
int                      n,
int                      nnzb,
const cusparseMatDescr_t  descrA,
const float*             bsrSortedVal,
const int*               bsrSortedRowPtr,
const int*               bsrSortedColInd,
int                      blockDim,
bsrsm2Info_t             info,
cusparseSolvePolicy_t    policy,
void*                    pBuffer)

cusparseStatus_t
cusparseDbsrsm2_analysis(cusparseHandle_t         handle,
cusparseDirection_t      dirA,
cusparseOperation_t      transA,
cusparseOperation_t      transX,
int                      mb,
int                      n,
int                      nnzb,
const cusparseMatDescr_t  descrA,
const double*            bsrSortedVal,
const int*               bsrSortedRowPtr,
const int*               bsrSortedColInd,
int                      blockDim,
bsrsm2Info_t             info,
cusparseSolvePolicy_t    policy,
void*                    pBuffer)

cusparseStatus_t
cusparseCbsrsm2_analysis(cusparseHandle_t         handle,
cusparseDirection_t      dirA,
cusparseOperation_t      transA,
cusparseOperation_t      transX,
int                      mb,
int                      n,
int                      nnzb,
const cusparseMatDescr_t  descrA,
const cuComplex*         bsrSortedVal,
const int*               bsrSortedRowPtr,
const int*               bsrSortedColInd,
int                      blockDim,
bsrsm2Info_t             info,
cusparseSolvePolicy_t    policy,
void*                    pBuffer)

cusparseStatus_t
cusparseZbsrsm2_analysis(cusparseHandle_t         handle,
cusparseDirection_t      dirA,
cusparseOperation_t      transA,
cusparseOperation_t      transX,
int                      mb,
int                      n,
int                      nnzb,
const cusparseMatDescr_t  descrA,
const cuDoubleComplex*   bsrSortedVal,
const int*               bsrSortedRowPtr,
const int*               bsrSortedColInd,
This function performs the analysis phase of `bsrsm2()`, a new sparse triangular linear system \( \alpha \text{op}(A) \ast \text{op}(X) = \text{op}(B) \).

\( A \) is an \((mb \ast \text{blockDim}) \times (mb \ast \text{blockDim})\) sparse matrix that is defined in BSR storage format by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA`; \( B \) and \( X \) are the right-hand-side and the solution matrices; \( \alpha \) is a scalar; and

\[
\text{op}(A) = \begin{cases} 
A^T & \text{if } \text{trans} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^H & \text{if } \text{trans} = \text{CUSPARSE_OPERATION_TRANSPOSE}
\end{cases}
\]

and

\[
\text{op}(X) = \begin{cases} 
X^T & \text{if } \text{transX} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
X^H & \text{if } \text{transX} = \text{CUSPARSE_OPERATION_TRANSPOSE}
\end{cases}
\]

and \( \text{op}(B) \) and \( \text{op}(X) \) are equal.

The block of BSR format is of size \( \text{blockDim} \ast \text{blockDim} \), stored in column-major or row-major as determined by parameter \( \text{dirA} \), which is either \text{CUSPARSE_DIRECTION_ROW} or \text{CUSPARSE_DIRECTION_COLUMN}. The matrix type must be \text{CUSPARSE_MATRIX_TYPE_GENERAL}, and the fill mode and diagonal type are ignored.

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires the buffer size returned by `bsrsm2_bufferSize()`. The address of \( \text{pBuffer} \) must be multiple of 128 bytes. If not, \text{CUSPARSE_STATUS_INVALID_VALUE} is returned.

Function `bsrsm2_analysis()` reports a structural zero and computes the level information stored in opaque structure `info`. The level information can extract more parallelism during a triangular solver. However `bsrsm2_solve()` can be done without level information. To disable level information, the user needs to specify the policy of the triangular solver as \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL}.

Function `bsrsm2_analysis()` always reports the first structural zero, even if the parameter `policy` is \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL}. Besides, no structural zero is reported if \text{CUSPARSE_DIAG_TYPE_UNIT} is specified, even if block \( A(j,j) \) is missing for some \( j \). The user must call `cusparseXbsrsm2_query_zero_pivot()` to know where the structural zero is.

If `bsrsm2_analysis()` reports a structural zero, the solve will return a numerical zero in the same position as the structural zero but this result \( X \) is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>dirA</code></td>
<td>storage format of blocks, either \text{CUSPARSE_DIRECTION_ROW} or \text{CUSPARSE_DIRECTION_COLUMN}.</td>
</tr>
<tr>
<td>transA</td>
<td>the operation ( \text{op}(A) ).</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>transX</td>
<td>the operation ( \text{op}(B) ) and ( \text{op}(X) ).</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows of matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>number of columns of matrix ( \text{op}(B) ) and ( \text{op}(X) ).</td>
</tr>
<tr>
<td>nnzb</td>
<td>number of non-zero blocks of matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.</td>
</tr>
<tr>
<td>bsrValA</td>
<td>(&lt;\text{type}&gt;) array of ( \text{nnzb} = bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of ( \text{mb} + 1 ) elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of ( \text{nnzb} = bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix A; larger than zero.</td>
</tr>
<tr>
<td>info</td>
<td>structure initialized using cusparseCreateBsrsm2Info.</td>
</tr>
<tr>
<td>policy</td>
<td>The supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user; the size is return by bsrsm2_bufferSize().</td>
</tr>
</tbody>
</table>

**Output**

| info   | structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged). |

See cusparseStatus_t for the description of the return status
9.4. cusparse<t>bsrsm2_solve()

cusparseStatus_t
cusparseBsrsm2_solve(cusparseHandle_t         handle,
cusparseDirection_t      dirA,
cusparseOperation_t      transA,
cusparseOperation_t      transX,
int                      mb,
int                      n,
int                      nnzb,
const float*             alpha,
const cusparseMatDescr_t descrA,
const float*             bsrSortedVal,
const int*               bsrSortedRowPtr,
const int*               bsrSortedColInd,
int                      blockDim,
bsrsm2Info_t             info,
const float*             B,
int                      ldb,
float*                   X,
int                      ldx,
cusparseSolvePolicy_t    policy,
void*                    pBuffer)

cusparseStatus_t
cusparseDbsrsm2_solve(cusparseHandle_t         handle,
cusparseDirection_t      dirA,
cusparseOperation_t      transA,
cusparseOperation_t      transX,
int                      mb,
int                      n,
int                      nnzb,
const double*            alpha,
const cusparseMatDescr_t descrA,
const double*            bsrSortedVal,
const int*               bsrSortedRowPtr,
const int*               bsrSortedColInd,
int                      blockDim,
bsrsm2Info_t             info,
const double*            B,
int                      ldb,
double*                  X,
int                      ldx,
cusparseSolvePolicy_t    policy,
void*                    pBuffer)

cusparseStatus_t
cusparseCbsrsm2_solve(cusparseHandle_t         handle,
cusparseDirection_t      dirA,
cusparseOperation_t      transA,
cusparseOperation_t      transX,
int                      mb,
int                      n,
int                      nnzb,
const cuComplex*         alpha,
const cusparseMatDescr_t descrA,
const cuComplex*         bsrSortedVal,
const int*               bsrSortedRowPtr,
const int*               bsrSortedColInd,
int                      blockDim,
bsrsm2Info_t             info,
const cuComplex*         B,
int                      ldb,
cuComplex*               X,
int                      ldx,
cusparseSolvePolicy_t    policy,
void*                    pBuffer)

cusparseStatus_t
cusparseZbsrsm2_solve(cusparseHandle_t         handle,
cusparseDirection_t      dirA,
cusparseOperation_t      transA,
cusparseOperation_t      transX,
int                      mb,
int                      n,
int                      nnzb,
const cuDoubleComplex*   alpha,
const cusparseMatDescr_t descrA,
const cuDoubleComplex*   bsrSortedVal,
const int*               bsrSortedRowPtr,
const int*               bsrSortedColInd,
int                      blockDim,
bsrsm2Info_t             info,
const cuDoubleComplex*   B,
int                      ldb,
cuDoubleComplex*         X,
int                      ldx,
cusparseSolvePolicy_t    policy,
void*                    pBuffer)
This function performs the solve phase of the solution of a sparse triangular linear system:

\[ \text{op}(A) \times \text{op}(X) = \alpha \times \text{op}(B) \]

\( A \) is an \((mb \times \text{blockDim}) \times (mb \times \text{blockDim})\) sparse matrix that is defined in BSR storage format by the three arrays \( \text{bsrValA} \), \( \text{bsrRowPtrA} \), and \( \text{bsrColIndA} \); \( B \) and \( X \) are the right-hand-side and the solution matrices; \( \alpha \) is a scalar, and

\[
\text{op}(A) = \begin{cases} 
A & \text{if } \text{transA} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if } \text{transA} = \text{CUSPARSE_OPERATION_TRANSPOSE} \\
A^H & \text{if } \text{transA} = \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} 
\end{cases}
\]

and

\[
\text{op}(X) = \begin{cases} 
X & \text{if } \text{transX} = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
X^T & \text{if } \text{transX} = \text{CUSPARSE_OPERATION_TRANSPOSE} \\
X^H & \text{not supported}
\end{cases}
\]

Only \( \text{op}(A) = A \) is supported. 

\( \text{op}(B) \) and \( \text{op}(X) \) must be performed in the same way. In other words, if \( \text{op}(B) = B \), \( \text{op}(X) = X \).

The block of BSR format is of size \( \text{blockDim} \times \text{blockDim} \), stored as column-major or row-major as determined by parameter \( \text{dirA} \), which is either \text{CUSPARSE_DIRECTION_ROW} or \text{CUSPARSE_DIRECTION_COLUMN}. The matrix type must be \text{CUSPARSE_MATRIX_TYPE_GENERAL}, and the fill mode and diagonal type are ignored. Function \text{bsrsm02_solve()} can support an arbitrary \text{blockDim}.

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires the buffer size returned by \text{bsrsm2_bufferSize()}. The address of \( \text{pBuffer} \) must be multiple of 128 bytes. If it is not, \text{CUSPARSE_STATUS_INVALID_VALUE} is returned.

Although \text{bsrsm2_solve()} can be done without level information, the user still needs to be aware of consistency. If \text{bsrsm2_analysis()} is called with policy \text{CUSPARSE_SOLVE_POLICY_USE_LEVEL}, \text{bsrsm2_solve()} can be run with or without levels. On the other hand, if \text{bsrsm2_analysis()} is called with \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL}, \text{bsrsm2_solve()} can only accept \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL}; otherwise, \text{CUSPARSE_STATUS_INVALID_VALUE} is returned.

Function \text{bsrsm02_solve()} has the same behavior as \text{bsrsv02_solve()}, reporting the first numerical zero, including a structural zero. The user must call \text{cusparseXbsrsm2_query_zero_pivot()} to know where the numerical zero is.

The motivation of \text{transpose}(X) is to improve the memory access of matrix \( X \). The computational pattern of \text{transpose}(X) with matrix \( X \) in column-major order is equivalent to \( X \) with matrix \( X \) in row-major order.

In-place is supported and requires that \( B \) and \( X \) point to the same memory block, and \( ldb = ldx \).

The function supports the following properties if \( \text{pBuffer} \neq \text{NULL} \)
The routine requires no extra storage
The routine supports asynchronous execution
The routine supports CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>dirA</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>transA</td>
<td>the operation op(A).</td>
</tr>
<tr>
<td>transX</td>
<td>the operation op(B) and op(X).</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows of matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>number of columns of matrix op(B) and op(X).</td>
</tr>
<tr>
<td>nnzb</td>
<td>number of non-zero blocks of matrix A.</td>
</tr>
<tr>
<td>alpha</td>
<td>scalar used for multiplication.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.</td>
</tr>
<tr>
<td>bsrValA</td>
<td>array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0)) non-zero blocks of matrix A.</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of mb + 1 elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0)) column indices of the nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix A; larger than zero.</td>
</tr>
<tr>
<td>info</td>
<td>structure initialized using cusparseCreateBsrsm2Info().</td>
</tr>
<tr>
<td>B</td>
<td>right-hand-side array.</td>
</tr>
<tr>
<td>ldb</td>
<td>leading dimension of B. If op(B)=B, ldb &gt;= (mb*blockDim); otherwise, ldb &gt;= n.</td>
</tr>
<tr>
<td>ldx</td>
<td>leading dimension of X. If op(X)=X, then ldx &gt;= (mb*blockDim). Otherwise, ldx &gt;= n.</td>
</tr>
<tr>
<td>policy</td>
<td>the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user; the size is returned by bsrsm2_bufferSize().</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>solution array with leading dimensions ldx.</td>
</tr>
</tbody>
</table>
9.5. `cusparseXbsrsm2_zeroPivot()`

```c
 cusparseStatus_t
cusparseXbsrsm2_zeroPivot(cusparseHandle_t handle,
                          bsrsm2Info_t info,
                          int* position)
```

If the returned error code is `CUSPARSE_STATUS_ZERO_PIVOT`, `position=j` means $A(j,j)$ is either a structural zero or a numerical zero (singular block). Otherwise `position=-1`.

The `position` can be 0-base or 1-base, the same as the matrix.

Function `cusparseXbsrsm2_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The `position` can be in the host memory or device memory. The user can set the proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>info</td>
<td>info contains a structural zero or a numerical zero if the user already called <code>bsrsm2_analysis()</code> or <code>bsrsm2_solve()</code>.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>position</td>
<td>if no structural or numerical zero, <code>position</code> is -1; otherwise, if $A(j,j)$ is missing or $U(j,j)$ is zero, <code>position</code> = j.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
### 9.6. `cusparse<t>csrsm2_bufferSizeExt()`

<table>
<thead>
<tr>
<th>Function</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>cusparseStatus_t</code></td>
<td>handle, algo, transA, transB, m, nrhs, nnz, alpha, descrA, csrSortedValA, csrSortedRowPtrA,csrSortedColIndA, B, ldb, info, policy, pBufferSize</td>
</tr>
<tr>
<td><code>cusparseScsrsm2_bufferSizeExt</code></td>
<td>handle, algo, transA, transB, m, nrhs, nnz, alpha, descrA, csrSortedValA, csrSortedRowPtrA,csrSortedColIndA, B, ldb, info, policy, pBufferSize</td>
</tr>
<tr>
<td><code>cusparseDcsrsm2_bufferSizeExt</code></td>
<td>handle, algo, transA, transB, m, nrhs, nnz, alpha, descrA, csrSortedValA, csrSortedRowPtrA,csrSortedColIndA, B, ldb, info, policy, pBufferSize</td>
</tr>
<tr>
<td><code>cusparseCcsrsm2_bufferSizeExt</code></td>
<td>handle, algo, transA, transB, m, nrhs, nnz, alpha, descrA, csrSortedValA, csrSortedRowPtrA,csrSortedColIndA, B, ldb, info, policy, pBufferSize</td>
</tr>
<tr>
<td><code>cusparseZcsrsm2_bufferSizeExt</code></td>
<td>handle, algo, transA, transB, m, nrhs, nnz, alpha, descrA, csrSortedValA, csrSortedRowPtrA,csrSortedColIndA, B, ldb, info, policy, pBufferSize</td>
</tr>
</tbody>
</table>
This function returns the size of the buffer used in \texttt{csrsm2}, a sparse triangular linear system \( \text{op}(A) \times \text{op}(X) = \alpha \text{op}(B) \).

\( A \) is an \( m \times m \) sparse matrix that is defined in CSR storage format by the three arrays \texttt{csrValA}, \texttt{csrRowPtrA}, and \texttt{csrColIndA}; \( B \) and \( X \) are the right-hand-side matrix and the solution matrix; \( \alpha \) is a scalar; and

\[
\text{op}(A) = \begin{cases} 
A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\
A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\
A & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} 
\end{cases}
\]

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

\textbf{Input}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>algo</td>
<td>algo = 0 is non-block version; algo = 1 is block version.</td>
</tr>
<tr>
<td>transA</td>
<td>the operation ( \text{op}(A) ).</td>
</tr>
<tr>
<td>transB</td>
<td>the operation ( \text{op}(B) ).</td>
</tr>
<tr>
<td>( m )</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>nrhs</td>
<td>number of columns of right hand side matrix ( \text{op}(B) ).</td>
</tr>
<tr>
<td>nnz</td>
<td>number of nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>alpha</td>
<td>(&lt;\text{type}&gt;) scalar used for multiplication.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix ( A ). The supported matrix type is \text{CUSPARSE_MATRIX_TYPE_GENERAL}, while the supported diagonal types are \text{CUSPARSE_DIAG_TYPE_UNIT} and \text{CUSPARSE_DIAG_TYPE_NON_UNIT}.</td>
</tr>
<tr>
<td>\texttt{csrValA}</td>
<td>(&lt;\text{type}&gt;) array of ( \text{nnz} = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0) ) nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>\texttt{csrRowPtrA}</td>
<td>integer array of ( m + 1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>\texttt{csrColIndA}</td>
<td>integer array of ( \text{nnz} = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0) ) column indices of the nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>( B )</td>
<td>(&lt;\text{type}&gt;) right-hand-side matrix. ( \text{op}(B) ) is of size ( m \text{-by}\text{-} \text{nrhs} ).</td>
</tr>
<tr>
<td>ldb</td>
<td>leading dimension of ( B ) and ( X ).</td>
</tr>
<tr>
<td>info</td>
<td>structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).</td>
</tr>
<tr>
<td>policy</td>
<td>The supported policies are \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL} and \text{CUSPARSE_SOLVE_POLICY_USE_LEVEL}.</td>
</tr>
</tbody>
</table>
## Output

<table>
<thead>
<tr>
<th>info</th>
<th>record of internal states based on different algorithms.</th>
</tr>
</thead>
<tbody>
<tr>
<td>pBufferSize</td>
<td>number of bytes of the buffer used in the csrsm2_analysis and csrsm2_solve.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
9.7. cusparse<t>csrsm2_analysis()

```c
cusparseStatus_t
cusparseScsrsm2_analysis(cusparseHandle_t handle,
    int algorithm,    // algo,
    cusparseOperation_t transA,
    cusparseOperation_t transB,
    int m,
    int nrhs,
    int nnz,
    const float* alpha,
    const cusparseMatDescr_t descrA,
    const float* csrSortedValA,
    const int* csrSortedRowPtrA,
    const int* csrSortedColIndA,
    const float* B,
    int ldb,
    csrsm2Info_t info,
    cusparseSolvePolicy_t policy,
    void* pBuffer)

cusparseStatus_t
cusparseDcsrsm2_analysis(cusparseHandle_t handle,
    int algorithm,    // algo,
    cusparseOperation_t transA,
    cusparseOperation_t transB,
    int m,
    int nrhs,
    int nnz,
    const double* alpha,
    const cusparseMatDescr_t descrA,
    const double* csrSortedValA,
    const int* csrSortedRowPtrA,
    const int* csrSortedColIndA,
    const double* B,
    int ldb,
    csrsm2Info_t info,
    cusparseSolvePolicy_t policy,
    void* pBuffer)

cusparseStatus_t
cusparseCcsrsm2_analysis(cusparseHandle_t handle,
    int algorithm,    // algo,
    cusparseOperation_t transA,
    cusparseOperation_t transB,
    int m,
    int nrhs,
    int nnz,
    const cuComplex* alpha,
    const cusparseMatDescr_t descrA,
    const cuComplex* csrSortedValA,
    const int* csrSortedRowPtrA,
    const int* csrSortedColIndA,
    const cuComplex* B,
    int ldb,
    csrsm2Info_t info,
    cusparseSolvePolicy_t policy,
    void* pBuffer)

cusparseStatus_t
cusparseZcsrsm2_analysis(cusparseHandle_t handle,
    int algorithm,    // algo,
    cusparseOperation_t transA,
    cusparseOperation_t transB,
    int m,
    int nrhs,
    int nnz,
    const cuDoubleComplex* alpha,
    const cusparseMatDescr_t descrA,
    const cuDoubleComplex* csrSortedValA,
    const int* csrSortedRowPtrA,
    const int* csrSortedColIndA,
    const cuDoubleComplex* B,
    int ldb,
    csrsm2Info_t info,
    cusparseSolvePolicy_t policy,
    void* pBuffer)
```
This function performs the analysis phase of \( \text{csrsm2} \), a sparse triangular linear system
\[
\text{op}(A) \cdot \text{op}(X) = \alpha \cdot \text{op}(B).
\]

\( A \) is an \( m \times m \) sparse matrix that is defined in CSR storage format by the three arrays
\( \text{csrValA}, \text{csrRowPtrA}, \) and \( \text{csrColIndA} \); \( B \) and \( X \) are the right-hand-side matrix and
the solution matrix; \( \alpha \) is a scalar; and
\[
\text{op}(A) = \begin{cases} 
A & \text{if trans} == \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if trans} == \text{CUSPARSE_OPERATION_TRANSPOSE} \\
A^H & \text{if trans} == \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE}
\end{cases}
\]

It is expected that this function will be executed only once for a given matrix and a
particular operation type.

This function requires a buffer size returned by \( \text{csrsm2\_bufferSize()} \). The address of
\( pBuffer \) must be multiple of 128 bytes. If it is not, \text{CUSPARSE\_STATUS\_INVALID\_VALUE}
is returned.

Function \( \text{csrsm2\_analysis()} \) reports a structural zero and computes level information
that is stored in opaque structure \( \text{info} \). The level information can extract more
parallelism for a triangular solver. However \( \text{csrsm2\_solve()} \) can be done without
level information. To disable level information, the user needs to specify the policy of the
triangular solver as \text{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL}.

Function \( \text{csrsm2\_analysis()} \) always reports the first structural zero, even if the
policy is \text{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL}. No structural zero is reported if
\text{CUSPARSE\_DIAG\_TYPE\_UNIT} is specified, even if \( A(j,j) \) is missing for some \( j \). The
user needs to call \( \text{cusparseXcsrsm2\_zeroPivot()} \) to know where the structural zero
is.

It is the user’s choice whether to call \( \text{csrsm2\_solve()} \) if \( \text{csrsm2\_analysis()} \) reports
a structural zero. In this case, the user can still call \( \text{csrsm2\_solve()} \) which will return
a numerical zero in the same position as the structural zero. However the result \( X \) is
meaningless.

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>algo</td>
<td>algo = 0 is non-block version; algo = 1 is block version.</td>
</tr>
<tr>
<td>transA</td>
<td>the operation ( \text{op}(A) ).</td>
</tr>
<tr>
<td>transB</td>
<td>the operation ( \text{op}(B) ).</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>nrhs</td>
<td>number of columns of matrix ( \text{op}(B) ).</td>
</tr>
<tr>
<td>nnz</td>
<td>number of nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>alpha</td>
<td>(&lt;\text{type}&gt;) scalar used for multiplication.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix ( A ). The supported matrix type is \text{CUSPARSE_MATRIX_TYPE_GENERAL}, while the supported diagonal types are \text{CUSPARSE_DIAG_TYPE_UNIT} and \text{CUSPARSE_DIAG_TYPE_NON_UNIT}.</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
</tr>
<tr>
<td>-------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>csrValA</td>
<td></td>
</tr>
<tr>
<td></td>
<td><code>&lt;type&gt;</code> array of (\text{nnz} = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0)) nonzero elements of matrix (A).</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of (m + 1) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of (\text{nnz} = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0)) column indices of the nonzero elements of matrix (A).</td>
</tr>
<tr>
<td>B</td>
<td><code>&lt;type&gt;</code> right-hand-side matrix. (\text{op}(B)) is of size (m)-by-(\text{nrhs}).</td>
</tr>
<tr>
<td>ldb</td>
<td>leading dimension of (B) and (X).</td>
</tr>
<tr>
<td>info</td>
<td>structure initialized using \text{cusparseCreateCsrsv2Info}().</td>
</tr>
<tr>
<td>policy</td>
<td>The supported policies are \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL} and \text{CUSPARSE_SOLVE_POLICY_USE_LEVEL}.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user, the size is returned by \text{csrsm2_bufferSize}().</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>info</td>
<td>structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged).</td>
</tr>
</tbody>
</table>

See \text{cusparseStatus\_t} for the description of the return status.
9.8. cusparse<t>csrsm2_solve()

cusparseStatus_t
cusparseScsrsm2_solve(cusparseHandle_t handle, int algo, cusparseOperation_t transA, cusparseOperation_t transB, int m, int nrhs, int nnz, const float* alpha, const cusparseMatDescr_t descrA, const float* csrSortedValA, const int* csrSortedRowPtrA, const int* csrSortedColIndA, float* B, int ldb, csrsm2Info_t info, cusparseSolvePolicy_t policy, void* pBuffer)

cusparseStatus_t
cusparseDcsrsm2_solve(cusparseHandle_t handle, int algo, cusparseOperation_t transA, cusparseOperation_t transB, int m, int nrhs, int nnz, const double* alpha, const cusparseMatDescr_t descrA, const double* csrSortedValA, const int* csrSortedRowPtrA, const int* csrSortedColIndA, double* B, int ldb, csrsm2Info_t info, cusparseSolvePolicy_t policy, void* pBuffer)

cusparseStatus_t
cusparseCcsrsm2_solve(cusparseHandle_t handle, int algo, cusparseOperation_t transA, cusparseOperation_t transB, int m, int nrhs, int nnz, const cuComplex* alpha, const cusparseMatDescr_t descrA, const cuComplex* csrSortedValA, const int* csrSortedRowPtrA, const int* csrSortedColIndA, cuComplex* B, int ldb, csrsm2Info_t info, cusparseSolvePolicy_t policy, void* pBuffer)

cusparseStatus_t
cusparseZcsrsm2_solve(cusparseHandle_t handle, int algo, cusparseOperation_t transA, cusparseOperation_t transB, int m, int nrhs, int nnz, const cuDoubleComplex* alpha, const cusparseMatDescr_t descrA, const cuDoubleComplex* csrSortedValA, const int* csrSortedRowPtrA, const int* csrSortedColIndA, cuDoubleComplex* B, int ldb, csrsm2Info_t info, cusparseSolvePolicy_t policy, void* pBuffer)
This function performs the solve phase of \( \text{csrsm2} \), a sparse triangular linear system
\[
\text{op}(A) \ast \text{op}(X) = \alpha \text{op}(B).
\]

\( A \) is an \( m \times m \) sparse matrix that is defined in CSR storage format by the three arrays \( \text{csrValA}, \text{csrRowPtrA}, \text{and csrColIndA} \); \( B \) and \( X \) are the right-hand-side matrix and the solution matrix; \( \alpha \) is a scalar; and
\[
\text{op}(A) = \begin{cases} A & \text{if transA} = \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if transA} = \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if transA} = \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}
\]

\( \text{transB} \) acts on both matrix \( B \) and matrix \( X \), only \( \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \) and \( \text{CUSPARSE\_OPERATION\_TRANSPOSE} \). The operation is in-place, matrix \( B \) is overwritten by matrix \( X \).

\( \text{ldb} \) must be not less than \( m \) if \( \text{transB} = \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \). Otherwise, \( \text{ldb} \) must be not less than \( \text{nrhs} \).

This function requires the buffer size returned by \( \text{csrsm2\_bufferSize()} \). The address of \( \text{ pBuffer} \) must be multiple of 128 bytes. If it is not, \( \text{CUSPARSE\_STATUS\_INVALID\_VALUE} \) is returned.

Although \( \text{csrsm2\_solve()} \) can be done without level information, the user still needs to be aware of consistency. If \( \text{csrsm2\_analysis()} \) is called with policy \( \text{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL} \), \( \text{csrsm2\_solve()} \) can be run with or without levels. On the contrary, if \( \text{csrsm2\_analysis()} \) is called with \( \text{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL} \), \( \text{csrsm2\_solve()} \) can only accept \( \text{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL} \); otherwise, \( \text{CUSPARSE\_STATUS\_INVALID\_VALUE} \) is returned.

The level information may not improve the performance but spend extra time doing analysis. For example, a tridiagonal matrix has no parallelism.

In this case, \( \text{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL} \) performs better than \( \text{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL} \). If the user has an iterative solver, the best approach is to do \( \text{csrsm2\_analysis()} \) with \( \text{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL} \) once. Then do \( \text{csrsm2\_solve()} \) with \( \text{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL} \) in the first run and with \( \text{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL} \) in the second run, picking faster one to perform the remaining iterations.

Function \( \text{csrsm2\_solve()} \) reports the first numerical zero, including a structural zero. If \( \text{status} \) is 0, no numerical zero was found. Furthermore, no numerical zero is reported if \( \text{CUSPARSE\_DIAG\_TYPE\_UNIT} \) is specified, even if \( A(j,j) \) is zero for some \( j \). The user needs to call \( \text{cusparseXcsrsm2\_zeroPivot()} \) to know where the numerical zero is.

\( \text{csrsm2} \) provides two algorithms specified by the parameter \( \text{algo} \). \( \text{algo}=0 \) is non-block version and \( \text{algo}=1 \) is block version. non-block version is memory-bound, limited by bandwidth. block version partitions the matrix into small tiles and applies desne operations. Although it has more flops than non-block version, it may be faster if non-block version already reaches maximum bandwidth.

Appendix section shows an example of \( \text{csrsm2} \).

The function supports the following properties if \( \text{ pBuffer} \neq \text{ NULL} \)

- The routine requires no extra storage
The routine supports asynchronous execution
The routine supports CUDA graph capture

Input

<table>
<thead>
<tr>
<th>Input</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>algo</td>
<td>algo = 0 is non-block version; algo = 1 is block version.</td>
</tr>
<tr>
<td>transA</td>
<td>the operation ( \text{op}(A) ).</td>
</tr>
<tr>
<td>transB</td>
<td>the operation ( \text{op}(B) ).</td>
</tr>
<tr>
<td>m</td>
<td>number of rows and columns of matrix ( A ).</td>
</tr>
<tr>
<td>nrhs</td>
<td>number of columns of matrix ( \text{op}(B) ).</td>
</tr>
<tr>
<td>nnz</td>
<td>number of nonzeros of matrix ( A ).</td>
</tr>
<tr>
<td>alpha</td>
<td>(&lt;\text{type}&gt;) scalar used for multiplication.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix ( A ). The supported matrix type is \text{CUSPARSE_MATRIX_TYPE_GENERAL}, while the supported diagonal types are \text{CUSPARSE_DIAG_TYPE_UNIT} and \text{CUSPARSE_DIAG_TYPE_NON_UNIT}.</td>
</tr>
<tr>
<td>csrValA</td>
<td>(&lt;\text{type}&gt;) array of ( \text{nnz} = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0) ) nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of ( m+1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of ( \text{nnz} = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0) ) column indices of the nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>B</td>
<td>(&lt;\text{type}&gt;) right-hand-side matrix. ( \text{op}(B) ) is of size ( m )-by-( \text{nrhs} ).</td>
</tr>
<tr>
<td>ldb</td>
<td>leading dimension of ( B ) and ( X ).</td>
</tr>
<tr>
<td>info</td>
<td>structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).</td>
</tr>
<tr>
<td>policy</td>
<td>The supported policies are \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL} and \text{CUSPARSE_SOLVE_POLICY_USE_LEVEL}.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user, the size is returned by \text{csrsm2_bufferSize}.</td>
</tr>
</tbody>
</table>

Output

<table>
<thead>
<tr>
<th>Output</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>(&lt;\text{type}&gt;) solution matrix, ( \text{op}(X) ) is of size ( m )-by-( \text{nrhs} ).</td>
</tr>
</tbody>
</table>

See \text{cusparseStatus\_t} for the description of the return status.
9.9. cusparseXcsrsm2_zeroPivot()

```c
cusparseStatus_t
cusparseXcsrsm2_zeroPivot(cusparseHandle_t handle,
                         csrsm2Info_t info,
                         int* position)
```

If the returned error code is **CUSPARSE_STATUS_ZERO_PIVOT**, `position=j` means $A(j,j)$ has either a structural zero or a numerical zero. Otherwise `position=-1`.

The `position` can be 0-based or 1-based, the same as the matrix.

Function `cusparseXcsrsm2_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The `position` can be in the host memory or device memory. The user can set the proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine does **not** support asynchronous execution
- The routine does **not** support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>info</td>
<td>info contains structural zero or numerical zero if the user already called csrsm2_analysis() or csrsm2_solve.</td>
</tr>
</tbody>
</table>

**Output**

| position | if no structural or numerical zero, position is -1; otherwise, if $A(j,j)$ is missing or $U(j,j)$ is zero, position=j. |

See `cusparseStatus_t` for the description of the return status.
9.10. cuSparse<t>gemmi() [DEPRECATED]

[[DEPRECATED]] use cuSparseSpMM() instead. The routine will be removed in the next major release.

cusparseStatus_t
cusparseSgemmi(cusparseHandle_t handle,
    int m,
    int n,
    int k,
    int nnz,
    const float* alpha,
    const float* A,
    int lda,
    const float* cscValB,
    const int* cscColPtrB,
    const int* cscRowIndB,
    const float* beta,
    float* C,
    int ldc)

cusparseStatus_t
cusparseDgemmi(cusparseHandle_t handle,
    int m,
    int n,
    int k,
    int nnz,
    const double* alpha,
    const double* A,
    int lda,
    const double* cscValB,
    const int* cscColPtrB,
    const int* cscRowIndB,
    const double* beta,
    double* C,
    int ldc)

cusparseStatus_t
cusparseCgemmi(cusparseHandle_t handle,
    int m,
    int n,
    int k,
    int nnz,
    const cuComplex* alpha,
    const cuComplex* A,
    int lda,
    const cuComplex* cscValB,
    const int* cscColPtrB,
    const int* cscRowIndB,
    const cuComplex* beta,
    cuComplex* C,
    int ldc)

cusparseStatus_t
cusparseZgemmi(cusparseHandle_t handle,
    int m,
    int n,
    int k,
    int nnz,
    const cuDoubleComplex* alpha,
    const cuDoubleComplex* A,
    int lda,
    const cuDoubleComplex* cscValB,
    const int* cscColPtrB,
    const int* cscRowIndB,
This function performs the following matrix-matrix operations:

\[ C = \alpha \cdot A \cdot B + \beta \cdot C \]

\(A\) and \(C\) are dense matrices; \(B\) is a \(k \times n\) sparse matrix that is defined in CSC storage format by the three arrays \(\text{cscValB}, \text{cscColPtrB}, \text{and cscRowIndB}\); \(\alpha\) and \(\beta\) are scalars; and

Remark: \(B\) is base-0.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>(m)</td>
<td>number of rows of matrix (A).</td>
</tr>
<tr>
<td>(n)</td>
<td>number of columns of matrices (B) and (C).</td>
</tr>
<tr>
<td>(k)</td>
<td>number of columns of matrix (A).</td>
</tr>
<tr>
<td>(\text{nnz})</td>
<td>number of nonzero elements of sparse matrix (B).</td>
</tr>
<tr>
<td>(\text{alpha})</td>
<td>&lt;type&gt; scalar used for multiplication.</td>
</tr>
<tr>
<td>(A)</td>
<td>array of dimensions ( (\text{lda}, k) ).</td>
</tr>
<tr>
<td>(\text{lda})</td>
<td>leading dimension of (A). It must be at least (m).</td>
</tr>
<tr>
<td>(\text{cscValB})</td>
<td>&lt;type&gt; array of (\text{nnz} = \text{cscColPtrB}(k) - \text{cscColPtrB}(0) ) nonzero elements of matrix (B).</td>
</tr>
<tr>
<td>(\text{cscColPtrB})</td>
<td>integer array of (k + 1) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>(\text{cscRowIndB})</td>
<td>integer array of (\text{nnz} = \text{cscColPtrB}(k) - \text{cscColPtrB}(0) ) column indices of the nonzero elements of matrix (B).</td>
</tr>
<tr>
<td>(\text{beta})</td>
<td>&lt;type&gt; scalar used for multiplication. If (\text{beta}) is zero, (C) does not have to be a valid input.</td>
</tr>
<tr>
<td>(C)</td>
<td>array of dimensions ( (\text{ldc}, n) ).</td>
</tr>
<tr>
<td>(\text{ldc})</td>
<td>leading dimension of (C). It must be at least (m).</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C)</td>
<td>&lt;type&gt; updated array of dimensions ( (\text{ldc}, n) ).</td>
</tr>
</tbody>
</table>

See \(\text{cusparseStatus\_t}\) for the description of the return status.
This chapter describes the extra routines used to manipulate sparse matrices.
10.1. `cusparse<t>csrgeam2()`

```c
int
const float* alpha,
const cusparseMatDescr_t descrA,
int
const float* csrSortedValA,
const int* csrSortedColIndA,
const int* csrSortedRowPtrA,
const double* beta,
const cusparseMatDescr_t descrB,
int
const double* csrSortedValB,
const int* csrSortedColIndB,
const int* csrSortedRowPtrB,
const double* beta,
const cusparseMatDescr_t descrC,
int
const double* csrSortedValC,
const int* csrSortedColIndC,
const int* csrSortedRowPtrC,
```

```c
size_t*
```

```c
cusparseStatus_t
cusparseScsrgeam2_bufferSizeExt(cusparseHandle_t handle,
int m,
int n,
const float* alpha,
const cusparseMatDescr_t descrA,
int nnzA,
const float* csrSortedValA,
const int* csrSortedColIndA,
const int* csrSortedRowPtrA,
const int* csrSortedColIndB,
const int* csrSortedRowPtrB,
```

```c
const int* csrSortedColIndC,
```

```c
const int* csrSortedRowPtrB,
```

```c
const double* beta,
const cusparseMatDescr_t descrB,
int nnzB,
const double* csrSortedValB,
```

```c
const int* csrSortedColIndB,
```

```c
const double* csrSortedValC,
const int* csrSortedColIndC,
```

```c
const int* csrSortedRowPtrC,
```

```c
```

```c
cusparseStatus_t
cusparseDcsrgeam2_bufferSizeExt(cusparseHandle_t handle,
int m,
int n,
const double* alpha,
const cusparseMatDescr_t descrA,
int nnzA,
const double* csrSortedValA,
const int* csrSortedColIndA,
const int* csrSortedRowPtrA,
const int* csrSortedColIndB,
const int* csrSortedRowPtrB,
```

```c
const int* csrSortedRowPtrB,
```

```c
const double* beta,
const cusparseMatDescr_t descrB,
int nnzB,
const double* csrSortedValB,
```

```c
const int* csrSortedColIndB,
```

```c
const double* csrSortedValC,
const int* csrSortedColIndC,
```

```c
const int* csrSortedRowPtrC,
```

```c
```

```c
cusparseStatus_t
cusparseCcsrgeam2_bufferSizeExt(cusparseHandle_t handle,
int m,
int n,
const cuComplex* alpha,
const cusparseMatDescr_t descrA,
int nnzA,
const cuComplex* csrSortedValA,
const int* csrSortedColIndA,
const int* csrSortedRowPtrA,
const int* csrSortedColIndB,
const int* csrSortedRowPtrB,
```

```c
const int* csrSortedRowPtrB,
```

```c
const cuComplex* beta,
const cusparseMatDescr_t descrB,
int nnzB,
const cuComplex* csrSortedValB,
const int* csrSortedColIndB,
```

```c
const int* csrSortedColIndC,
```

```c
const int* csrSortedRowPtrC,
```

```c
const cuComplex* csrSortedValC,
const int* csrSortedColIndC,
```

```c
```

```c
cusparseStatus_t
cusparseZcsrgeam2_bufferSizeExt(cusparseHandle_t handle,
int m,
int n,
const cuDoubleComplex* alpha,
const cusparseMatDescr_t descrA,
int nnzA,
const cuDoubleComplex* csrSortedValA,
const int* csrSortedColIndA,
```

```c
const int* csrSortedRowPtrA,
```

```c
const cuDoubleComplex* beta,
```
This function performs following matrix-matrix operation

\[ C = \alpha A + \beta B \]

where \( A, B, \) and \( C \) are \( m \times n \) sparse matrices (defined in CSR storage format by the three 
arrays \( \text{csrValA}, \text{csrValB}, \text{csrValC}, \text{csrRowPtrA}, \text{csrRowPtrB}, \text{csrRowPtrC}, \) and 
\( \text{csrColIndA}, \text{csrColIndB}, \text{csrColIndC} \) respectively), and \( \alpha \) and \( \beta \) are scalars. 
Since \( A \) and \( B \) have different sparsity patterns, cuSPARSE adopts a two-step approach 
to complete sparse matrix \( C \). In the first step, the user allocates \( \text{csrRowPtrC} \) of \( m+1 \) elements 
and uses function \( \text{cusparseXcsrgeam2Nnz()} \) to determine \( \text{csrRowPtrC} \) and the total number of nonzero elements. 
In the second step, the user gathers \( \text{nnzC} \) (number of nonzero elements of matrix \( C \)) from either 
(\( \text{nnzC} = *\text{nnzTotalDevHostPtr} \)) or 
(\( \text{nnzC} = \text{csrRowPtrC}(m) - \text{csrRowPtrC}(0) \)) and allocates \( \text{csrValC}, \text{csrColIndC} \) 
of \( \text{nnzC} \) elements respectively, then finally calls function \( \text{cusparse[S|D|C|Z]csrgeam2()} \) to complete matrix \( C \).

The general procedure is as follows:

```c
int baseC, nnzC;
/* alpha, nnzTotalDevHostPtr points to host memory */
size_t BufferSizeInBytes;
char *buffer = NULL;
int *nnzTotalDevHostPtr = &nnzC;
cusparseSetPointerMode(handle, CUSPARSE_POINTER_MODE_HOST);
cudaMalloc((void**)&csrRowPtrC, sizeof(int)*(m+1));
/* prepare buffer */
cusparseScsrgeam2_bufferSizeExt(handle, m, n,
    alpha,
    descrA, nnzA,
    csrValA, csrRowPtrA, csrColIndA,
    beta,
    descrB, nnzB,
    csrValB, csrRowPtrB, csrColIndB,
    descrC,
    csrValC, csrRowPtrC, csrColIndC
    &BufferSizeInBytes);
    cudaMalloc((void**)&buffer, sizeof(char)*BufferSizeInBytes);
cusparseXcsrgeam2Nnz(handle, m, n,
    descrA, nnzA, csrRowPtrA, csrColIndA,
    descrB, nnzB, csrRowPtrB, csrColIndB,
    descrC, csrRowPtrC, nnzTotalDevHostPtr,
    buffer);
if (NULL != nnzTotalDevHostPtr){
    nnzC = *nnzTotalDevHostPtr;
} else{
    cudaMemcpy(&nnzC, csrRowPtrC+m, sizeof(int), cudaMemcpyDeviceToHost);
    cudaMemcpy(&baseC, csrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
    nnzC -= baseC;
}
cudaMalloc((void**)&csrColIndC, sizeof(int)*nnzC);
cudaMalloc((void**)&csrValC, sizeof(float)*nnzC);
cusparseScsrgeam2(handle, m, n,
    alpha,
    descrA, nnzA,
    csrValA, csrRowPtrA, csrColIndA,
    beta,
    descrB, nnzB,
    csrValB, csrRowPtrB, csrColIndB,
    descrC,
    csrValC, csrRowPtrC, csrColIndC
    buffer);
```
Several comments on `csrgeam2()`:

- The other three combinations, NT, TN, and TT, are not supported by cuSPARSE. In order to do any one of the three, the user should use the routine `csr2csc()` to convert \( A|B \) to \( A^T|B^T \).
- Only `CUSPARSE_MATRIX_TYPE_GENERAL` is supported. If either \( A \) or \( B \) is symmetric or Hermitian, then the user must extend the matrix to a full one and reconfigure the `MatrixType` field of the descriptor to `CUSPARSE_MATRIX_TYPE_GENERAL`.
- If the sparsity pattern of matrix \( C \) is known, the user can skip the call to function `cusparseXcsrgeam2Nnz()`.
- The pointers \( \alpha \) and \( \beta \) must be valid.
- When \( \alpha \) or \( \beta \) is zero, it is not considered a special case by cuSPARSE. The sparsity pattern of \( C \) is independent of the value of \( \alpha \) and \( \beta \). If the user wants \( C = 0 \times A + 1 \times B^T \), then `csr2csc()` is better than `csrgeam2()`.
- `csrgeam2()` is the same as `csrgeam()` except `csrgeam2()` needs explicit buffer where `csrgeam()` allocates the buffer internally.
- This function requires temporary extra storage that is allocated internally.
- The routine does **not** support asynchronous execution.
- The routine does **not** support CUDA graph capture.

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>m</code></td>
<td>number of rows of sparse matrix ( A, B, C ).</td>
</tr>
<tr>
<td><code>n</code></td>
<td>number of columns of sparse matrix ( A, B, C ).</td>
</tr>
<tr>
<td><code>alpha</code></td>
<td>(&lt;\text{type}&gt;) scalar used for multiplication.</td>
</tr>
<tr>
<td><code>descrA</code></td>
<td>the descriptor of matrix ( A ). The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code> only.</td>
</tr>
<tr>
<td><code>nnzA</code></td>
<td>number of nonzero elements of sparse matrix ( A ).</td>
</tr>
<tr>
<td><code>csrValA</code></td>
<td>(&lt;\text{type}&gt;) array of <code>nnzA</code> (= <code>csrRowPtrA(m)</code> - <code>csrRowPtrA(0)</code> ) nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td><code>csrRowPtrA</code></td>
<td>integer array of ( m + 1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td><code>csrColIndA</code></td>
<td>integer array of <code>nnzA</code> (= <code>csrRowPtrA(m)</code> - <code>csrRowPtrA(0)</code> ) column indices of the nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td><code>beta</code></td>
<td>(&lt;\text{type}&gt;) scalar used for multiplication. If <code>beta</code> is zero, ( y ) does not have to be a valid input.</td>
</tr>
<tr>
<td><code>descrB</code></td>
<td>the descriptor of matrix ( B ). The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code> only.</td>
</tr>
<tr>
<td><code>nnzB</code></td>
<td>number of nonzero elements of sparse matrix ( B ).</td>
</tr>
</tbody>
</table>
### csrValB

<type> array of \( \text{nnzB} = \text{csrRowPtrB}(m) - \text{csrRowPtrB}(0) \) nonzero elements of matrix B.

### csrRowPtrB

integer array of \( m + 1 \) elements that contains the start of every row and the end of the last row plus one.

### csrColIndB

integer array of \( \text{nnzB} = \text{csrRowPtrB}(m) - \text{csrRowPtrB}(0) \) column indices of the nonzero elements of matrix B.

### descrC

the descriptor of matrix C. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL` only.

### Output

<table>
<thead>
<tr>
<th>Description</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrValC</td>
<td>&lt;type&gt; array of ( \text{nnzC} = \text{csrRowPtrC}(m) - \text{csrRowPtrC}(0) ) nonzero elements of matrix C.</td>
</tr>
<tr>
<td>csrRowPtrC</td>
<td>integer array of ( m + 1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndC</td>
<td>integer array of ( \text{nnzC} = \text{csrRowPtrC}(m) - \text{csrRowPtrC}(0) ) column indices of the nonzero elements of matrix C.</td>
</tr>
<tr>
<td>nnzTotalDevHostPtr</td>
<td>total number of nonzero elements in device or host memory. It is equal to ( (\text{csrRowPtrC}(m) - \text{csrRowPtrC}(0)) ).</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
10.2. cusparse<t>csrgemm2() [DEPRECATED]

[[DEPRECATED]] use cusparseSpGEMM() instead. The routine will be removed in the next major release

```c

 cusparseStatus_t
 cusparseScsrgemm2_bufferSizeExt(cusparseHandle_t handle,
                   int m,
                   int n,
                   int k,
                   const float* alpha,
                   const cusparseMatDescr_t descrA,
                   int nnzA,
                   const int* csrRowPtrA,
                   const int* csrColIndA,
                   const cusparseMatDescr_t descrB,
                   int nnzB,
                   const int* csrRowPtrB,
                   const int* csrColIndB,
                   const float* beta,
                   const cusparseMatDescr_t descrC,
                   int nnzC,
                   const int* csrRowPtrC,
                   const int* csrColIndC,
                   const int* csrRowPtrD,
                   const int* csrColIndD,
                   const cusparseMatDescr_t descrD,
                   const int* csrRowPtrD,
                   const int* csrColIndD,
                   const cusparseMatDescr_t descrE,
                   const int* csrRowPtrE,
                   const int* csrColIndE,
                   const cusparseMatDescr_t descrF,
                   const int* csrRowPtrF,
                   const int* csrColIndF,
                   size_t* pBufferSizeInBytes)
```

```c

 cusparseStatus_t
 cusparseDcsrgemm2_bufferSizeExt(cusparseHandle_t handle,
                   int m,
                   int n,
                   int k,
                   const double* alpha,
                   const cusparseMatDescr_t descrA,
                   int nnzA,
                   const int* csrRowPtrA,
                   const int* csrColIndA,
                   const cusparseMatDescr_t descrB,
                   int nnzB,
                   const int* csrRowPtrB,
                   const int* csrColIndB,
                   const double* beta,
                   const cusparseMatDescr_t descrC,
                   int nnzC,
                   const int* csrRowPtrC,
                   const int* csrColIndC,
                   const cusparseMatDescr_t descrD,
                   int nnzD,
                   const int* csrRowPtrD,
                   const int* csrColIndD,
                   const cusparseMatDescr_t descrE,
                   const int* csrRowPtrE,
                   const int* csrColIndE,
                   const cusparseMatDescr_t descrF,
                   const int* csrRowPtrF,
                   const int* csrColIndF,
                   size_t* pBufferSizeInBytes)
```

```c

 cusparseStatus_t
 cusparseXcsrgemm2Nnz(cusparseHandle_t handle,
                      const cusparseMatDescr_t descrA,
                      const int* csrRowPtrA,
                      const int* csrColIndA,
                      const double* alpha,
                      const cusparseMatDescr_t descrB,
                      const int* csrRowPtrB,
                      const int* csrColIndB,
                      const double* beta,
                      const cusparseMatDescr_t descrC,
                      const int* csrRowPtrC,
                      const int* csrColIndC,
                      size_t* nnz)
```

```c

 cusparseStatus_t
 cusparseZcsrgemm2_bufferSizeExt(cusparseHandle_t handle,
                      const cusparseMatDescr_t descrA,
                      const int* csrRowPtrA,
                      const int* csrColIndA,
                      const cuDoubleComplex* alpha,
                      const cusparseMatDescr_t descrB,
                      const int* csrRowPtrB,
                      const int* csrColIndB,
                      const cuDoubleComplex* beta,
                      const cusparseMatDescr_t descrC,
                      const int* csrRowPtrC,
                      const int* csrColIndC,
                      const cuDoubleComplex* descD,
                      const int* csrRowPtrD,
                      const int* csrColIndD,
                      size_t* pBufferSizeInBytes)
```

```c

 cusparseStatus_t
 cusparseCsrgemm2_bufferSizeExt(cusparseHandle_t handle,
                      const cusparseMatDescr_t descrA,
                      const int* csrRowPtrA,
                      const int* csrColIndA,
                      const cuComplex* alpha,
                      const cusparseMatDescr_t descrB,
                      const int* csrRowPtrB,
                      const int* csrColIndB,
                      const cuComplex* beta,
                      const cusparseMatDescr_t descrC,
                      const int* csrRowPtrC,
                      const int* csrColIndC,
                      size_t* pBufferSizeInBytes)
```
This function performs following matrix-matrix operation:

\[ C = \alpha \* A \* B + \beta \* D \]

where \( A, B, D \) and \( C \) are \( m \times k, k \times n, m \times n \) and \( m \times n \) sparse matrices (defined in CSR storage format by the three arrays \( \text{csrValA} | \text{csrValB} | \text{csrValD} | \text{csrValC}, \text{csrRowPtrA} | \text{csrRowPtrB} | \text{csrRowPtrD} | \text{csrRowPtrC}, \text{csrColIndA} | \text{csrColIndB} | \text{csrColIndD} | \text{csrColIndC} \) respectively.

The \text{csrgemm2} uses \( \alpha \) and \( \beta \) to support the following operations:

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>NULL</td>
<td>NULL</td>
<td>invalid</td>
</tr>
<tr>
<td>NULL</td>
<td>!NULL</td>
<td>( C = \beta * D, A ) and ( B ) are used</td>
</tr>
<tr>
<td>!NULL</td>
<td>NULL</td>
<td>( C = \alpha * A * B, D ) is not used</td>
</tr>
<tr>
<td>!NULL</td>
<td>!NULL</td>
<td>( C = \alpha * A * B + \beta * D )</td>
</tr>
</tbody>
</table>

The numerical value of \( \alpha \) and \( \beta \) only affects the numerical values of \( C \), not its sparsity pattern. For example, if \( \alpha \) and \( \beta \) are not zero, the sparsity pattern of \( C \) is union of \( A \* B \) and \( D \), independent of numerical value of \( \alpha \) and \( \beta \).

The following table shows different operations according to the value of \( m, n \) and \( k \):

<table>
<thead>
<tr>
<th>( m, n, k )</th>
<th>operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m &lt; 0 ) or ( n &lt; 0 ) or ( k &lt; 0 )</td>
<td>invalid</td>
</tr>
<tr>
<td>( m ) is 0 or ( n ) is 0</td>
<td>do nothing</td>
</tr>
<tr>
<td>( m &gt; 0 ) and ( n &gt; 0 ) and ( k ) is 0</td>
<td>invalid if ( \beta ) is zero; ( C = \beta * D ) if ( \beta ) is not zero.</td>
</tr>
<tr>
<td>( m &gt; 0 ) and ( n &gt; 0 ) and ( k ) &gt; 0</td>
<td>( C = \beta * D ) if ( \alpha ) is zero. ( C = \alpha * A * B ) if ( \beta ) is zero. ( C = \alpha * A * B + \beta * D ) if ( \alpha ) and ( \beta ) are not zero.</td>
</tr>
</tbody>
</table>

This function requires the buffer size returned by \text{csrgemm2_bufferSizeExt()}.
The address of \text{pBuffer} must be multiple of 128 bytes. If it is not, \text{CUSPARSE_STATUS_INVALID_VALUE} is returned.

The cuSPARSE library adopts a two-step approach to complete sparse matrix. In the first step, the user allocates \( \text{csrRowPtrC} \) of \( m+1 \) elements and uses the function \text{cusparseXcsrgemm2Nnz()} to determine \( \text{csrRowPtrC} \) and the total number of nonzero elements. In the second step, the user gathers \( \text{nnzC} \) (the number of nonzero elements of matrix \( C \)) from either \text{(nnzC=nnzTotalDevHostPtr)} or \text{(nnzC=csrRowPtrC(m)-csrRowPtrC(0))} and allocates \( \text{csrValC} \) and \( \text{csrColIndC} \) of \( \text{nnzC} \) elements respectively, then finally calls function \text{cusparse[S|D|C|Z]csrgemm2()} to evaluate matrix \( C \).
The general procedure of \( C = -A \times B + D \) is as follows:

```c
// assume matrices A, B and D are ready.
int baseC, nnzC;
csr2info_t info = NULL;
size_t bufferSize;
void *buffer = NULL;

// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzC;
double alpha = -1.0;
double beta  =  1.0;
cusparseSetPointerMode(handle, CUSPARSE_POINTER_MODE_HOST);

// step 1: create an opaque structure
cusparseCreateCsr2info(&info);

// step 2: allocate buffer for csr2nnz and csr2

cusparseDcsr2_bufferSizeExt(handle, m, n, k, &alpha,
   descrA, nnzA, csrRowPtrA, csrColIndA,
   descrB, nnzB, csrRowPtrB, csrColIndB,
   &beta,
   descrD, nnzD, csrRowPtrD, csrColIndD,
   info,
   &bufferSize);
cudaMalloc(&buffer, bufferSize);

// step 3: compute csrRowPtrC

cudaMalloc((void**)&csrRowPtrC, sizeof(int)*(m+1));
cusparseXcsr2nnz(handle, m, n, k,
   descrA, nnzA, csrRowPtrA, csrColIndA,
   descrB, nnzB, csrRowPtrB, csrColIndB,
   descrD, nnzD, csrRowPtrD, csrColIndD,
   descrC, csrRowPtrC, nnzTotalDevHostPtr,
   info, buffer);
if (NULL != nnzTotalDevHostPtr){
   nnzC = *nnzTotalDevHostPtr;
} else{
   cudaMemcpy(&nnzC, csrRowPtrC+m, sizeof(int), cudaMemcpyDeviceToHost);
   cudaMemcpy(&baseC, csrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
   nnzC -= baseC;
}

// step 4: finish sparsity pattern and value of C

cudaMalloc((void**)&csrColIndC, sizeof(int)*nnzC);
cudaMalloc((void**)&csrValC, sizeof(double)*nnzC);

// Remark: set csrValC to null if only sparsity pattern is required.
cusparseDcsr2(handle, m, n, k, &alpha,
   descrA, nnzA, csrValA, csrRowPtrA, csrColIndA,
   descrB, nnzB, csrValB, csrRowPtrB, csrColIndB,
   &beta,
   descrD, nnzD, csrValD, csrRowPtrD, csrColIndD,
   descrC, csrValC, csrRowPtrC, csrColIndC,
   info, buffer);

// step 5: destroy the opaque structure
cusparseDestroyCsr2info(info);
```

Several comments on `csrgemm2()`:

- Only the NN version is supported. For other modes, the user has to transpose \( A \) or \( B \) explicitly.

- Only `CUSPARSE_MATRIX_TYPE_GENERAL` is supported. If either \( A \) or \( B \) is symmetric or Hermitian, the user must extend the matrix to a full one and reconfigure the `MatrixType` field descriptor to `CUSPARSE_MATRIX_TYPE_GENERAL`. 

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cuSPARSE Library
if `csrValC` is zero, only sparsity pattern of C is calculated.

The functions `cusparseXcsrgem2Nnz()` and `cusparse<t>csrgem2()` supports the following properties if `pBuffer != NULL`

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine does *not* support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of sparse matrix A, D and C.</td>
</tr>
<tr>
<td>n</td>
<td>number of columns of sparse matrix B, D and C.</td>
</tr>
<tr>
<td>k</td>
<td>number of columns/rows of sparse matrix A / B.</td>
</tr>
<tr>
<td>alpha</td>
<td><code>&lt;type&gt;</code> scalar used for multiplication.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.</td>
</tr>
<tr>
<td>nnzA</td>
<td>number of nonzero elements of sparse matrix A.</td>
</tr>
<tr>
<td>csrValA</td>
<td><code>&lt;type&gt;</code> array of nnzA nonzero elements of matrix A.</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of m+1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of nnzA column indices of the nonzero elements of matrix A.</td>
</tr>
<tr>
<td>descrB</td>
<td>the descriptor of matrix B. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.</td>
</tr>
<tr>
<td>nnzB</td>
<td>number of nonzero elements of sparse matrix B.</td>
</tr>
<tr>
<td>csrValB</td>
<td><code>&lt;type&gt;</code> array of nnzB nonzero elements of matrix B.</td>
</tr>
<tr>
<td>csrRowPtrB</td>
<td>integer array of k+1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndB</td>
<td>integer array of nnzB column indices of the nonzero elements of matrix B.</td>
</tr>
<tr>
<td>descrD</td>
<td>the descriptor of matrix D. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.</td>
</tr>
<tr>
<td>nnzD</td>
<td>number of nonzero elements of sparse matrix D.</td>
</tr>
<tr>
<td>csrValD</td>
<td><code>&lt;type&gt;</code> array of nnzD nonzero elements of matrix D.</td>
</tr>
<tr>
<td>csrRowPtrD</td>
<td>integer array of m+1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndD</td>
<td>integer array of nnzD column indices of the nonzero elements of matrix D.</td>
</tr>
</tbody>
</table>
### Input

<table>
<thead>
<tr>
<th><strong>beta</strong></th>
<th>(&lt;\text{type}) scalar used for multiplication.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>descrC</strong></td>
<td>the descriptor of matrix C. The supported matrix type is \text{CUSPARSE_MATRIX_TYPE_GENERAL} only.</td>
</tr>
<tr>
<td><strong>info</strong></td>
<td>structure with information used in \text{csrgemm2Nnz} and \text{csrgemm2}.</td>
</tr>
<tr>
<td><strong>pBuffer</strong></td>
<td>buffer allocated by the user; the size is returned by \text{csrgemm2_bufferSizeExt}.</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th><strong>csrValC</strong></th>
<th>(&lt;\text{type}) array of (\text{nnzC}) nonzero elements of matrix C.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>csrRowPtrC</strong></td>
<td>integer array of (m+1) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td><strong>csrColIndC</strong></td>
<td>integer array of (\text{nnzC}) column indices of the nonzero elements of matrix C.</td>
</tr>
<tr>
<td><strong>pBufferSizeInBytes</strong></td>
<td>number of bytes of the buffer used in \text{csrgemm2Nnz} and \text{csrgemm2}.</td>
</tr>
<tr>
<td><strong>nnzTotalDevHostPtr</strong></td>
<td>total number of nonzero elements in device or host memory. It is equal to ((\text{csrRowPtrC}(m) - \text{csrRowPtrC}(0))).</td>
</tr>
</tbody>
</table>

See \text{cusparseStatus\_t} for the description of the return status
Chapter 11.
CUSPARSE PRECONDITIONERS REFERENCE

This chapter describes the routines that implement different preconditioners.

11.1. Incomplete Cholesky Factorization: level 0

Different algorithms for ic0 are discussed in this section.
11.1.1. cusparse<t>csric02_bufferSize()

This function returns size of buffer used in computing the incomplete-Cholesky factorization with 0 fill-in and no pivoting:

\[ A \approx LL^H \]

\( A \) is an \( m \times m \) sparse matrix that is defined in CSR storage format by the three arrays \( \text{csrValA}, \text{csrRowPtrA}, \text{and csrColIndA}. \)

The buffer size depends on dimension \( m \) and \( nnz \), the number of nonzeros of the matrix. If the user changes the matrix, it is necessary to call \text{csric02_bufferSize()} again to have the correct buffer size; otherwise, a segmentation fault may occur.
The routine requires no extra storage
The routine supports asynchronous execution
The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>m</code></td>
<td>number of rows and columns of matrix ( A ).</td>
</tr>
<tr>
<td><code>nnz</code></td>
<td>number of nonzeros of matrix ( A ).</td>
</tr>
<tr>
<td><code>descrA</code></td>
<td>the descriptor of matrix ( A ). The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code>. Also, the supported index bases are <code>CUSPARSE_INDEX_BASE_ZERO</code> and <code>CUSPARSE_INDEX_BASE_ONE</code>.</td>
</tr>
<tr>
<td><code>csrValA</code></td>
<td><code>&lt;type&gt;</code> array of ( nnz = csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td><code>csrRowPtrA</code></td>
<td>integer array of ( m ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td><code>csrColIndA</code></td>
<td>integer array of ( nnz = csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix ( A ).</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>info</code></td>
<td>record internal states based on different algorithms.</td>
</tr>
<tr>
<td><code>pBufferSizeInBytes</code></td>
<td>number of bytes of the buffer used in <code>csric02_analysis()</code> and <code>csric02()</code>.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
11.1.2. `cusparse<t>csric02_analysis()`

```c
cusparseStatus_t
cusparseScsric02_analysis(cusparseHandle_t handle, int m, int nnz, const cusparseMatDescr_t descrA, const float* csrValA, const int* csrRowPtrA, const int* csrColIndA, csric02Info_t info, cusparseSolvePolicy_t policy, void* pBuffer);

cusparseStatus_t
cusparseDcsric02_analysis(cusparseHandle_t handle, int m, int nnz, const cusparseMatDescr_t descrA, const double* csrValA, const int* csrRowPtrA, const int* csrColIndA, csric02Info_t info, cusparseSolvePolicy_t policy, void* pBuffer);

cusparseStatus_t
cusparseCcsric02_analysis(cusparseHandle_t handle, int m, int nnz, const cusparseMatDescr_t descrA, const cuComplex* csrValA, const int* csrRowPtrA, const int* csrColIndA, csric02Info_t info, cusparseSolvePolicy_t policy, void* pBuffer);

cusparseStatus_t
cusparseZcsric02_analysis(cusparseHandle_t handle, int m, int nnz, const cusparseMatDescr_t descrA, const cuDoubleComplex* csrValA, const int* csrRowPtrA, const int* csrColIndA, csric02Info_t info, cusparseSolvePolicy_t policy, void* pBuffer);
```

This function performs the analysis phase of the incomplete-Cholesky factorization with 0 fill-in and no pivoting:

$$A \approx LL^H$$

$A$ is an $m \times m$ sparse matrix that is defined in CSR storage format by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA`. 

This function requires a buffer size returned by `csric02_bufferSize()`. The address of `pBuffer` must be multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `csric02_analysis()` reports a structural zero and computes level information stored in the opaque structure `info`. The level information can extract more parallelism during incomplete Cholesky factorization. However `csric02()` can be done without level information. To disable level information, the user must specify the policy of `csric02_analysis()` and `csric02()` as `CUSPARSE_SOLVE_POLICY_NO_LEVEL`.

Function `csric02_analysis()` always reports the first structural zero, even if the policy is `CUSPARSE_SOLVE_POLICY_NO_LEVEL`. The user needs to call `cusparseXcsric02_zeroPivot()` to know where the structural zero is.

It is the user's choice whether to call `csric02()` if `csric02_analysis()` reports a structural zero. In this case, the user can still call `csric02()`, which will return a numerical zero at the same position as the structural zero. However the result is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>number of rows and columns of matrix A.</td>
</tr>
<tr>
<td>nnz</td>
<td>number of nonzeros of matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code>. Also, the supported index bases are <code>CUSPARSE_INDEX_BASE_ZERO</code> and <code>CUSPARSE_INDEX_BASE_ONE</code>.</td>
</tr>
<tr>
<td>csrValA</td>
<td><code>&lt;type&gt;</code> array of <code>nnz (= csrRowPtrA(m) - csrRowPtrA(0) )</code> nonzero elements of matrix A.</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of <code>m + 1</code> elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of <code>nnz (= csrRowPtrA(m) - csrRowPtrA(0) )</code> column indices of the nonzero elements of matrix A.</td>
</tr>
<tr>
<td>info</td>
<td>structure initialized using <code>cusparseCreateCsric02Info()</code>.</td>
</tr>
<tr>
<td>policy</td>
<td>the supported policies are <code>CUSPARSE_SOLVE_POLICY_NO_LEVEL</code> and <code>CUSPARSE_SOLVE_POLICY_USE_LEVEL</code>.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user; the size is returned by <code>csric02_bufferSize()</code></td>
</tr>
</tbody>
</table>

### Output
This function performs the solve phase of the computing the incomplete-Cholesky factorization with 0 fill-in and no pivoting:
This function requires a buffer size returned by `csr02_bufferSize()`. The address of `pBuffer` must be a multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Although `csric02()` can be done without level information, the user still needs to be aware of consistency. If `csric02_analysis()` is called with policy `CUSPARSE_SOLVE_POLICY_USE_LEVEL`, `csric02()` can be run with or without levels. On the other hand, if `csric02_analysis()` is called with `CUSPARSE_SOLVE_POLICY_NO_LEVEL`, `csric02()` can only accept `CUSPARSE_SOLVE_POLICY_NO_LEVEL`; otherwise, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `csric02()` reports the first numerical zero, including a structural zero. The user must call `cusparseXcsric02_zeroPivot()` to know where the numerical zero is.

Function `csric02()` only takes the lower triangular part of matrix `A` to perform factorization. The matrix type must be `CUSPARSE_MATRIX_TYPE_GENERAL`, the fill mode and diagonal type are ignored, and the strictly upper triangular part is ignored and never touched. It does not matter if `A` is Hermitian or not. In other words, from the point of view of `csric02()` `A` is Hermitian and only the lower triangular part is provided.

---

**In practice, a positive definite matrix may not have incomplete cholesky factorization. To the best of our knowledge, only matrix `M` can guarantee the existence of incomplete cholesky factorization. If `csric02()` failed cholesky factorization and reported a numerical zero, it is possible that incomplete cholesky factorization does not exist.**

For example, suppose `A` is a real `m × m` matrix, the following code solves the precondition system `M*y = x` where `M` is the product of Cholesky factorization `L` and its transpose.
// Suppose that $A$ is $m \times m$ sparse matrix represented by CSR format,
// Assumption:
// - handle is already created by cusparseCreate(),
// - $(d\_csrRowPtr, d\_csrColInd, d\_csrVal)$ is CSR of $A$ on device memory,
// - $d\_x$ is right hand side vector on device memory,
// - $d\_y$ is solution vector on device memory.
// - $d\_z$ is intermediate result on device memory.

cusparseMatDescr_t descr_M = 0;
cusparseMatDescr_t descr_L = 0;
csric02Info_t info_M = 0;
csrsv2Info_t info_L = 0;
csrsv2Info_t info_Lt = 0;
int pBufferSize_M;
int pBufferSize_L;
int pBufferSize_Lt;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy_M = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_L = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_Lt = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans_L = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_Lt = CUSPARSE_OPERATION_TRANSPOSE;

// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has non-unit diagonal
cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_M, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseCreateMatDescr(&descr_L);
cusparseSetMatIndexBase(descr_L, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_NON_UNIT);

// step 2: create a empty info structure
// we need one info for csric02 and two info's for csrsv2
cusparseCreateCsric02Info(&info_M);
cusparseCreateCsrsv2Info(&info_L);
cusparseCreateCsrsv2Info(&info_Lt);

// step 3: query how much memory used in csric02 and csrsv2, and allocate the buffer
cusparseDcsric02_bufferSize(handle, m, nnz,
    descr_M, d_csrVal, d_csrRowPtr, d_csrColInd, info_M, &pBufferSize_M);
cusparseDcsrsv2_bufferSize(handle, trans_L, m, nnz,
    descr_L, d_csrVal, d_csrRowPtr, d_csrColInd, info_L, &pBufferSize_L);
cusparseDcsrsv2_bufferSize(handle, trans_Lt, m, nnz,
    descr_L, d_csrVal, d_csrRowPtr, d_csrColInd, info_Lt, &pBufferSize_Lt);
pBufferSize = max(pBufferSize_M, max(pBufferSize_L, pBufferSize_Lt));

// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis of incomplete Cholesky on M
// perform analysis of triangular solve on L
// The lower triangular part of M has the same sparsity pattern as L, so
// we can do analysis of csric02 and csrsv2 simultaneously.
cusparseDcsric02_analysis(handle, m, nnz, descr_M,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_M, policy_M, pBuffer);
status = cusparseXcsric02_zeroPivot(handle, info_M, &structural_zero);

if (CUSPARSE_STATUS_ZERO_PIVOT == status){
    printf("A(%d,%d) is missing\n", structural_zero, structural_zero);
}

cusparseDcsrsv2_analysis(handle, trans_L, m, nnz, &alpha, descr_L,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_L, policy_L, pBuffer);

cusparseDcsrsv2_analysis(handle, trans_Lt, m, nnz, &alpha, descr_L,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_Lt, policy_Lt, pBuffer);

// step 5: M = L * L'
cusparseDcsric02(handle, m, nnz, descr_M,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_M, policy_M, pBuffer);
status = cusparseXcsric02_zeroPivot(handle, info_M, &numerical_zero);

if (CUSPARSE_STATUS_ZERO_PIVOT == status){
    printf("L(%d,%d) is zero\n", numerical_zero, numerical_zero);
}

// step 6: solve L*z = x
cusparseDcsrsv2_solve(handle, trans_L, m, nnz, &alpha, descr_L,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_L, d_x, d_z, policy_L, pBuffer);

// step 7: solve L'*y = z
cusparseDcsrsv2_solve(handle, trans_Lt, m, nnz, &alpha, descr_L,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_Lt, d_z, d_y, policy_Lt, pBuffer);

// step 6: free resources
cudaFree(pBuffer);
cusparseDestroyMatDescr(descr_M);
cusparseDestroyMatDescr(descr_L);
cusparseDestroyCsric02Info(info_M);
cusparseDestroyCsrsv2Info(info_L);
cusparseDestroyCsrsv2Info(info_Lt);
cusparseDestroy(handle);
The function supports the following properties if `pBuffer != NULL`

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

### Input

<table>
<thead>
<tr>
<th><strong>handle</strong></th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>m</strong></td>
<td>number of rows and columns of matrix A.</td>
</tr>
<tr>
<td><strong>nnz</strong></td>
<td>number of nonzeros of matrix A.</td>
</tr>
<tr>
<td><strong>descrA</strong></td>
<td>the descriptor of matrix A. The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code>. Also, the supported index bases are <code>CUSPARSE_INDEX_BASE_ZERO</code> and <code>CUSPARSE_INDEX_BASE_ONE</code>.</td>
</tr>
<tr>
<td><strong>csrValA_valM</strong></td>
<td><code>&lt;type&gt; array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) nonzero elements of matrix A.</code></td>
</tr>
<tr>
<td><strong>csrRowPtrA</strong></td>
<td>integer array of <code>m + 1</code> elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td><strong>csrColIndA</strong></td>
<td>integer array of <code>nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column indices of the nonzero elements of matrix A.</code></td>
</tr>
<tr>
<td><strong>info</strong></td>
<td>structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).</td>
</tr>
<tr>
<td><strong>policy</strong></td>
<td>the supported policies are <code>CUSPARSE_SOLVE_POLICY_NO_LEVEL</code> and <code>CUSPARSE_SOLVE_POLICY_USE_LEVEL</code>.</td>
</tr>
</tbody>
</table>
| **pBuffer** | buffer allocated by the user; the size is returned by `csric02_bufferSize()`.

### Output

| **csrValA_valM** | `<type> matrix containing the incomplete-Cholesky lower triangular factor.` |

See `cusparseStatus_t` for the description of the return status

#### 11.1.4. `cusparseXcsric02_zeroPivot()`

```c
cusparseStatus_t
cusparseXcsric02_zeroPivot(cusparseHandle_t handle,        
                           csric02Info_t info,          
                           int* position)
```

If the returned error code is `CUSPARSE_STATUS_ZERO_PIVOT`, `position=j` means `A(j,j)` has either a structural zero or a numerical zero; otherwise, `position=-1`.

The `position` can be 0-based or 1-based, the same as the matrix.
Function `cusparseXcsric02_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The position can be in the host memory or device memory. The user can set proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>info</td>
<td>info contains structural zero or numerical zero if the user already called <code>csric02_analysis()</code> or <code>csric02()</code>.</td>
</tr>
</tbody>
</table>

**Output**

| position | if no structural or numerical zero, position is -1; otherwise, if \( A(j,j) \) is missing or \( L(j,j) \) is zero, position=j. |

See `cusparseStatus_t` for the description of the return status.
11.1.5. 

This function returns the size of a buffer used in computing the incomplete-Cholesky factorization with 0 fill-in and no pivoting

\[ A \approx LL^H \]
**A** is an \((mb \times blockDim) \times (mb \times blockDim)\) sparse matrix that is defined in BSR storage format by the three arrays \(\text{bsrValA}, \text{bsrRowPtrA},\) and \(\text{bsrColIndA}\.\)

The buffer size depends on the dimensions of \(mb, blockDim,\) and the number of nonzero blocks of the matrix \(nnzb\.\) If the user changes the matrix, it is necessary to call \(\text{bsric02_bufferSize()}\) again to have the correct buffer size; otherwise, a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>dirA</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows and block columns of matrix (A.)</td>
</tr>
<tr>
<td>nnzb</td>
<td>number of nonzero blocks of matrix (A.)</td>
</tr>
<tr>
<td>descrA</td>
<td>descriptor of matrix (A.) The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>bsrValA</td>
<td>&lt;type&gt; array of (nnzb = bsrRowPtrA(mb) - bsrRowPtrA(0)) nonzero blocks of matrix (A.)</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of (mb + 1) elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of (nnzb = bsrRowPtrA(mb) - bsrRowPtrA(0)) column indices of the nonzero blocks of matrix (A.)</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix (A,) larger than zero.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>info</td>
<td>record internal states based on different algorithms.</td>
</tr>
<tr>
<td>pBufferSizeInBytes</td>
<td>number of bytes of the buffer used in (\text{bsric02_analysis()}) and (\text{bsric02()}).</td>
</tr>
</tbody>
</table>

See \(\text{cusparseStatus_t}\) for the description of the return status.
11.1.6. cusparse<t>bsric02_analysis()

cusparseStatus_t
cusparseSbsric02_analysis(cusparseHandle_t handle,
cusparseDirection_t dirA,
int mb,
int nnzb,
const cusparseMatDescr_t descrA,
const float* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
bsric02Info_t info,
cusparseSolvePolicy_t policy,
void* pBuffer)

cusparseStatus_t
cusparseDbsric02_analysis(cusparseHandle_t handle,
cusparseDirection_t dirA,
int mb,
int nnzb,
const cusparseMatDescr_t descrA,
const double* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
bsric02Info_t info,
cusparseSolvePolicy_t policy,
void* pBuffer)

cusparseStatus_t
cusparseCbsric02_analysis(cusparseHandle_t handle,
cusparseDirection_t dirA,
int mb,
int nnzb,
const cusparseMatDescr_t descrA,
const cuComplex* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
bsric02Info_t info,
cusparseSolvePolicy_t policy,
void* pBuffer)

cusparseStatus_t
cusparseZbsric02_analysis(cusparseHandle_t handle,
cusparseDirection_t dirA,
int mb,
int nnzb,
const cusparseMatDescr_t descrA,
const cuDoubleComplex* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
bsric02Info_t info,
cusparseSolvePolicy_t policy,
void* pBuffer)
This function performs the analysis phase of the incomplete-Cholesky factorization with 0 fill-in and no pivoting

\[ A \approx LL^H \]

\[ A \text{ is an } (mb \times blockDim) \times (mb \times blockDim) \text{ sparse matrix that is defined in BSR storage format by the three arrays } bsrValA, bsrRowPtrA, \text{ and } bsrColIndA. \]

The block in BSR format is of size blockDim \times blockDim, stored as column-major or row-major as determined by parameter dirA, which is either CUSPARSE_DIRECTION_COLUMN or CUSPARSE_DIRECTION_ROW. The matrix type must be CUSPARSE_MATRIX_TYPE_GENERAL, and the fill mode and diagonal type are ignored.

This function requires a buffer size returned by bsric02_bufferSize90. The address of pBuffer must be a multiple of 128 bytes. If it is not, CUSPARSE_STATUS_INVALID_VALUE is returned.

Function bsric02_analysis() reports structural zero and computes level information stored in the opaque structure info. The level information can extract more parallelism during incomplete Cholesky factorization. However bsric02() can be done without level information. To disable level information, the user needs to specify the parameter policy of bsric02[_analysis| ] as CUSPARSE_SOLVE_POLICY_NO_LEVEL.

Function bsric02_analysis always reports the first structural zero, even when parameter policy is CUSPARSE_SOLVE_POLICY_NO_LEVEL. The user must call cusparseXbsric02_zeroPivot() to know where the structural zero is.

It is the user’s choice whether to call bsric02() if bsric02_analysis() reports a structural zero. In this case, the user can still call bsric02(), which returns a numerical zero in the same position as the structural zero. However the result is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

Input

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>dirA</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows and block columns of matrix A.</td>
</tr>
<tr>
<td>nnzb</td>
<td>number of nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>bsrValA</td>
<td>&lt;type&gt; array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</td>
</tr>
</tbody>
</table>
### bsrRowPtrA
Integer array of `mb + 1` elements that contains the start of every block row and the end of the last block row plus one.

### bsrColIndA
Integer array of `nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0))` column indices of the nonzero blocks of matrix `A`.

### blockDim
Block dimension of sparse matrix `A`; must be larger than zero.

### info
Structure initialized using `cusparseCreateBsric02Info()`.

### policy
The supported policies are `CUSPARSE_SOLVE_POLICY_NO_LEVEL` and `CUSPARSE_SOLVE_POLICY_USE_LEVEL`.

### pBuffer
Buffer allocated by the user; the size is returned by `bsric02_bufferSize()`.

### Output

| info | Structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged). |

See `cusparseStatus_t` for the description of the return status.
11.1.7. cusparse<t>bsric02()

```c
cusparseStatus_t cusparseBsric02(cusparseHandle_t         handle,
                          cusparseDirection_t      dirA,
                          int                      mb,
                          int                      nnzb,
                          const cusparseMatDescr_t descrA,
                          float*                   bsrValA,
                          const int*               bsrRowPtrA,
                          const int*               bsrColIndA,
                          int                      blockDim,
                          bsric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)
```

```c
cusparseStatus_t cusparseDbsric02(cusparseHandle_t         handle,
                          cusparseDirection_t      dirA,
                          int                      mb,
                          int                      nnzb,
                          const cusparseMatDescr_t descrA,
                          double*                  bsrValA,
                          const int*               bsrRowPtrA,
                          const int*               bsrColIndA,
                          int                      blockDim,
                          bsric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)
```

```c
cusparseStatus_t cusparseCbsric02(cusparseHandle_t         handle,
                          cusparseDirection_t      dirA,
                          int                      mb,
                          int                      nnzb,
                          const cusparseMatDescr_t descrA,
                          cuComplex*               bsrValA,
                          const int*               bsrRowPtrA,
                          const int*               bsrColIndA,
                          int                      blockDim,
                          bsric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)
```

```c
cusparseStatus_t cusparseZbsric02(cusparseHandle_t         handle,
                          cusparseDirection_t      dirA,
                          int                      mb,
                          int                      nnzb,
                          const cusparseMatDescr_t descrA,
                          cuDoubleComplex*         bsrValA,
                          const int*               bsrRowPtrA,
                          const int*               bsrColIndA,
                          int                      blockDim,
                          bsric02Info_t            info,
                          cusparseSolvePolicy_t    policy,
                          void*                    pBuffer)
```
This function performs the solve phase of the incomplete-Cholesky factorization with 0 fill-in and no pivoting

\[ A \approx LL^H \]

\( A \) is an \((mb*blockDim) \times (mb*blockDim)\) sparse matrix that is defined in BSR storage format by the three arrays \( bsrValA, bsrRowPtrA, \) and \( bsrColIndA \). The block in BSR format is of size \( blockDim*blockDim \), stored as column-major or row-major as determined by parameter \( \text{dirA} \), which is either \( \text{CUSPARSE_DIRECTION_COLUMN} \) or \( \text{CUSPARSE_DIRECTION_ROW} \). The matrix type must be \( \text{CUSPARSE_MATRIX_TYPE_GENERAL} \), and the fill mode and diagonal type are ignored.

This function requires a buffer size returned by \( \text{bsric02_bufferSize()} \). The address of \( pBuffer \) must be a multiple of 128 bytes. If it is not, \( \text{CUSPARSE_STATUS_INVALID_VALUE} \) is returned.

Although \( \text{bsric02()} \) can be done without level information, the user must be aware of consistency. If \( \text{bsric02\_analysis()} \) is called with policy \( \text{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL} \), \( \text{bsric02()} \) can be run with or without levels. On the other hand, if \( \text{bsric02\_analysis()} \) is called with \( \text{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL} \), \( \text{bsric02()} \) can only accept \( \text{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL} \); otherwise, \( \text{CUSPARSE\_STATUS\_INVALID\_VALUE} \) is returned.

Function \( \text{bsric02()} \) has the same behavior as \( \text{csric02()} \). That is, \( \text{bsr2csr(\text{bsric02}(A)) = csric02(\text{bsr2csr}(A))} \). The numerical zero of \( \text{csric02()} \) means there exists some zero \( L(j,j) \). The numerical zero of \( \text{bsric02()} \) means there exists some block \( L(j,j) \) that is not invertible.

Function \( \text{bsric02()} \) reports the first numerical zero, including a structural zero. The user must call \( \text{cusparseXbsric02\_zeroPivot()} \) to know where the numerical zero is.

The \( \text{bsric02()} \) function only takes the lower triangular part of matrix \( A \) to perform factorization. The strictly upper triangular part is ignored and never touched. It does not matter if \( A \) is Hermitian or not. In other words, from the point of view of \( \text{bsric02()} \), \( A \) is Hermitian and only the lower triangular part is provided. Moreover, the imaginary part of diagonal elements of diagonal blocks is ignored.

For example, suppose \( A \) is a real \( m \)-by-\( m \) matrix, where \( m=mb*blockDim \). The following code solves precondition system \( M*y = x \), where \( M \) is the product of Cholesky factorization \( L \) and its transpose.
\[ M = LL^H \]

// Suppose that A is m x m sparse matrix represented by BSR format,
// The number of block rows/columns is mb, and
// the number of nonzero blocks is nnzb.
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_bsrRowPtr, d_bsrColInd, d_bsrVal) is BSR of A on device memory,
// - d_x is right hand side vector on device memory,
// - d_y is solution vector on device memory.
// - d_z is intermediate result on device memory.
// - d_x, d_y and d_z are of size m.
cusparseMatDescr_t descr_M = 0;
cusparseMatDescr_t descr_L = 0;
bsric02Info_t info_M = 0;
bsrsv2Info_t info_L = 0;
bsrsv2Info_t info_Lt = 0;
int pBufferSize_M;
int pBufferSize_L;
int pBufferSize_Lt;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy_M = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_L = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_Lt = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans_L = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_Lt = CUSPARSE_OPERATION_TRANSPOSE;
const cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;

// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has non-unit diagonal
 cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_M, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseCreateMatDescr(&descr_L);
cusparseSetMatIndexBase(descr_L, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_NON_UNIT);

// step 2: create a empty info structure
// we need one info for bsric02 and two info's for bsrsv2
 cusparseCreateBsric02Info(&info_M);
cusparseCreateBsrsv2Info(&info_L);
cusparseCreateBsrsv2Info(&info_Lt);

// step 3: query how much memory used in bsric02 and bsrsv2, and allocate the
// buffer
 cusparseDbsric02_bufferSize(handle, dir, mb, nnzb,
 descr_M, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_M,
 &bufferSize_M);
cusparseDbsrsv2_bufferSize(handle, dir, trans_L, mb, nnzb,
 descr_L, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_L,
 &pBufferSize_L);
cusparseDbsrsv2_bufferSize(handle, dir, trans_Lt, mb, nnzb,
 descr_L, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_Lt,
 &pBufferSize_Lt);
pBufferSize = max(bufferSize_M, max(pBufferSize_L, pBufferSize_Lt));

// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis of incomplete Cholesky on M
// perform analysis of triangular solve on L
// The lower triangular part of M has the same sparsity pattern as L, so
// cusparseDbsric02_analysis(handle, dir, mb, nnzb, descr_M,
// d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_M,
// policy_M, pBuffer);
status = cusparseXbsric02_zeroPivot(handle, info_M, &structural_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
 printf("A(%d,%d) is missing\n", structural_zero, structural_zero);
}

// step 5: perform analysis of triangular solve on L
 cusparseDbsrsv2_analysis(handle, dir, trans_L, mb, nnzb, descr_L,
 d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_L,
 policy_L, pBuffer);
 cusparseDbsrsv2_analysis(handle, dir, trans_Lt, mb, nnzb, descr_L,
 d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_Lt,
 policy_Lt, pBuffer);

// step 6: free resources
 cudaFree(pBuffer);
cusparseDestroyMatDescr(descr_M);
cusparseDestroyMatDescr(descr_L);
cusparseDestroyBsric02Info(info_M);
cusparseDestroyBsrsv2Info(info_L);
cusparseDestroyBsrsv2Info(info_Lt);
cusparseDestroy(handle);
The function supports the following properties if `pBuffer != NULL`

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>dirA</code></td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td><code>mb</code></td>
<td>number of block rows and block columns of matrix A.</td>
</tr>
<tr>
<td><code>nnzb</code></td>
<td>number of nonzero blocks of matrix A.</td>
</tr>
<tr>
<td><code>descrA</code></td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td><code>bsrValA</code></td>
<td><code>&lt;type&gt;</code> array of <code>nnzb</code> (= <code>bsrRowPtrA(mb) - bsrRowPtrA(0)</code>) nonzero blocks of matrix A.</td>
</tr>
<tr>
<td><code>bsrRowPtrA</code></td>
<td>integer array of <code>mb</code> + 1 elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td><code>bsrColIndA</code></td>
<td>integer array of <code>nnzb</code> (= <code>bsrRowPtrA(mb) - bsrRowPtrA(0)</code>) column indices of the nonzero blocks of matrix A.</td>
</tr>
<tr>
<td><code>blockDim</code></td>
<td>block dimension of sparse matrix A, larger than zero.</td>
</tr>
<tr>
<td><code>info</code></td>
<td>structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).</td>
</tr>
<tr>
<td><code>policy</code></td>
<td>the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.</td>
</tr>
<tr>
<td><code>pBuffer</code></td>
<td>buffer allocated by the user, the size is returned by <code>bsric02_bufferSize()</code>.</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>bsrValA</code></td>
<td><code>&lt;type&gt;</code> matrix containing the incomplete-Cholesky lower triangular factor.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
11.1.8. cusparseXbsric02_zeroPivot()

cusparseStatus_t
cusparseXbsric02_zeroPivot(cusparseHandle_t handle,
bsric02Info_t info,
int* position)

If the returned error code is CUSPARSE_STATUS_ZERO_PIVOT, position=j means
A(j,j) has either a structural zero or a numerical zero (the block is not positive
definite). Otherwise position=-1.

The position can be 0-based or 1-based, the same as the matrix.

Function cusparseXbsric02_zeroPivot() is a blocking call. It calls
cudaDeviceSynchronize() to make sure all previous kernels are done.

The position can be in the host memory or device memory. The user can set the proper
mode with cusparseSetPointerMode().

- The routine requires no extra storage
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

Input

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>info</td>
<td>info contains a structural zero or a</td>
</tr>
<tr>
<td></td>
<td>numerical zero if the user already called</td>
</tr>
<tr>
<td></td>
<td>bsric02_analysis() OR bsric02().</td>
</tr>
</tbody>
</table>

Output

<table>
<thead>
<tr>
<th>position</th>
<th>if no structural or numerical zero, position is -1,</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>otherwise if A(j,j) is missing or L(j,j) is not</td>
</tr>
<tr>
<td></td>
<td>positive definite, position=j.</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status

11.2. Incomplete LU Factorization: level 0

Different algorithms for ilu0 are discussed in this section.
11.2.1. **cusparse\textless t\textgreater csrilu02_numericBoost()**

```c
 cusparseStatus_t
cusparseScsrilu02_numericBoost(cusparseHandle_t handle,
                              csrilu02Info_t   info,
                              int              enable_boost,
                              double*         tol,
                              float*          boost_val)

cusparseStatus_t
cusparseDcsrilu02_numericBoost(cusparseHandle_t handle,
                              csrilu02Info_t   info,
                              int              enable_boost,
                              double*         tol,
                              double*         boost_val)

cusparseStatus_t
cusparseCcsrilu02_numericBoost(cusparseHandle_t handle,
                              csrilu02Info_t   info,
                              int              enable_boost,
                              double*         tol,
                              cuComplex*      boost_val)

cusparseStatus_t
cusparseZcsrilu02_numericBoost(cusparseHandle_t handle,
                              csrilu02Info_t   info,
                              int              enable_boost,
                              double*         tol,
                              cuDoubleComplex* boost_val)
```

The user can use a boost value to replace a numerical value in incomplete LU factorization. The $\texttt{tol}$ is used to determine a numerical zero, and the $\texttt{boost\_val}$ is used to replace a numerical zero. The behavior is

if $\texttt{tol} \geq \text{fabs}(A(j,j))$, then $A(j,j)=$boost\_val.

To enable a boost value, the user has to set parameter $\texttt{enable\_boost}$ to 1 before calling $\texttt{csrilu02()}$. To disable a boost value, the user can call $\texttt{csrilu02\_numericBoost()}$ again with parameter $\texttt{enable\_boost}=0$.

If $\texttt{enable\_boost}=0$, $\texttt{tol}$ and $\texttt{boost\_val}$ are ignored.

Both $\texttt{tol}$ and $\texttt{boost\_val}$ can be in the host memory or device memory. The user can set the proper mode with $\texttt{cusparseSetPointerMode()}$.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>info</td>
<td>structure initialized using $\texttt{cusparseCreateCsrilu02Info()}$.</td>
</tr>
<tr>
<td>enable_boost</td>
<td>disable boost by $\texttt{enable_boost}=0$; otherwise, boost is enabled.</td>
</tr>
</tbody>
</table>
tol

- tolerance to determine a numerical zero.

boost_val

- boost value to replace a numerical zero.

See **cusparseStatus_t** for the description of the return status

### 11.2.2. cusparse<t>csrilu02_bufferSize()

```c

cusparseStatus_t
cusparseScsrilu02_bufferSize(cusparseHandle_t handle, int m, int

const cusparseMatDescr_t descrA, float*

csrValA, const int*

csrRowPtrA, const int*

csrColIndA, cusrilu02Info_t info,

int*

pBufferSizeInBytes)

cusparseStatus_t
cusparseDcsrilu02_bufferSize(cusparseHandle_t handle, int m, int

const cusparseMatDescr_t descrA, double*

csrValA, const int*

csrRowPtrA, const int*

csrColIndA, cusrilu02Info_t info,

int*

pBufferSizeInBytes)

cusparseStatus_t
cusparseCcsrilu02_bufferSize(cusparseHandle_t handle, int m, int

const cusparseMatDescr_t descrA, cuComplex*

csrValA, const int*

csrRowPtrA, const int*

csrColIndA, cusrilu02Info_t info,

int*

pBufferSizeInBytes)

cusparseStatus_t
cusparseZcsrilu02_bufferSize(cusparseHandle_t handle, int m, int

const cusparseMatDescr_t descrA, cuDoubleComplex*

csrValA, const int*

csrRowPtrA, const int*

csrColIndA, cusrilu02Info_t info,

int*

pBufferSizeInBytes)
```

This function returns size of the buffer used in computing the incomplete-LU factorization with 0 fill-in and no pivoting:
A is an $m \times m$ sparse matrix that is defined in CSR storage format by the three arrays $\text{csrValA}$, $\text{csrRowPtrA}$, and $\text{csrColIndA}$.

The buffer size depends on the dimension $m$ and $\text{nnz}$, the number of nonzeros of the matrix. If the user changes the matrix, it is necessary to call $\text{csrilu02_bufferSize()}$ again to have the correct buffer size; otherwise, a segmentation fault may occur.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>number of rows and columns of matrix $A$.</td>
</tr>
<tr>
<td>$\text{nnz}$</td>
<td>number of nonzeros of matrix $A$.</td>
</tr>
<tr>
<td>$\text{descrA}$</td>
<td>the descriptor of matrix $A$. The supported matrix type is $\text{CUSPARSE_MATRIX_TYPE_GENERAL}$. Also, the supported index bases are $\text{CUSPARSE_INDEX_BASE_ZERO}$ and $\text{CUSPARSE_INDEX_BASE_ONE}$.</td>
</tr>
<tr>
<td>$\text{csrValA}$</td>
<td>$&lt;$type$&gt;$ array of $\text{nnz}(=\text{csrRowPtrA}(m) - \text{csrRowPtrA}(0))$ nonzero elements of matrix $A$.</td>
</tr>
<tr>
<td>$\text{csrRowPtrA}$</td>
<td>integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>$\text{csrColIndA}$</td>
<td>integer array of $\text{nnz}(=\text{csrRowPtrA}(m) - \text{csrRowPtrA}(0))$ column indices of the nonzero elements of matrix $A$.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>$\text{info}$</th>
<th>record internal states based on different algorithms.</th>
</tr>
</thead>
<tbody>
<tr>
<td>p$\text{BufferSizeInBytes}$</td>
<td>number of bytes of the buffer used in $\text{csrilu02}()$ and $\text{csrilu02_bufferSize()}$</td>
</tr>
</tbody>
</table>

See $\text{cusparseStatus_t}$ for the description of the return status.
11.2.3. cusparse<t>csrilu02_analysis()

This function performs the analysis phase of the incomplete-LU factorization with fill-in and no pivoting:

\[ A \approx LU \]

\( A \) is an \( m \times m \) sparse matrix that is defined in CSR storage format by the three arrays \( \text{csrValA} \), \( \text{csrRowPtrA} \), and \( \text{csrColIndA} \).
This function requires the buffer size returned by `csrilu02_bufferSize()`. The address of `pBuffer` must be a multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

Function `csrilu02_analysis()` reports a structural zero and computes level information stored in the opaque structure `info`. The level information can extract more parallelism during incomplete LU factorization; however `csrilu02()` can be done without level information. To disable level information, the user must specify the policy of `csrilu02()` as `CUSPARSE_SOLVE_POLICY_NO_LEVEL`.

It is the user’s choice whether to call `csrilu02()` if `csrilu02_analysis()` reports a structural zero. In this case, the user can still call `csrilu02()`, which will return a numerical zero at the same position as the structural zero. However the result is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine does **not** support asynchronous execution
- The routine does **not** support CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>m</code></td>
<td>number of rows and columns of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>nnz</code></td>
<td>number of nonzeros of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>descrA</code></td>
<td>the descriptor of matrix <code>A</code>. The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code>. Also, the supported index bases are <code>CUSPARSE_INDEX_BASE_ZERO</code> and <code>CUSPARSE_INDEX_BASE_ONE</code>.</td>
</tr>
<tr>
<td><code>csrValA</code></td>
<td><code>&lt;type&gt;</code> array of <code>nnz</code> (= <code>csrRowPtrA(m) - csrRowPtrA(0)</code> ) nonzero elements of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>csrRowPtrA</code></td>
<td>integer array of <code>m + 1</code> elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td><code>csrColIndA</code></td>
<td>integer array of <code>nnz</code> (= <code>csrRowPtrA(m) - csrRowPtrA(0)</code> ) column indices of the nonzero elements of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>info</code></td>
<td>structure initialized using <code>cusparseCreateCsrilu02Info()</code>.</td>
</tr>
<tr>
<td><code>policy</code></td>
<td>the supported policies are <code>CUSPARSE_SOLVE_POLICY_NO_LEVEL</code> and <code>CUSPARSE_SOLVE_POLICY_USE_LEVEL</code>.</td>
</tr>
<tr>
<td><code>pBuffer</code></td>
<td>buffer allocated by the user, the size is returned by <code>csrilu02_bufferSize()</code>.</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>info</code></td>
<td>structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged).</td>
</tr>
</tbody>
</table>


See `cusparseStatus_t` for the description of the return status

### 11.2.4. cusparse<t>csrilu02()

```
cusparseStatus_t
cusparseScsrilu02(cusparseHandle_t handle, int m, int nnz,
               const cusparseMatDescr_t descrA,
               float* csrValA_valM,
               const int* csrRowPtrA,
               const int* csrColIndA,
               csrilu02Info_t info,
               cusparseSolvePolicy_t policy,
               void* pBuffer)
```

```
cusparseStatus_t
cusparseDcsrilu02(cusparseHandle_t handle, int m, int nnz,
                const cusparseMatDescr_t descrA,
                double* csrValA_valM,
                const int* csrRowPtrA,
                const int* csrColIndA,
                csrilu02Info_t info,
                cusparseSolvePolicy_t policy,
                void* pBuffer)
```

```
cusparseStatus_t
cusparseCcsrilu02(cusparseHandle_t handle, int m, int nnz,
                 const cusparseMatDescr_t descrA,
                 cuComplex* csrValA_valM,
                 const int* csrRowPtrA,
                 const int* csrColIndA,
                 csrilu02Info_t info,
                 cusparseSolvePolicy_t policy,
                 void* pBuffer)
```

```
cusparseStatus_t
cusparseZcsrilu02(cusparseHandle_t handle, int m, int nnz,
                 const cusparseMatDescr_t descrA,
                 cuDoubleComplex* csrValA_valM,
                 const int* csrRowPtrA,
                 const int* csrColIndA,
                 csrilu02Info_t info,
                 cusparseSolvePolicy_t policy,
                 void* pBuffer)
```

This function performs the solve phase of the incomplete-LU factorization with 0 fill-in and no pivoting:

\[ A \approx LU \]

\( A \) is an \( m \times m \) sparse matrix that is defined in CSR storage format by the three arrays \( \text{csrValA}_\text{valM} \), \( \text{csrRowPtrA} \), and \( \text{csrColIndA} \).
This function requires a buffer size returned by \texttt{csrilu02\_bufferSize()}. The address of \texttt{pBuffer} must be a multiple of 128 bytes. If not, \texttt{CUSPARSE\_STATUS\_INVALID\_VALUE} is returned.

The matrix type must be \texttt{CUSPARSE\_MATRIX\_TYPE\_GENERAL}. The fill mode and diagonal type are ignored.

Although \texttt{csrilu02()} can be done without level information, the user still needs to be aware of consistency. If \texttt{csrilu02\_analysis()} is called with policy \texttt{CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL}, \texttt{csrilu02()} can be run with or without levels. On the other hand, if \texttt{csrilu02\_analysis()} is called with \texttt{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL}, \texttt{csrilu02()} can only accept \texttt{CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL}; otherwise, \texttt{CUSPARSE\_STATUS\_INVALID\_VALUE} is returned.

Function \texttt{csrilu02()} reports the first numerical zero, including a structural zero. The user must call \texttt{cusparseXcsrilu02\_zeroPivot()} to know where the numerical zero is.
For example, suppose $A$ is a real $m \times m$ matrix, the following code solves precondition system $M^*y = x$ where $M$ is the product of LU factors $L$ and $U$.

```
// Suppose that A is m x m sparse matrix represented by CSR format,
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_csrRowPtr, d_csrColInd, d_csrVal) is CSR of A on device memory,
// - d_x is right hand side vector on device memory,
// - d_y is solution vector on device memory.
// - d_z is intermediate result on device memory.

cusparseMatDescr_t descr_M = 0;
cusparseMatDescr_t descr_L = 0;
cusparseMatDescr_t descr_U = 0;
csrilu02Info_t info_M = 0;
crsrv2Info_t info_L = 0;
crsrv2Info_t info_U = 0;
int pBufferSize_M;
int pBufferSize_L;
int pBufferSize_U;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy_M = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_L = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_U = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans_L = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_U = CUSPARSE_OPERATION_NON_TRANSPOSE;

// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has unit diagonal
// - matrix U is base-1
// - matrix U is upper triangular
// - matrix U has non-unit diagonal
cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_M, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseCreateMatDescr(&descr_L);
cusparseSetMatIndexBase(descr_L, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_UNIT);
cusparseCreateMatDescr(&descr_U);
cusparseSetMatIndexBase(descr_U, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_U, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_U, CUSPARSE_FILL_MODE_UPPER);
cusparseSetMatDiagType(descr_U, CUSPARSE_DIAG_TYPE_NON_UNIT);

// step 2: create a empty info structure
// we need one info for csrilu02 and two info's for csrsv2
cusparseCreateCsrilu02Info(&info_M);
cusparseCreateCrsrv2Info(&info_L);
cusparseCreateCrsrv2Info(&info_U);

// step 3: query how much memory used in csrilu02 and csrsv2, and allocate the buffer
 cusparseDcsrilu02_bufferSize(handle, m, nnz, descr_M, d_csrVal, d_csrRowPtr, d_csrColInd, info_M, &pBufferSize_M);
cusparseDcsrv2_bufferSize(handle, trans_L, m, nnz, descr_L, d_csrVal, d_csrRowPtr, d_csrColInd, info_L, &pBufferSize_L);
cusparseDcsrv2_bufferSize(handle, trans_U, m, nnz, descr_U, d_csrVal, d_csrRowPtr, d_csrColInd, info_u, &pBufferSize_U);
pBufferSize = max(pBufferSize_M, max(pBufferSize_L, pBufferSize_U));

// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);

// step 4: perform analysis of incomplete Cholesky on M
 cusparseDcsrilu02_analysis(handle, m, nnz, descr_M, d_csrVal, d_csrRowPtr, d_csrColInd, info_M, policy_M, pBuffer);

status = cusparseXcsrilu02_zeroPivot(handle, info_M, &structural_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
    printf("A(%d,%d) is missing\n", structural_zero, structural_zero);
}

cusparseDcsrsv2_analysis(handle, trans_L, m, nnz, descr_L, d_csrVal, d_csrRowPtr, d_csrColInd, info_L, policy_L, pBuffer);

// step 5: M = L * U
 cusparseDcsrilu02(handle, m, nnz, descr_M, d_csrVal, d_csrRowPtr, d_csrColInd, info_M, policy_M, pBuffer);

status = cusparseXcsrilu02_zeroPivot(handle, info_M, &numerical_zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status){
    printf("U(%d,%d) is zero\n", numerical_zero, numerical_zero);
}

// step 6: solve L*z = x
 cusparseDcsrsv2_solve(handle, trans_L, m, nnz, &alpha, descr_L, d_csrVal, d_csrRowPtr, d_csrColInd, info_L, d_x, d_z, policy_L, pBuffer);

// step 7: solve U*y = z
 cusparseDcsrsv2_solve(handle, trans_U, m, nnz, &alpha, descr_U, d_csrVal, d_csrRowPtr, d_csrColInd, info_U, d_z, d_y, policy_U, pBuffer);

// step 6: free resources
 cudaFree(pBuffer);
cusparseDestroyMatDescr(descr_M);
cusparseDestroyMatDescr(descr_L);
cusparseDestroyMatDescr(descr_U);
cusparseDestroyCsrilu02Info(info_M);
cusparseDestroyCsrsv2Info(info_L);
cusparseDestroyCsrsv2Info(info_U);
cusparseDestroy(handle);
```
The function supports the following properties if pBuffer != NULL

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>number of rows and columns of matrix A.</td>
</tr>
<tr>
<td>nnz</td>
<td>number of nonzeros of matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>csrValA_valM</td>
<td>&lt;type&gt; array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) nonzero elements of matrix A.</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of m + 1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column indices of the nonzero elements of matrix A.</td>
</tr>
<tr>
<td>info</td>
<td>structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).</td>
</tr>
<tr>
<td>policy</td>
<td>the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user; the size is returned by csrilu02_bufferSize().</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrValA_valM</td>
<td>&lt;type&gt; matrix containing the incomplete-LU lower and upper triangular factors.</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status

### 11.2.5. cusparseXcsrilu02_zeroPivot()

cusparseStatus_t

cusparseXcsrilu02_zeroPivot(cusparseHandle_t handle,    
csrilu02Info_t info,  
int* position)

If the returned error code is CUSPARSE_STATUS_ZERO_PIVOT, position=j means A(j,j) has either a structural zero or a numerical zero; otherwise, position=-1.

The position can be 0-based or 1-based, the same as the matrix.
Function `cusparseXcsrilu02_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The position can be in the host memory or device memory. The user can set proper mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>info</td>
<td>info contains structural zero or numerical zero if the user already called <code>csrilu02_analysis()</code> or <code>csrilu02()</code>.</td>
</tr>
</tbody>
</table>

### Output

| position | if no structural or numerical zero, position is -1; otherwise if $A(j,j)$ is missing or $U(j,j)$ is zero, position=j. |

See `cusparseStatus_t` for the description of the return status

### 11.2.6. `cusparse<t>bsrilu02_numericBoost()`

```c

cusparseStatus_t
cusparseSbsrilu02_numericBoost(cusparseHandle_t handle,
                        bsrilu02Info_t   info,
                        int              enable_boost,
                        double*         tol,
                        float*          boost_val)

cusparseStatus_t
cusparseDbsrilu02_numericBoost(cusparseHandle_t handle,
                        bsrilu02Info_t   info,
                        int              enable_boost,
                        double*         tol,
                        double*         boost_val)

cusparseStatus_t
cusparseCbsrilu02_numericBoost(cusparseHandle_t handle,
                        bsrilu02Info_t   info,
                        int              enable_boost,
                        double*         tol,
                        cuComplex*      boost_val)

cusparseStatus_t
cusparseZbsrilu02_numericBoost(cusparseHandle_t handle,
                        bsrilu02Info_t   info,
                        int              enable_boost,
                        double*         tol,
                        cuDoubleComplex* boost_val)
```
The user can use a boost value to replace a numerical value in incomplete LU factorization. Parameter tol is used to determine a numerical zero, and boost_val is used to replace a numerical zero. The behavior is as follows:

if tol >= fabs(A(j,j)), then reset each diagonal element of block A(j,j) by boost_val.

To enable a boost value, the user sets parameter enable_boost to 1 before calling bsrilu02(). To disable the boost value, the user can call bsrilu02_numericBoost() with parameter enable_boost=0.

If enable_boost=0, tol and boost_val are ignored.

Both tol and boost_val can be in host memory or device memory. The user can set the proper mode with cusparseSetPointerMode().

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>info</td>
<td>structure initialized using cusparseCreateBsrilu02Info().</td>
</tr>
<tr>
<td>enable_boost</td>
<td>disable boost by setting enable_boost=0. Otherwise, boost is enabled.</td>
</tr>
<tr>
<td>tol</td>
<td>tolerance to determine a numerical zero.</td>
</tr>
<tr>
<td>boost_val</td>
<td>boost value to replace a numerical zero.</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status
11.2.7. cusparse<t>bsrilu02_bufferSize()

This function returns the size of the buffer used in computing the incomplete-LU factorization with 0 fill-in and no pivoting.
$A \approx LU$

$A$ is an $(mb \times blockDim) \times (mb \times blockDim)$ sparse matrix that is defined in BSR storage format by the three arrays $\text{bsrValA}$, $\text{bsrRowPtrA}$, and $\text{bsrColIndA}$.

The buffer size depends on the dimensions of $mb$, $blockDim$, and the number of nonzero blocks of the matrix $\text{nnzb}$. If the user changes the matrix, it is necessary to call $\text{bsrilu02_bufferSize()}$ again to have the correct buffer size; otherwise, a segmentation fault may occur.

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>dirA</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows and columns of matrix $A$.</td>
</tr>
<tr>
<td>nnzb</td>
<td>number of nonzero blocks of matrix $A$.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix $A$. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>$\text{bsrValA}$</td>
<td>$\text{&lt;type&gt; array of } \text{nnzb} (= \text{bsrRowPtrA}(mb) - \text{bsrRowPtrA}(0) )$ nonzero blocks of matrix $A$.</td>
</tr>
<tr>
<td>$\text{bsrRowPtrA}$</td>
<td>integer array of $mb + 1$ elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>$\text{bsrColIndA}$</td>
<td>integer array of $\text{nnzb} (= \text{bsrRowPtrA}(mb) - \text{bsrRowPtrA}(0) )$ column indices of the nonzero blocks of matrix $A$.</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix $A$, larger than zero.</td>
</tr>
</tbody>
</table>

**Output**

| info                                        | record internal states based on different algorithms. |
| pBufferSizeInBytes                          | number of bytes of the buffer used in $\text{bsrilu02_analysis()}$ and $\text{bsrilu02()}$. |

**Status Returned**

<p>| CUSPARSE_STATUS_SUCCESS                     | the operation completed successfully. |
| CUSPARSE_STATUS_NOT_INITIALIZED             | the library was not initialized. |
| CUSPARSE_STATUS_ALLOC_FAILED               | the resources could not be allocated. |
| CUSPARSE_STATUS_INVALID_VALUE              | invalid parameters were passed ($mb$, $\text{nnzb} \leq 0$), base index is not 0 or 1. |</p>
<table>
<thead>
<tr>
<th>Status Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_STATUS_ARCH_MISMATCH</td>
<td>the device only supports compute capability 2.0 and above.</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_INTERNAL_ERROR</td>
<td>an internal operation failed.</td>
</tr>
<tr>
<td>CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED</td>
<td>the matrix type is not supported.</td>
</tr>
</tbody>
</table>
### 11.2.8. `cusparse<t>bsrilu02_analysis()`

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>cusparseStatus_t</code></td>
<td><code>cusparseSbsrilu02_analysis()</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseHandle_t</code> handle,</td>
</tr>
<tr>
<td></td>
<td><code>cusparseDirection_t</code> dirA,</td>
</tr>
<tr>
<td></td>
<td>int <code>mb</code>,</td>
</tr>
<tr>
<td></td>
<td>int <code>nnzb</code>,</td>
</tr>
<tr>
<td></td>
<td>const <code>cusparseMatDescr_t descrA,</code></td>
</tr>
<tr>
<td></td>
<td><code>float* bsrValA,</code></td>
</tr>
<tr>
<td></td>
<td><code>const int* bsrRowPtrA,</code></td>
</tr>
<tr>
<td></td>
<td><code>const int* bsrColIndA,</code></td>
</tr>
<tr>
<td></td>
<td>int <code>blockDim</code>,</td>
</tr>
<tr>
<td></td>
<td><code>bsrilu02Info_t info,</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseSolvePolicy_t policy,</code></td>
</tr>
<tr>
<td></td>
<td><code>void* pBuffer)</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseStatus_t</code> <code>cusparseDbsrilu02_analysis()</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseHandle_t</code> handle,</td>
</tr>
<tr>
<td></td>
<td><code>cusparseDirection_t</code> dirA,</td>
</tr>
<tr>
<td></td>
<td>int <code>mb</code>,</td>
</tr>
<tr>
<td></td>
<td>int <code>nnzb</code>,</td>
</tr>
<tr>
<td></td>
<td>const <code>cusparseMatDescr_t descrA,</code></td>
</tr>
<tr>
<td></td>
<td><code>double* bsrValA,</code></td>
</tr>
<tr>
<td></td>
<td><code>const int* bsrRowPtrA,</code></td>
</tr>
<tr>
<td></td>
<td><code>const int* bsrColIndA,</code></td>
</tr>
<tr>
<td></td>
<td>int <code>blockDim</code>,</td>
</tr>
<tr>
<td></td>
<td><code>bsrilu02Info_t info,</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseSolvePolicy_t policy,</code></td>
</tr>
<tr>
<td></td>
<td><code>void* pBuffer)</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseStatus_t</code> <code>cusparseCbsrilu02_analysis()</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseHandle_t</code> handle,</td>
</tr>
<tr>
<td></td>
<td><code>cusparseDirection_t</code> dirA,</td>
</tr>
<tr>
<td></td>
<td>int <code>mb</code>,</td>
</tr>
<tr>
<td></td>
<td>int <code>nnzb</code>,</td>
</tr>
<tr>
<td></td>
<td>const <code>cusparseMatDescr_t descrA,</code></td>
</tr>
<tr>
<td></td>
<td><code>cuComplex* bsrValA,</code></td>
</tr>
<tr>
<td></td>
<td><code>const int* bsrRowPtrA,</code></td>
</tr>
<tr>
<td></td>
<td><code>const int* bsrColIndA,</code></td>
</tr>
<tr>
<td></td>
<td>int <code>blockDim</code>,</td>
</tr>
<tr>
<td></td>
<td><code>bsrilu02Info_t info,</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseSolvePolicy_t policy,</code></td>
</tr>
<tr>
<td></td>
<td><code>void* pBuffer)</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseStatus_t</code> <code>cusparseZbsrilu02_analysis()</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseHandle_t</code> handle,</td>
</tr>
<tr>
<td></td>
<td><code>cusparseDirection_t</code> dirA,</td>
</tr>
<tr>
<td></td>
<td>int <code>mb</code>,</td>
</tr>
<tr>
<td></td>
<td>int <code>nnzb</code>,</td>
</tr>
<tr>
<td></td>
<td>const <code>cusparseMatDescr_t descrA,</code></td>
</tr>
<tr>
<td></td>
<td><code>cuDoubleComplex* bsrValA,</code></td>
</tr>
<tr>
<td></td>
<td><code>const int* bsrRowPtrA,</code></td>
</tr>
<tr>
<td></td>
<td><code>const int* bsrColIndA,</code></td>
</tr>
<tr>
<td></td>
<td>int <code>blockDim</code>,</td>
</tr>
<tr>
<td></td>
<td><code>bsrilu02Info_t info,</code></td>
</tr>
<tr>
<td></td>
<td><code>cusparseSolvePolicy_t policy,</code></td>
</tr>
<tr>
<td></td>
<td><code>void* pBuffer)</code></td>
</tr>
</tbody>
</table>
This function performs the analysis phase of the incomplete-LU factorization with 0 fill-in and no pivoting

\[ A \approx LU \]

\( A \) is an \((mb*blockDim) \times (mb*blockDim)\) sparse matrix that is defined in BSR storage format by the three arrays \( \text{bsrValA}, \text{bsrRowPtrA}, \) and \( \text{bsrColIndA} \). The block in BSR format is of size \( \text{blockDim*blockDim} \), stored as column-major or row-major as determined by parameter \( \text{dirA} \), which is either \text{CUSPARSE_DIRECTION_COLUMN} or \text{CUSPARSE_DIRECTION_ROW}. The matrix type must be \text{CUSPARSE_MATRIX_TYPE_GENERAL}, and the fill mode and diagonal type are ignored.

This function requires a buffer size returned by \( \text{bsrilu02_bufferSize()} \). The address of \( \text{pBuffer} \) must be multiple of 128 bytes. If it is not, \text{CUSPARSE_STATUS_INVALID_VALUE} is returned.

Function \( \text{bsrilu02_analysis()} \) reports a structural zero and computes level information stored in the opaque structure \( \text{info} \). The level information can extract more parallelism during incomplete LU factorization. However \( \text{bsrilu02()} \) can be done without level information. To disable level information, the user needs to specify the parameter \( \text{policy} \) of \( \text{bsrilu02[\_analysis]} \) as \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL}.

Function \( \text{bsrilu02_analysis()} \) always reports the first structural zero, even with parameter \( \text{policy} \) is \text{CUSPARSE_SOLVE_POLICY_NO_LEVEL}. The user must call \( \text{cusparseXbsrilu02_zeroPivot()} \) to know where the structural zero is.

It is the user’s choice whether to call \( \text{bsrilu02()} \) if \( \text{bsrilu02_analysis()} \) reports a structural zero. In this case, the user can still call \( \text{bsrilu02()} \), which will return a numerical zero at the same position as the structural zero. However the result is meaningless.

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>dirA</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows and block columns of matrix ( A ).</td>
</tr>
<tr>
<td>nnzb</td>
<td>number of nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix ( A ). The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>bsrValA</td>
<td>(&lt;\text{type}&gt;) array of ( \text{nnzb} = \text{bsrRowPtrA}(\text{mb}) - \text{bsrRowPtrA}(0) ) nonzero blocks of matrix ( A ).</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of $mb + 1$ elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of $nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0))$ column indices of the nonzero blocks of matrix $A$.</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix $A$, larger than zero.</td>
</tr>
<tr>
<td>info</td>
<td>structure initialized using <code>cusparseCreateBsrilu02Info()</code>.</td>
</tr>
<tr>
<td>policy</td>
<td>the supported policies are <code>CUSPARSE_SOLVE_POLICY_NO_LEVEL</code> and <code>CUSPARSE_SOLVE_POLICY_USE_LEVEL</code>.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user, the size is returned by <code>bsrilu02_bufferSize()</code>.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>info</td>
<td>structure filled with information collected during the analysis phase (that should be passed to the solve phase unchanged).</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
11.2.9. `cusparse<t>bsrilu02()`

cusparseStatus_t cusparseSbsrilu02(cusparseHandle_t handle, cusparseDirection_t dirA, int mb, int nnzb, const cusparseMatDescr_t descrY, float* bsrValA, const int* bsrRowPtrA, const int* bsrColIndA, int blockDim, bsrilu02Info_t info, cusparseSolvePolicy_t policy, void* pBuffer)

cusparseStatus_t cusparseDbsrilu02(cusparseHandle_t handle, cusparseDirection_t dirA, int mb, int nnzb, const cusparseMatDescr_t descrY, double* bsrValA, const int* bsrRowPtrA, const int* bsrColIndA, int blockDim, bsrilu02Info_t info, cusparseSolvePolicy_t policy, void* pBuffer)

cusparseStatus_t cusparseCbsrilu02(cusparseHandle_t handle, cusparseDirection_t dirA, int mb, int nnzb, const cusparseMatDescr_t descrY, cuComplex* bsrValA, const int* bsrRowPtrA, const int* bsrColIndA, int blockDim, bsrilu02Info_t info, cusparseSolvePolicy_t policy, void* pBuffer)

cusparseStatus_t cusparseZbsrilu02(cusparseHandle_t handle, cusparseDirection_t dirA, int mb, int nnzb, const cusparseMatDescr_t descrY, cuDoubleComplex* bsrValA, const int* bsrRowPtrA, const int* bsrColIndA, int blockDim, bsrilu02Info_t info, cusparseSolvePolicy_t policy, void* pBuffer)
This function performs the solve phase of the incomplete-LU factorization with 0 fill-in and no pivoting

\[ A \approx LU \]

\( A \) is an \((m\times blockDim) \times (m\times blockDim)\) sparse matrix that is defined in BSR storage format by the three arrays \( bsrValA, bsrRowPtrA, \) and \( bsrColIndA. \) The block in BSR format is of size \( blockDim\times blockDim, \) stored as column-major or row-major determined by parameter \( dirA, \) which is either \texttt{CUSPARSE_DIRECTION_COLUMN} or \texttt{CUSPARSE_DIRECTION_ROW}. The matrix type must be \texttt{CUSPARSE_MATRIX_TYPE_GENERAL}, and the fill mode and diagonal type are ignored. Function \texttt{bsrilu02()} supports an arbitrary \texttt{blockDim}.

This function requires a buffer size returned by \texttt{bsrilu02_bufferSize()}. The address of \( pBuffer \) must be a multiple of 128 bytes. If it is not, \texttt{CUSPARSE_STATUS_INVALID_VALUE} is returned.

Although \texttt{bsrilu02()} can be used without level information, the user must be aware of consistency. If \texttt{bsrilu02_analysis()} is called with policy \texttt{CUSPARSE_SOLVE_POLICY_USE_LEVEL}, \texttt{bsrilu02()} can be run with or without levels. On the other hand, if \texttt{bsrilu02_analysis()} is called with \texttt{CUSPARSE_SOLVE_POLICY_NO_LEVEL}, \texttt{bsrilu02()} can only accept \texttt{CUSPARSE_SOLVE_POLICY_NO_LEVEL}; otherwise, \texttt{CUSPARSE_STATUS_INVALID_VALUE} is returned.

Function \texttt{bsrilu02()} has the same behavior as \texttt{csrilu02()}. That is, \( bsr2csr(bsrilu02(A)) = csrilu02(bsr2csr(A)). \) The numerical zero of \texttt{csrilu02()} means there exists some zero \( U(j,j). \) The numerical zero of \texttt{bsrilu02()} means there exists some block \( U(j,j) \) that is not invertible.

Function \texttt{bsrilu02} reports the first numerical zero, including a structural zero. The user must call \texttt{cusparseXbsrilu02_zeroPivot()} to know where the numerical zero is.
For example, suppose $A$ is a real $m$-by-$m$ matrix where $m=mb*\text{blockDim}$. The following code solves precondition system $M*y = x$, where $M$ is the product of LU factors $L$ and $U$.

```
// Suppose that $A$ is $m \times m$ sparse matrix represented by BSR format,
// the number of block rows/columns is $mb$, and
// the number of nonzero blocks is $nnzb$.
// Assumption:
// - handle is already created by cusparseCreate(),
// - ($d_{\text{bsrRowPtr}}$, $d_{\text{bsrColInd}}$, $d_{\text{bsrVal}}$) is BSR of $A$ on device memory,
// - $d_x$ is right hand side vector on device memory.
// - $d_y$ is solution vector on device memory.
// - $d_z$ is intermediate result on device memory.
// - $d_x$, $d_y$ and $d_z$ are of size $m$,

int structural_zero;
int numerical_zero;
const double alpha = 1.;
const cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
const cusparseOperation_t trans_U = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_L = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseSolvePolicy_t policy_U = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseSolvePolicy_t policy_L = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_M = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
constcusparseOperation_t trans_U = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_L = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;

```
The function supports the following properties if `pBuffer != NULL`:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>dirA</td>
<td>storage format of blocks: either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows and block columns of matrix A.</td>
</tr>
<tr>
<td>nnzb</td>
<td>number of nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>bsrValA</td>
<td><code>&lt;type&gt;</code> array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0)) nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of mb + 1 elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0)) column indices of the nonzero blocks of matrix A.</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix A; must be larger than zero.</td>
</tr>
<tr>
<td>info</td>
<td>structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).</td>
</tr>
<tr>
<td>policy</td>
<td>the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user; the size is returned by bsrilu02_bufferSize().</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bsrValA</td>
<td><code>&lt;type&gt;</code> matrix containing the incomplete-LU lower and upper triangular factors.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
11.2.10. cusparseXbsrilu02_zeroPivot()

cusparseStatus_t

`cusparseXbsrilu02_zeroPivot(cusparseHandle_t handle,`
`bsrilu02Info_t info,`
`int* position)`

If the returned error code is `CUSPARSE_STATUS_ZERO_PIVOT`, `position=j` means `A(j,j)` has either a structural zero or a numerical zero (the block is not invertible). Otherwise `position=-1`.

The `position` can be 0-based or 1-based, the same as the matrix.

Function `cusparseXbsrilu02_zeroPivot()` is a blocking call. It calls `cudaDeviceSynchronize()` to make sure all previous kernels are done.

The `position` can be in the host memory or device memory. The user can set proper the mode with `cusparseSetPointerMode()`.

- The routine requires no extra storage
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
</table>
| info   | info contains structural zero or numerical zero if the user already called `bsrilu02_analysis()` or `bsrilu02()`.

**Output**

| position | if no structural or numerical zero, position is -1; otherwise if `A(j,j)` is missing or `U(j,j)` is not invertible, position=j. |

See `cusparseStatus_t` for the description of the return status

11.3. Tridiagonal Solve

Different algorithms for tridiagonal solve are discussed in this section.
11.3.1. `cusparse<t>gtsv2_buffSizeExt()`

```c
 cusparseStatus_t
cusparseSgtsv2_bufferSizeExt(cusparseHandle_t handle,
    int m,
    int n,
    const float* dl,
    const float* d,
    const float* du,
    const float* B,
    int ldb,
    size_t* bufferSizeInBytes)
```

```c
 cusparseStatus_t
cusparseDgtsv2_bufferSizeExt(cusparseHandle_t handle,
    int m,
    int n,
    const double* dl,
    const double* d,
    const double* du,
    const double* B,
    int ldb,
    size_t* bufferSizeInBytes)
```

```c
 cusparseStatus_t
cusparseCgtsv2_bufferSizeExt(cusparseHandle_t handle,
    int m,
    int n,
    const cuComplex* dl,
    const cuComplex* d,
    const cuComplex* du,
    const cuComplex* B,
    int ldb,
    size_t* bufferSizeInBytes)
```

```c
 cusparseStatus_t
cusparseZgtsv2_bufferSizeExt(cusparseHandle_t handle,
    int m,
    int n,
    const cuDoubleComplex* dl,
    const cuDoubleComplex* d,
    const cuDoubleComplex* du,
    const cuDoubleComplex* B,
    int ldb,
    size_t* bufferSizeInBytes)
```

This function returns the size of the buffer used in `gtsv2` which computes the solution of a tridiagonal linear system with multiple right-hand sides.

$$ \mathbf{A} \mathbf{X} = \mathbf{B} $$

The coefficient matrix \( \mathbf{A} \) of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (\( \mathbf{dl} \)), main (\( \mathbf{d} \)), and upper (\( \mathbf{du} \)) matrix diagonals; the right-hand sides are stored in the dense matrix \( \mathbf{B} \). Notice that solution \( \mathbf{X} \) overwrites right-hand-side matrix \( \mathbf{B} \) on exit.

- The routine requires no extra storage
The routine supports asynchronous execution
The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>the size of the linear system (must be ≥ 3).</td>
</tr>
<tr>
<td>n</td>
<td>number of right-hand sides, columns of matrix B.</td>
</tr>
<tr>
<td>dl</td>
<td>&lt;type&gt; dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.</td>
</tr>
<tr>
<td>d</td>
<td>&lt;type&gt; dense array containing the main diagonal of the tri-diagonal linear system.</td>
</tr>
<tr>
<td>du</td>
<td>&lt;type&gt; dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.</td>
</tr>
<tr>
<td>B</td>
<td>&lt;type&gt; dense right-hand-side array of dimensions (ldb, n).</td>
</tr>
<tr>
<td>ldb</td>
<td>leading dimension of B (that is ≥ max(1, m)).</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>pBufferSizeInBytes</td>
<td>number of bytes of the buffer used in the gtsv2.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
11.3.2. *cusparse*<t>gtsv2()<br />

```
cusparseStatus_t
 cusparseSgtsv2(cusparseHandle_t handle,
            int              m,
            int              n,
            const float*     dl,
            const float*     d,
            const float*     du,
            float*           B,
            int              ldb,
            void             pBuffer)

cusparseStatus_t
 cusparseDgtsv2(cusparseHandle_t handle,
            int              m,
            int              n,
            const double*    dl,
            const double*    d,
            const double*    du,
            double*          B,
            int              ldb,
            void             pBuffer)

cusparseStatus_t
 cusparseCgtsv2(cusparseHandle_t handle,
            int              m,
            int              n,
            const cuComplex* dl,
            const cuComplex* d,
            const cuComplex* du,
            cuComplex*       B,
            int              ldb,
            void             pBuffer)

cusparseStatus_t
 cusparseZgtsv2(cusparseHandle_t handle,
            int              m,
            int              n,
            const cuDoubleComplex* dl,
            const cuDoubleComplex* d,
            const cuDoubleComplex* du,
            cuDoubleComplex* B,
            int              ldb,
            void             pBuffer)
```

This function computes the solution of a tridiagonal linear system with multiple right-hand sides:

\[ A \times X = B \]

The coefficient matrix \( A \) of each of these tridiagonal linear system is defined with three vectors corresponding to its lower (\( dl \)), main (\( d \)), and upper (\( du \)) matrix diagonals; the right-hand sides are stored in the dense matrix \( B \). Notice that solution \( X \) overwrites right-hand-side matrix \( B \) on exit.

Assuming \( A \) is of size \( m \) and base-1, \( dl \), \( d \) and \( du \) are defined by the following formula:
\[ dl(i) := A(i, i-1) \text{ for } i=1,2,\ldots,m \]

The first element of \( dl \) is out-of-bound (\( dl(1) := A(1,0) \)), so \( dl(1) = 0 \).

\[ d(i) = A(i,i) \text{ for } i=1,2,\ldots,m \]

\[ du(i) = A(i,i+1) \text{ for } i=1,2,\ldots,m \]

The last element of \( du \) is out-of-bound (\( du(m) := A(m,m+1) \)), so \( du(m) = 0 \).

The routine does perform pivoting, which usually results in more accurate and more stable results than \( \text{cusparse}<t>\text{gtsv_nopivot}() \) or \( \text{cusparse}<t>\text{gtsv2_nopivot}() \) at the expense of some execution time.

This function requires a buffer size returned by \( \text{gtsv2_bufferSizeExt()} \).

The address of \( pBuffer \) must be multiple of 128 bytes. If it is not, \( \text{CUSPARSE_STATUS_INVALID_VALUE} \) is returned.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>( \text{handle} )</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>the size of the linear system (must be ( \geq 3 )).</td>
</tr>
<tr>
<td>( n )</td>
<td>number of right-hand sides, columns of matrix ( B ).</td>
</tr>
<tr>
<td>( dl )</td>
<td>(&lt;\text{type}&gt;) dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.</td>
</tr>
<tr>
<td>( d )</td>
<td>(&lt;\text{type}&gt;) dense array containing the main diagonal of the tri-diagonal linear system.</td>
</tr>
<tr>
<td>( du )</td>
<td>(&lt;\text{type}&gt;) dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.</td>
</tr>
<tr>
<td>( B )</td>
<td>(&lt;\text{type}&gt;) dense right-hand-side array of dimensions ( (ldb, n) ).</td>
</tr>
<tr>
<td>( ldb )</td>
<td>leading dimension of ( B ) (that is ( \geq \max(1, m) )).</td>
</tr>
<tr>
<td>( pBuffer )</td>
<td>buffer allocated by the user, the size is return by ( \text{gtsv2_bufferSizeExt}() ).</td>
</tr>
</tbody>
</table>

**Output**

| \( B \) | \(<\text{type}>\) dense solution array of dimensions \( (ldb, n) \). |

See \( \text{cusparseStatus_t} \) for the description of the return status.
11.3.3. cusparse<t>gtsv2_nopivot_bufferSizeExt()

This function returns the size of the buffer used in gtsv2_nopivot which computes the solution of a tridiagonal linear system with multiple right-hand sides.

\[ A \times X = B \]

The coefficient matrix \( A \) of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (\( d_1 \)), main (\( d \)), and upper (\( d_u \)) matrix diagonals; the right-hand sides are stored in the dense matrix \( B \). Notice that solution \( X \) overwrites right-hand-side matrix \( B \) on exit.

- The routine requires no extra storage
The routine supports asynchronous execution

The routine supports CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>the size of the linear system (must be ≥ 3).</td>
</tr>
<tr>
<td>n</td>
<td>number of right-hand sides, columns of matrix B.</td>
</tr>
<tr>
<td>dl</td>
<td>&lt;type&gt; dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.</td>
</tr>
<tr>
<td>d</td>
<td>&lt;type&gt; dense array containing the main diagonal of the tri-diagonal linear system.</td>
</tr>
<tr>
<td>du</td>
<td>&lt;type&gt; dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.</td>
</tr>
<tr>
<td>B</td>
<td>&lt;type&gt; dense right-hand-side array of dimensions (ldb, n).</td>
</tr>
<tr>
<td>ldb</td>
<td>leading dimension of B. (that is ≥ max(1, m)).</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>pBufferSizeInBytes</td>
<td>number of bytes of the buffer used in the gtsv2_nopivot.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
11.3.4. cusparse<t>gtsv2_nopivot()

This function computes the solution of a tridiagonal linear system with multiple right-hand sides:

\[ A \times X = B \]

The coefficient matrix \( A \) of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (\( dl \)), main (\( d \)), and upper (\( du \)) matrix diagonals; the right-hand sides are stored in the dense matrix \( B \). Notice that solution \( X \) overwrites right-hand-side matrix \( B \) on exit.
The routine does not perform any pivoting and uses a combination of the Cyclic Reduction (CR) and the Parallel Cyclic Reduction (PCR) algorithms to find the solution. It achieves better performance when \( m \) is a power of 2.

This function requires a buffer size returned by `gtsv2_nopivot_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If it is not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>the size of the linear system (must be ( \geq 3 )).</td>
</tr>
<tr>
<td>( n )</td>
<td>number of right-hand sides, columns of matrix ( B ).</td>
</tr>
<tr>
<td>( dl )</td>
<td><code>&lt;type&gt;</code> dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.</td>
</tr>
<tr>
<td>( d )</td>
<td><code>&lt;type&gt;</code> dense array containing the main diagonal of the tri-diagonal linear system.</td>
</tr>
<tr>
<td>( du )</td>
<td><code>&lt;type&gt;</code> dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.</td>
</tr>
<tr>
<td>( B )</td>
<td><code>&lt;type&gt;</code> dense right-hand-side array of dimensions ((ldb, n)).</td>
</tr>
<tr>
<td>( ldb )</td>
<td>leading dimension of ( B ). (that is ( \geq \max(1, m) )).</td>
</tr>
<tr>
<td>( pBuffer )</td>
<td>buffer allocated by the user, the size is return by <code>gtsv2_nopivot_bufferSizeExt</code>.</td>
</tr>
</tbody>
</table>

### Output

| \( B \)         | `<type>` dense solution array of dimensions \((ldb, n)\). |

See `cusparseStatus_t` for the description of the return status

### 11.4. Batched Tridiagonal Solve

Different algorithms for batched tridiagonal solve are discussed in this section.
11.4.1. `cusparse<t>gtsv2StridedBatch_bufferSizeExt()`

This function returns the size of the buffer used in `gtsv2StridedBatch` which computes the solution of multiple tridiagonal linear systems for \( i=0, \ldots, \text{batchCount} \):

\[
A^{(i)} \cdot y^{(i)} = x^{(i)}
\]
The coefficient matrix $A$ of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower ($dl$), main ($d$), and upper ($du$) matrix diagonals; the right-hand sides are stored in the dense matrix $X$. Notice that solution $Y$ overwrites right-hand-side matrix $X$ on exit. The different matrices are assumed to be of the same size and are stored with a fixed $batchStride$ in memory.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>n</td>
<td>the size of the linear system (must be $\geq 3$).</td>
</tr>
<tr>
<td>dl</td>
<td>&lt;type&gt; dense array containing the lower diagonal of the tri-diagonal linear system. The lower diagonal $dl[i]$ that corresponds to the $i^{th}$ linear system starts at location $dl+batchStride\times i$ in memory. Also, the first element of each lower diagonal must be zero.</td>
</tr>
<tr>
<td>d</td>
<td>&lt;type&gt; dense array containing the main diagonal of the tri-diagonal linear system. The main diagonal $d[i]$ that corresponds to the $i^{th}$ linear system starts at location $d+batchStride\times i$ in memory.</td>
</tr>
<tr>
<td>du</td>
<td>&lt;type&gt; dense array containing the upper diagonal of the tri-diagonal linear system. The upper diagonal $du[i]$ that corresponds to the $i^{th}$ linear system starts at location $du+batchStride\times i$ in memory. Also, the last element of each upper diagonal must be zero.</td>
</tr>
<tr>
<td>x</td>
<td>&lt;type&gt; dense array that contains the right-hand-side of the tri-diagonal linear system. The right-hand-side $x[i]$ that corresponds to the $i^{th}$ linear system starts at location $x+batchStride\times i$ in memory.</td>
</tr>
<tr>
<td>batchCount</td>
<td>number of systems to solve.</td>
</tr>
<tr>
<td>batchStride</td>
<td>stride (number of elements) that separates the vectors of every system (must be at least $m$).</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>pBufferSizeInBytes</td>
<td>number of bytes of the buffer used in the $gtsv2StridedBatch$.</td>
</tr>
</tbody>
</table>

See $cusparseStatus_t$ for the description of the return status
11.4.2. cusparse<t>gtsv2StridedBatch()

```
cusparseStatus_t
cusparseSgtsv2StridedBatch(cusparseHandle_t handle,
    int m,
    const float* dl,
    const float* d,
    const float* du,
    float* x,
    int batchCount,
    int batchStride,
    void* pBuffer)

cusparseStatus_t
cusparseDgtsv2StridedBatch(cusparseHandle_t handle,
    int m,
    const double* dl,
    const double* d,
    const double* du,
    double* x,
    int batchCount,
    int batchStride,
    void* pBuffer)

cusparseStatus_t
cusparseCgtsv2StridedBatch(cusparseHandle_t handle,
    int m,
    const cuComplex* dl,
    const cuComplex* d,
    const cuComplex* du,
    cuComplex* x,
    int batchCount,
    int batchStride,
    void* pBuffer)

cusparseStatus_t
cusparseZgtsv2StridedBatch(cusparseHandle_t handle,
    int m,
    const cuDoubleComplex* dl,
    const cuDoubleComplex* d,
    const cuDoubleComplex* du,
    cuDoubleComplex* x,
    int batchCount,
    int batchStride,
    void* pBuffer)
```

This function computes the solution of multiple tridiagonal linear systems for \(i=0, \ldots, \text{batchCount}\):

\[
A^{(i)} \cdot y^{(i)} = x^{(i)}
\]

The coefficient matrix \(A\) of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (\(d_l\)), main (\(d\)), and upper (\(d_u\)) matrix diagonals; the right-hand sides are stored in the dense matrix \(X\). Notice that solution \(Y\) overwrites right-hand-side matrix \(X\) on exit. The different matrices are assumed to be of the same size and are stored with a fixed \text{batchStride} in memory.
The routine does not perform any pivoting and uses a combination of the Cyclic Reduction (CR) and the Parallel Cyclic Reduction (PCR) algorithms to find the solution. It achieves better performance when $m$ is a power of 2.

This function requires a buffer size returned by \texttt{gtsv2StridedBatch_bufferSizeExt()}. The address of \texttt{pBuffer} must be multiple of 128 bytes. If it is not, \texttt{CUSPARSE_STATUS_INVALID_VALUE} is returned.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>the size of the linear system (must be ≥ 3).</td>
</tr>
<tr>
<td>dl</td>
<td>$&lt;$type$&gt;$ dense array containing the lower diagonal of the tri-diagonal linear system. The lower diagonal $d_l(i)$ that corresponds to the $i^{th}$ linear system starts at location $dl + \text{batchStride} \times i$ in memory. Also, the first element of each lower diagonal must be zero.</td>
</tr>
<tr>
<td>d</td>
<td>$&lt;$type$&gt;$ dense array containing the main diagonal of the tri-diagonal linear system. The main diagonal $d(i)$ that corresponds to the $i^{th}$ linear system starts at location $d + \text{batchStride} \times i$ in memory.</td>
</tr>
<tr>
<td>du</td>
<td>$&lt;$type$&gt;$ dense array containing the upper diagonal of the tri-diagonal linear system. The upper diagonal $d_u(i)$ that corresponds to the $i^{th}$ linear system starts at location $du + \text{batchStride} \times i$ in memory. Also, the last element of each upper diagonal must be zero.</td>
</tr>
<tr>
<td>x</td>
<td>$&lt;$type$&gt;$ dense array that contains the right-hand-side of the tri-diagonal linear system. The right-hand-side $x(i)$ that corresponds to the $i^{th}$ linear system starts at location $x + \text{batchStride} \times i$ in memory.</td>
</tr>
<tr>
<td>batchCount</td>
<td>number of systems to solve.</td>
</tr>
<tr>
<td>batchStride</td>
<td>stride (number of elements) that separates the vectors of every system (must be at least $n$).</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user, the size is return by \texttt{gtsv2StridedBatch_bufferSizeExt}.</td>
</tr>
</tbody>
</table>

**Output**

| x | $<$type$>$ dense array that contains the solution of the tri-diagonal linear system. The solution $x(i)$ that corresponds to the $i^{th}$ linear system starts at location $x + \text{batchStride} \times i$ in memory. |

See \texttt{cusparseStatus_t} for the description of the return status
11.4.3. `cusparse<t>gtsvInterleavedBatch()`

```c
cusparseStatus_t
cusparseSgtsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                          int              algo,
                                          int              m,
                                          const float*     dl,
                                          const float*     d,
                                          const float*     du,
                                          const float*     x,
                                          int              batchCount,
                                          size_t*          pBufferSizeInBytes)

cusparseStatus_t
cusparseDgtsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                           int              algo,
                                           int              m,
                                           const double*    dl,
                                           const double*    d,
                                           const double*    du,
                                           const double*    x,
                                           int              batchCount,
                                           size_t*          pBufferSizeInBytes)

cusparseStatus_t
cusparseCgtsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                          int              algo,
                                          int              m,
                                          const cuComplex* dl,
                                          const cuComplex* d,
                                          const cuComplex* du,
                                          const cuComplex* x,
                                          int              batchCount,
                                          size_t*          pBufferSizeInBytes)

cusparseStatus_t
cusparseZgtsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                          int              algo,
                                          int              m,
                                          const cuDoubleComplex* dl,
                                          const cuDoubleComplex* d,
                                          const cuDoubleComplex* du,
                                          const cuDoubleComplex* x,
                                          int              batchCount,
                                          size_t*          pBufferSizeInBytes)
```

```c
cusparseStatus_t
cusparseSgtsvInterleavedBatch(cusparseHandle_t handle,
                               int              algo,
                               int              m,
                               float*           dl,
                               float*           d,
                               float*           du,
                               float*           x,
                               int              batchCount,
                               void*            pBuffer)

cusparseStatus_t
cusparseDgtsvInterleavedBatch(cusparseHandle_t handle,
                               int              algo,
                               int              m,
                               double*          dl,
                               double*          d,
                               double*          du,
                               double*          x,
                               int              batchCount,
                               void*            pBuffer)

cusparseStatus_t
cusparseCgtsvInterleavedBatch(cusparseHandle_t handle,
                               int              algo,
                               int              m,
                               cuComplex*       dl,
                               cuComplex*       d,
                               cuComplex*       du,
                               cuComplex*       x,
                               int              batchCount,
                               void*            pBuffer)

cusparseStatus_t
cusparseZgtsvInterleavedBatch(cusparseHandle_t handle,
                               int              algo,
                               int              m,
                               cuDoubleComplex* dl,
                               cuDoubleComplex* d,
                               cuDoubleComplex* du,
                               cuDoubleComplex* x,
                               int              batchCount,
                               void*            pBuffer)
```
This function computes the solution of multiple tridiagonal linear systems for $i=0, \ldots, \text{batchCount}$:

$$A^0 x^0 = b^0$$

The coefficient matrix $A$ of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower ($dl$), main ($d$), and upper ($du$) matrix diagonals; the right-hand sides are stored in the dense matrix $B$. Notice that solution $X$ overwrites right-hand-side matrix $B$ on exit.

Assuming $A$ is of size $m$ and base-1, $dl$, $d$ and $du$ are defined by the following formula:

- $dl(i) := A(i, i-1)$ for $i=1, 2, \ldots, m$
- The first element of $dl$ is out-of-bound ($dl(1) := A(1, 0)$), so $dl(1) = 0$.
- $d(i) = A(i,i)$ for $i=1, 2, \ldots, m$
- $du(i) = A(i,i+1)$ for $i=1, 2, \ldots, m$
- The last element of $du$ is out-of-bound ($du(m) := A(m,m+1)$), so $du(m) = 0$.

The data layout is different from $\text{gtsvStridedBatch}$ which aggregates all matrices one after another. Instead, $\text{gtsvInterleavedBatch}$ gathers different matrices of the same element in a continuous manner. If $dl$ is regarded as a 2-D array of size $m \times \text{batchCount}$, $dl(:,j)$ to store $j$-th matrix. $\text{gtsvStridedBatch}$ uses column-major while $\text{gtsvInterleavedBatch}$ uses row-major.

The routine provides three different algorithms, selected by parameter $\text{algo}$. The first algorithm is $\text{cuThomas}$ provided by Barcelona Supercomputing Center. The second algorithm is LU with partial pivoting and last algorithm is QR. From stability perspective, $\text{cuThomas}$ is not numerically stable because it does not have pivoting. LU with partial pivoting and QR are stable. From performance perspective, LU with partial pivoting and QR is about 10% to 20% slower than $\text{cuThomas}$.

This function requires a buffer size returned by $\text{gtsvInterleavedBatch_bufferSizeExt()}$. The address of $\text{pBuffer}$ must be multiple of 128 bytes. If it is not, $\text{CUSPARSE\_STATUS\_INVALID\_VALUE}$ is returned.

If the user prepares aggregate format, one can use $\text{cublasXgeam}$ to get interleaved format. However such transformation takes time comparable to solver itself. To reach best performance, the user must prepare interleaved format explicitly.

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>algo</td>
<td>algo = 0: cuThomas (unstable algorithm); algo = 1: LU with pivoting (stable algorithm); algo = 2: QR (stable algorithm)</td>
</tr>
<tr>
<td>m</td>
<td>the size of the linear system.</td>
</tr>
</tbody>
</table>
#### dl
- <type> dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.

#### d
- <type> dense array containing the main diagonal of the tri-diagonal linear system.

#### du
- <type> dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.

#### x
- <type> dense right-hand-side array of dimensions (batchCount, n).

#### pBuffer
- buffer allocated by the user, the size is return by gtsvInterleavedBatch_bufferSizeExt.

### Output

#### x
- <type> dense solution array of dimensions (batchCount, n).

See cusparseStatus_t for the description of the return status

### 11.5. Batched Pentadiagonal Solve

Different algorithms for batched pentadiagonal solve are discussed in this section.
11.5.1. cuSparse<t>gpsvInterleavedBatch()

```c

cusparseStatus_t
cusparseSgpsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                   int algo,
                                   int m,
                                   const float* ds,
                                   const float* dl,
                                   const float* d,
                                   const float* du,
                                   const float* dw,
                                   const float* x,
                                   int batchSize,
                                   size_t* pBufferSizeInBytes)


cusparseStatus_t
cusparseDgpsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                   int algo,
                                   int m,
                                   const double* ds,
                                   const double* dl,
                                   const double* d,
                                   const double* du,
                                   const double* dw,
                                   const double* x,
                                   int batchSize,
                                   size_t* pBufferSizeInBytes)


cusparseStatus_t
cusparseCgpsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                   int algo,
                                   int m,
                                   const cuComplex* ds,
                                   const cuComplex* dl,
                                   const cuComplex* d,
                                   const cuComplex* du,
                                   const cuComplex* dw,
                                   const cuComplex* x,
                                   int batchSize,
                                   size_t* pBufferSizeInBytes)


cusparseStatus_t
cusparseZgpsvInterleavedBatch_bufferSizeExt(cusparseHandle_t handle,
                                   int algo,
                                   int m,
                                   const cuDoubleComplex* ds,
                                   const cuDoubleComplex* dl,
                                   const cuDoubleComplex* d,
                                   const cuDoubleComplex* du,
                                   const cuDoubleComplex* dw,
                                   const cuDoubleComplex* x,
                                   int batchSize,
                                   size_t* pBufferSizeInBytes)


cusparseStatus_t
cusparseSgpsvInterleavedBatch(cusparseHandle_t handle,
                                   int algo,
                                   int m,
                                   float* ds,
                                   float* dl,
                                   float* d,
                                   float* du,
                                   float* dw,
                                   float* x,
                                   int batchSize,
                                   void* pBuffer)


cusparseStatus_t
cusparseDgpsvInterleavedBatch(cusparseHandle_t handle,
                                   int algo,
                                   int m,
                                   double* ds,
                                   double* dl,
                                   double* d,
                                   double* du,
                                   double* dw,
                                   double* x,
                                   int batchSize,
                                   void* pBuffer)


cusparseStatus_t
cusparseCgpsvInterleavedBatch(cusparseHandle_t handle,
                                   int algo,
                                   int m,
                                   cuComplex* ds,
                                   cuComplex* dl,
                                   cuComplex* d,
                                   cuComplex* du,
                                   cuComplex* dw,
                                   cuComplex* x,
                                   int batchSize,
                                   void* pBuffer)


cusparseStatus_t
cusparseZgpsvInterleavedBatch(cusparseHandle_t handle,
                                   int algo,
                                   int m,
                                   cuDoubleComplex* ds,
                                   cuDoubleComplex* dl,
                                   cuDoubleComplex* d,
                                   cuDoubleComplex* du,
                                   cuDoubleComplex* dw,
                                   cuDoubleComplex* x,
                                   int batchSize,
                                   void* pBuffer)
```

This function computes the solution of multiple penta-diagonal linear systems for $i=0, \ldots, \text{batchCount}$:

$$A(i) x(i) = b(i)$$

The coefficient matrix $A$ of each of these penta-diagonal linear system is defined with five vectors corresponding to its lower ($ds$, $dl$), main ($d$), and upper ($du$, $dw$) matrix diagonals; the right-hand sides are stored in the dense matrix $B$. Notice that solution $X$ overwrites right-hand-side matrix $B$ on exit.

Assuming $A$ is of size $m$ and base-1, $ds$, $dl$, $d$, $du$ and $dw$ are defined by the following formula:

$ds(i) := A(i, i-2)$ for $i=1,2,\ldots,m$

The first two elements of $ds$ is out-of-bound ($ds(1) := A(1,-1), ds(2) := A(2,0)$), so $ds(1) = 0$ and $ds(2) = 0$.

$dl(i) := A(i, i-1)$ for $i=1,2,\ldots,m$

The first element of $dl$ is out-of-bound ($dl(1) := A(1,0)$), so $dl(1) = 0$.

$d(i) = A(i,i)$ for $i=1,2,\ldots,m$

$du(i) = A(i,i+1)$ for $i=1,2,\ldots,m$

The last element of $du$ is out-of-bound ($du(m) := A(m,m+1)$), so $du(m) = 0$.

$dw(i) = A(i,i+2)$ for $i=1,2,\ldots,m$

The last two elements of $dw$ is out-of-bound ($dw(m-1) := A(m-1,m+1), dw(m) := A(m,m+2)$), so $dw(m-1) = 0$ and $dw(m) = 0$.

The data layout is the same as gtsvStridedBatch.

The routine is numerically stable because it uses QR to solve the linear system.

This function requires a buffer size returned by gpsvInterleavedBatch_bufferSizeExt(). The address of pBuffer must be multiple of 128 bytes. If it is not, CUSPARSE_STATUS_INVALID_VALUE is returned.

Appendix section shows an example of gpsvInterleavedBatch. If the user prepares aggregate format, one can use cublasXgeam to get interleaved format. However such transformation takes time comparable to solver itself. To reach best performance, the user must prepare interleaved format explicitly.

The function supports the following properties if pBuffer != NULL

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>algo</td>
<td>only support algo = 0 (QR)</td>
</tr>
<tr>
<td>m</td>
<td>the size of the linear system.</td>
</tr>
</tbody>
</table>
### ds

- `<type>` dense array containing the lower diagonal (distance 2 to the diagonal) of the penta-diagonal linear system. The first two elements must be zero.

### d1

- `<type>` dense array containing the lower diagonal (distance 1 to the diagonal) of the penta-diagonal linear system. The first element must be zero.

### d

- `<type>` dense array containing the main diagonal of the penta-diagonal linear system.

### du

- `<type>` dense array containing the upper diagonal (distance 1 to the diagonal) of the penta-diagonal linear system. The last element must be zero.

### dw

- `<type>` dense array containing the upper diagonal (distance 2 to the diagonal) of the penta-diagonal linear system. The last two elements must be zero.

### x

- `<type>` dense right-hand-side array of dimensions (batchCount, n).

### pBuffer

- Buffer allocated by the user, the size is return by `gpsvInterleavedBatch_bufferSizeExt`.

### Output

- `<type>` dense solution array of dimensions (batchCount, n).

See `cusparseStatus_t` for the description of the return status.
This chapter describes the reordering routines used to manipulate sparse matrices.
12.1. cusparse<t>csrColor()

```c
 cusparseStatus_t
cusparseScsrsColor(cusparseHandle_t      handle,
                     int               m,
                     int              nnz,
                     const cusparseMatDescr_t descrA,
                     const float*    csrValA,
                     const int*      csrRowPtrA,
                     const int*      csrColIndA,
                     const float*    fractionToColor,
                     int*            ncolors,
                     int*            coloring,
                     int*            reordering,
                     cusparseColorInfo_t info)
```

```c
 cusparseStatus_t
cusparseDcsrcolor(cusparseHandle_t      handle,
                  int               m,
                  int              nnz,
                  const cusparseMatDescr_t descrA,
                  const double*   csrValA,
                  const int*      csrRowPtrA,
                  const int*      csrColIndA,
                  const double*   fractionToColor,
                  int*            ncolors,
                  int*            coloring,
                  int*            reordering,
                  cusparseColorInfo_t info)
```

```c
 cusparseStatus_t
cusparseCcsrcolor(cusparseHandle_t      handle,
                   int               m,
                   int            nnz,
                   const cusparseMatDescr_t descrA,
                   const cuComplex*   csrValA,
                   const int*      csrRowPtrA,
                   const int*      csrColIndA,
                   const cuComplex*   fractionToColor,
                   int*            ncolors,
                   int*            coloring,
                   int*            reordering,
                   cusparseColorInfo_t info)
```

```c
 cusparseStatus_t
cusparseZcsrcolor(cusparseHandle_t      handle,
                  int               m,
                  int            nnz,
                  const cusparseMatDescr_t descrA,
                  const cuDoubleComplex*   csrValA,
                  const int*      csrRowPtrA,
                  const int*      csrColIndA,
                  const cuDoubleComplex*   fractionToColor,
                  int*            ncolors,
                  int*            coloring,
                  int*            reordering,
                  cusparseColorInfo_t info)
```
This function performs the coloring of the adjacency graph associated with the matrix \( A \) stored in CSR format. The coloring is an assignment of colors (integer numbers) to nodes, such that neighboring nodes have distinct colors. An approximate coloring algorithm is used in this routine, and is stopped when a certain percentage of nodes has been colored. The rest of the nodes are assigned distinct colors (an increasing sequence of integers numbers, starting from the last integer used previously). The last two auxiliary routines can be used to extract the resulting number of colors, their assignment and the associated reordering. The reordering is such that nodes that have been assigned the same color are reordered to be next to each other.

The matrix \( A \) passed to this routine, must be stored as a general matrix and have a symmetric sparsity pattern. If the matrix is nonsymmetric the user should pass \( A + A^T \) as a parameter to this routine.

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>( m )</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>( nnz )</td>
<td>number of nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix ( A ). The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>csrValA</td>
<td>(&lt;\text{type}&gt;) array of ( nnz = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0) ) nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of ( m+1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of ( nnz = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0) ) column indices of the nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>fractionToColor</td>
<td>fraction of nodes to be colored, which should be in the interval ([0.0,1.0]), for example 0.8 implies that 80 percent of nodes will be colored.</td>
</tr>
<tr>
<td>info</td>
<td>structure with information to be passed to the coloring.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ncolors</td>
<td>The number of distinct colors used (at most the size of the matrix, but likely much smaller).</td>
</tr>
<tr>
<td>coloring</td>
<td>The resulting coloring permutation</td>
</tr>
<tr>
<td>reordering</td>
<td>The resulting reordering permutation (untouched if NULL)</td>
</tr>
</tbody>
</table>
See `cusparseStatus_t` for the description of the return status.
Chapter 13.
CUSPARSE FORMAT CONVERSION REFERENCE

This chapter describes the conversion routines between different sparse and dense storage formats.

`coosort`, `csrsort`, `cscsort`, and `csru2csr` are sorting routines without malloc inside, the following table estimates the buffer size

<table>
<thead>
<tr>
<th>routine</th>
<th>buffer size</th>
<th>maximum problem size if buffer is limited by 2GB</th>
</tr>
</thead>
<tbody>
<tr>
<td>coosort</td>
<td>&gt; 16*n bytes</td>
<td>125M</td>
</tr>
<tr>
<td>csrsort or cscsort</td>
<td>&gt; 20*n bytes</td>
<td>100M</td>
</tr>
<tr>
<td>csru2csr</td>
<td>'d' &gt; 28<em>n bytes ; 'z' &gt; 36</em>n bytes</td>
<td>71M for 'd' and 55M for 'z'</td>
</tr>
</tbody>
</table>
13.1. cusparse<t>bsr2csr()

cusparseStatus_t
cusparseSbsr2csr(cusparseHandle_t handle,
cusparseDirection_t dir,
int mb,
int nb,
const cusparseMatDescr_t descrA,
const float* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
const cusparseMatDescr_t descrC,
float* csrValC,
int* csrRowPtrC,
int* csrColIndC)

cusparseStatus_t
cusparseDbsr2csr(cusparseHandle_t handle,
cusparseDirection_t dir,
int mb,
int nb,
const cusparseMatDescr_t descrA,
const double* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
const cusparseMatDescr_t descrC,
double* csrValC,
int* csrRowPtrC,
int* csrColIndC)

cusparseStatus_t
cusparseCbsr2csr(cusparseHandle_t handle,
cusparseDirection_t dir,
int mb,
int nb,
const cusparseMatDescr_t descrA,
const cuComplex* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
const cusparseMatDescr_t descrC,
cuComplex* csrValC,
int* csrRowPtrC,
int* csrColIndC)

cusparseStatus_t
cusparseZbsr2csr(cusparseHandle_t handle,
cusparseDirection_t dir,
int mb,
int nb,
const cusparseMatDescr_t descrA,
const cuDoubleComplex* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int blockDim,
const cusparseMatDescr_t descrC,
cuDoubleComplex* csrValC,
int* csrRowPtrC,
int* csrColIndC)
This function converts a sparse matrix in BSR format that is defined by the three arrays \( \text{bsrValA}, \text{bsrRowPtrA}, \text{and bsrColIndA} \) into a sparse matrix in CSR format that is defined by arrays \( \text{csrValC}, \text{csrRowPtrC}, \text{and csrColIndC} \).

Let \( m(=mb\times\text{blockDim}) \) be the number of rows of \( A \) and \( n(=nb\times\text{blockDim}) \) be number of columns of \( A \), then \( A \) and \( C \) are \( m \times n \) sparse matrices. The BSR format of \( A \) contains \( nnzb(=\text{bsrRowPtrA}[mb] - \text{bsrRowPtrA}[0]) \) nonzero blocks, whereas the sparse matrix \( A \) contains \( nnz(=nnzb\times\text{blockDim}\times\text{blockDim}) \) elements. The user must allocate enough space for arrays \( \text{csrRowPtrC}, \text{csrColIndC}, \text{and csrValC} \). The requirements are as follows:

- \( \text{csrRowPtrC} \) of \( m+1 \) elements
- \( \text{csrValC} \) of \( nnz \) elements
- \( \text{csrColIndC} \) of \( nnz \) elements

The general procedure is as follows:

```c
// Given BSR format (bsrRowPtrA, bsrcolIndA, bsrValA) and
// blocks of BSR format are stored in column-major order.
int m = mb*blockDim;
int nnzb = bsrRowPtrA[mb] - bsrRowPtrA[0]; // number of blocks
int nnz  = nnzb * blockDim * blockDim; // number of elements

// Allocate the required memory blocks
cudaMalloc((void**)&csrRowPtrC, sizeof(int)*(m+1));
cudaMalloc((void**)&csrColIndC, sizeof(int)*nnz);
cudaMalloc((void**)&csrValC, sizeof(float)*nnz);

cusparseSbsr2csr(handle, dir, mb, nb,
    descrA,
    bsrValA, bsrRowPtrA, bsrColIndA,
    blockDim,
    descrC,
    csrValC, csrRowPtrC, csrColIndC);
```

- The routine requires no extra storage
- The routine does not support asynchronous execution if \( \text{blockDim} == 1 \)
- The routine does not support CUDA graph capture if \( \text{blockDim} == 1 \)

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
</tbody>
</table>
| dir           | storage format of blocks, either CUSPARSE_DIRECTION_ROW or
|               | CUSPARSE_DIRECTION_COLUMN                                                    |
| mb            | number of block rows of sparse matrix \( A \).                              |
| nb            | number of block columns of sparse matrix \( A \).                            |
| descrA        | the descriptor of matrix \( A \).                                           |
| bsrValA       | <type> array of \( nnzb\times\text{blockDim}\times\text{blockDim} \) nonzero
|               | elements of matrix \( A \).                                                |
| bsrRowPtrA    | integer array of \( mb+1 \) elements that contains the start of every
|               | block row and the end of the last block row plus one of matrix \( A \).      |
| bsrColIndA    | integer array of \( nnzb \) column indices of the nonzero blocks of matrix \( A \). |
**blockDim**  
block dimension of sparse matrix A.

**descrC**  
the descriptor of matrix C.

### Output

<table>
<thead>
<tr>
<th>csrValC</th>
<th>&lt;type&gt; array of ( nnz = \text{csrRowPtrC}[m] - \text{csrRowPtrC}[0] ) nonzero elements of matrix C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrRowPtrC</td>
<td>integer array of ( m+1 ) elements that contains the start of every row and the end of the last row plus one of matrix C.</td>
</tr>
<tr>
<td>csrColIndC</td>
<td>integer array of ( nnz ) column indices of the nonzero elements of matrix C.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
13.2. cusparse<t>gebsr2gebsc()

cusparseStatus_t
cusparseSgebsr2gebsc_bufferSize(cusparseHandle_t handle,
int mb,
int nb,
int nnzb,
const float* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int rowBlockDim,
int colBlockDim,
int* pBufferSize)

cusparseStatus_t
cusparseDgebsr2gebsc_bufferSize(cusparseHandle_t handle,
int mb,
int nb,
int nnzb,
const double* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int rowBlockDim,
int colBlockDim,
int* pBufferSize)

cusparseStatus_t
cusparseCgebsr2gebsc_bufferSize(cusparseHandle_t handle,
int mb,
int nb,
int nnzb,
const cuComplex* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int rowBlockDim,
int colBlockDim,
int* pBufferSize)

cusparseStatus_t
cusparseZgebsr2gebsc_bufferSize(cusparseHandle_t handle,
int mb,
int nb,
int nnzb,
const cuDoubleComplex* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int rowBlockDim,
int colBlockDim,
int* pBufferSize)

cusparseStatus_t
cusparseSgebsr2gebsc(cusparseHandle_t handle,
int mb,
int nb,
int nnzb,
const float* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int rowBlockDim,
int colBlockDim,
float* bscVal,
int* bscRowInd,
int* bscColPtr,
cusparseAction_t copyValues,
cusparseIndexBase_t baseIdx,
void* pBuffer)

cusparseStatus_t
cusparseDgebsr2gebsc(cusparseHandle_t handle,
int mb,
int nb,
int nnzb,
const double* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int rowBlockDim,
int colBlockDim,
double* bscVal,
int* bscRowInd,
int* bscColPtr,
cusparseAction_t copyValues,
cusparseIndexBase_t baseIdx,
void* pBuffer)

cusparseStatus_t
cusparseCgebsr2gebsc(cusparseHandle_t handle,
int mb,
int nb,
int nnzb,
const cuComplex* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int rowBlockDim,
int colBlockDim,
cuComplex* bscVal,
int* bscRowInd,
int* bscColPtr,
cusparseAction_t copyValues,
cusparseIndexBase_t baseIdx,
void* pBuffer)

cusparseStatus_t
cusparseZgebsr2gebsc(cusparseHandle_t handle,
int mb,
int nb,
int nnzb,
const cuDoubleComplex* bsrVal,
const int* bsrRowPtr,
const int* bsrColInd,
int rowBlockDim,
int colBlockDim,
cuDoubleComplex* bscVal,
int* bscRowInd,
int* bscColPtr,
cusparseAction_t copyValues,
cusparseIndexBase_t baseIdx,
void* pBuffer)
This function can be seen as the same as `csr2csc()` when each block of size `rowBlockDim*colBlockDim` is regarded as a scalar.

This sparsity pattern of the result matrix can also be seen as the transpose of the original sparse matrix, but the memory layout of a block does not change.

The user must call `gebsr2gebsc_bufferSize()` to determine the size of the buffer required by `gebsr2gebsc()`, allocate the buffer, and pass the buffer pointer to `gebsr2gebsc()`.

- The routine requires no extra storage if `pBuffer != NULL`
- The routine does **not** support asynchronous execution
- The routine does **not** support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>mb</code></td>
<td>number of block rows of sparse matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>nb</code></td>
<td>number of block columns of sparse matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>nnzb</code></td>
<td>number of nonzero blocks of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>bsrVal</code></td>
<td><code>&lt;type&gt;</code> array of <code>nnzb*rowBlockDim*colBlockDim</code> nonzero elements of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>bsrRowPtr</code></td>
<td>integer array of <code>mb+1</code> elements that contains the start of every block row and the end of the last block row plus one.</td>
</tr>
<tr>
<td><code>bsrColInd</code></td>
<td>integer array of <code>nnzb</code> column indices of the non-zero blocks of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>rowBlockDim</code></td>
<td>number of rows within a block of <code>A</code>.</td>
</tr>
<tr>
<td><code>colBlockDim</code></td>
<td>number of columns within a block of <code>A</code>.</td>
</tr>
<tr>
<td><code>copyValues</code></td>
<td><code>CUSPARSE_ACTION_SYMBOLIC</code> or <code>CUSPARSE_ACTION_NUMERIC</code>.</td>
</tr>
<tr>
<td><code>baseIdx</code></td>
<td><code>CUSPARSE_INDEX_BASE_ZERO</code> or <code>CUSPARSE_INDEX_BASE_ONE</code>.</td>
</tr>
<tr>
<td><code>pBufferSize</code></td>
<td>host pointer containing number of bytes of the buffer used in <code>gebsr2gebsc()</code>.</td>
</tr>
</tbody>
</table>
| `pBuffer` | buffer allocated by the user; the size is return by `gebsr2gebsc_bufferSize()`.

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>bscVal</code></td>
<td><code>&lt;type&gt;</code> array of <code>nnzb*rowBlockDim*colBlockDim</code> non-zero elements of matrix <code>A</code>. It is only filled-in if <code>copyValues</code> is set to <code>CUSPARSE_ACTION_NUMERIC</code>.</td>
</tr>
<tr>
<td><code>bscRowInd</code></td>
<td>integer array of <code>nnzb</code> row indices of the non-zero blocks of matrix <code>A</code>.</td>
</tr>
</tbody>
</table>
### bscColPtr

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>integer array of nb+1 elements that contains the start of every block column and the end of the last block column plus one.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
13.3. cusparse<

cusparseStatus_t
cusparseSgebsr2gebsr_bufferSize(cusparseHandle_t handle,
cusparseDirection_t dir,
int mb,
int nb,
int nnzb,
const cusparseMatDescr_t descrA,
const float* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int colBlockDimA,
int rowBlockDimA,
int bsrColIndC,
int bsrRowPtrC,
int* pBuffer)

cusparseStatus_t
cusparseDgebsr2gebsr_bufferSize(cusparseHandle_t handle,
cusparseDirection_t dir,
int mb,
int nb,
int nnzb,
const cusparseMatDescr_t descrA,
const double* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int colBlockDimA,
int rowBlockDimA,
int bsrColIndC,
int bsrRowPtrC,
int* pBuffer)

cusparseStatus_t
cusparseCgebsr2gebsr_bufferSize(cusparseHandle_t handle,
cusparseDirection_t dir,
int mb,
int nb,
int nnzb,
const cusparseMatDescr_t descrA,
const cuComplex* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int colBlockDimA,
int rowBlockDimA,
int bsrColIndC,
int bsrRowPtrC,
int* pBuffer)

cusparseStatus_t
cusparseZgebsr2gebsr_bufferSize(cusparseHandle_t handle,
cusparseDirection_t dir,
int mb,
int nb,
int nnzb,
const cusparseMatDescr_t descrA,
const cuDoubleComplex* bsrValA,
const int* bsrRowPtrA,
const int* bsrColIndA,
int colBlockDimA,
int rowBlockDimA,
int bsrColIndC,
int bsrRowPtrC,
int* pBuffer)

13.3. cusparse<t>gebsr2gebsr()
This function converts a sparse matrix in general BSR format that is defined by the three arrays \texttt{bsrValA}, \texttt{bsrRowPtrA}, and \texttt{bsrColIndA} into a sparse matrix in another general BSR format that is defined by arrays \texttt{bsrValC}, \texttt{bsrRowPtrC}, and \texttt{bsrColIndC}.

If \texttt{rowBlockDimA=1} and \texttt{colBlockDimA=1}, \texttt{cusparse[S|D|C|Z]gebsr2gebsr()} is the same as \texttt{cusparse[S|D|C|Z]csr2gebsr()}. If \texttt{rowBlockDimC=1} and \texttt{colBlockDimC=1}, \texttt{cusparse[S|D|C|Z]gebsr2gebsr()} is the same as \texttt{cusparse[S|D|C|Z]gebsr2csr()}. A is an \(m \times n\) sparse matrix where \(m (=mb \times \text{rowBlockDim})\) is the number of rows of \(A\), and \(n (=nb \times \text{colBlockDim})\) is the number of columns of \(A\). The general BSR format of \(A\) contains \(\text{nnzb} (=\text{bsrRowPtrA}[mb] - \text{bsrRowPtrA}[0])\) nonzero blocks. The matrix \(C\) is also general BSR format with a different block size, \(\text{rowBlockDimC} \times \text{colBlockDimC}\). If \(m\) is not a multiple of \(\text{rowBlockDimC}\), or \(n\) is not a multiple of \(\text{colBlockDimC}\), zeros are filled in. The number of block rows of \(C\) is \(mc = (m + \text{rowBlockDimC}-1)/\text{rowBlockDimC}\). The number of block rows of \(C\) is \(nc = (n + \text{colBlockDimC}-1)/\text{colBlockDimC}\). The number of nonzero blocks of \(C\) is \(\text{nnzc}\).

The implementation adopts a two-step approach to do the conversion. First, the user allocates \(\text{bsrRowPtrC}\) of \(mc+1\) elements and uses function \texttt{cusparseXgebsr2gebsrNnz()} to determine the number of nonzero block columns per block row of matrix \(C\). Second, the user gathers \(\text{nnzc}\) (number of nonzero block columns of matrix \(C\)) from either \(\text{nnzc} = \ast\text{nnzTotalDevHostPtr}\) or \(\text{nnzc} = \text{bsrRowPtrC}[mc] - \text{bsrRowPtrC}[0]\) and allocates \(\text{bsrValC}\) of \(\text{nnzc} \times \text{rowBlockDimC} \times \text{colBlockDimC}\) elements and \(\text{bsrColIndC}\) of \(\text{nnzc}\) integers. Finally the function \texttt{cusparse[S|D|C|Z]gebsr2gebsr()} is called to complete the conversion.

The user must call \texttt{gebsr2gebsr_bufferSize()} to know the size of the buffer required by \texttt{gebsr2gebsr()}, allocate the buffer, and pass the buffer pointer to \texttt{gebsr2gebsr()}. 
The general procedure is as follows:

```c
// Given general BSR format (bsrRowPtrA, bsrColIndA, bsrValA) and
// blocks of BSR format are stored in column-major order.

cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int base, nnzc;
int m = mb*rowBlockDimA;
int n = nb*colBlockDimA;
int mc = (m+rowBlockDimC-1)/rowBlockDimC;
int nc = (n+colBlockDimC-1)/colBlockDimC;
int bufferSize;
void *pBuffer;
cusparseSgebsr2gebsr_bufferSize(handle, dir, mb, nb, nnzb,
   descrA, bsrValA, bsrRowPtrA, bsrColIndA,
   rowBlockDimA, colBlockDimA,
   rowBlockDimC, colBlockDimC,
   &bufferSize);
cudaMalloc((void**)&pBuffer, bufferSize);
cudaMalloc((void**)&bsrRowPtrC, sizeof(int)*(mc+1));
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzc;
cusparseXgebsr2gebsrNnz(handle, dir, mb, nb, nnzb,
   descrA, bsrRowPtrC, bsrColIndA,
   rowBlockDimA, colBlockDimA,
   descrC, bsrRowPtrC,
   rowBlockDimC, colBlockDimC,
   nnzTotalDevHostPtr, pBuffer);
if (NULL != nnzTotalDevHostPtr){
nnzc = *nnzTotalDevHostPtr;
} else{
   cudaMemcpy(&nnzc, bsrRowPtrC+mc, sizeof(int), cudaMemcpyDeviceToHost);
   cudaMemcpy(&base, bsrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
   nnzc -= base;
}
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzc);
cudaMalloc((void**)&bsrValC, sizeof(float)*(rowBlockDimC*colBlockDimC)*nnzc);
cusparseSgebsr2gebsr(handle, dir, mb, nb, nnzb,
   descrA, bsrValA, bsrRowPtrA, bsrColIndA,
   rowBlockDimA, colBlockDimA,
   descrC, bsrValC, bsrRowPtrC, bsrColIndC,
   rowBlockDimC, colBlockDimC,
pBuffer);
```

- The routines require no extra storage if `pBuffer != NULL`
- The routines do **not** support asynchronous execution
- The routines do **not** support CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>dir</code></td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td><code>mb</code></td>
<td>number of block rows of sparse matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>nb</code></td>
<td>number of block columns of sparse matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>nnzb</code></td>
<td>number of nonzero blocks of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>descrA</code></td>
<td>the descriptor of matrix <code>A</code>. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are</td>
</tr>
</tbody>
</table>
CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bsrValA</td>
<td>&lt;type&gt; array of nnzb<em>rowBlockDimA</em>colBlockDimA non-zero elements of matrix A.</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix A.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of nnzb column indices of the non-zero blocks of matrix A.</td>
</tr>
<tr>
<td>rowBlockDimA</td>
<td>number of rows within a block of A.</td>
</tr>
<tr>
<td>colBlockDimA</td>
<td>number of columns within a block of A.</td>
</tr>
<tr>
<td>descrC</td>
<td>the descriptor of matrix C. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>rowBlockDimC</td>
<td>number of rows within a block of C.</td>
</tr>
<tr>
<td>colBlockDimC</td>
<td>number of columns within a block of C.</td>
</tr>
<tr>
<td>pBufferSize</td>
<td>host pointer containing number of bytes of the buffer used in gebsr2gebsr().</td>
</tr>
<tr>
<td>pBuffer</td>
<td>buffer allocated by the user; the size is returned by gebsr2gebsr_bufferSize().</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bsrValC</td>
<td>&lt;type&gt; array of nnzc<em>rowBlockDimC</em>colBlockDimC non-zero elements of matrix C.</td>
</tr>
<tr>
<td>bsrRowPtrC</td>
<td>integer array of mc+1 elements that contains the start of every block row and the end of the last block row plus one of matrix C.</td>
</tr>
<tr>
<td>bsrColIndC</td>
<td>integer array of nnzc block column indices of the nonzero blocks of matrix C.</td>
</tr>
<tr>
<td>nnzTotalDevHostPtr</td>
<td>total number of nonzero blocks of C. *nnzTotalDevHostPtr is the same as bsrRowPtrC[mc]-bsrRowPtrC[0].</td>
</tr>
</tbody>
</table>

See [cusparseStatus_t](#) for the description of the return status.
13.4. `cusparse<t>gebsr2csr()`

cusparseStatus_t cusparseSgebsr2csr(cusparseHandle_t handle, cusparseDirection_t dir, int mb, int nb, const cusparseMatDescr_t descrA, const float* bsrValA, const int* bsrRowPtrA, const int* bsrColIndA, int rowBlockDim, int colBlockDim, const cusparseMatDescr_t descrC, float* csrValC, int* csrRowPtrC, int* csrColIndC)

cusparseStatus_t cusparseDgebsr2csr(cusparseHandle_t handle, cusparseDirection_t dir, int mb, int nb, const cusparseMatDescr_t descrA, const double* bsrValA, const int* bsrRowPtrA, const int* bsrColIndA, int rowBlockDim, int colBlockDim, const cusparseMatDescr_t descrC, double* csrValC, int* csrRowPtrC, int* csrColIndC)

cusparseStatus_t cusparseCgebsr2csr(cusparseHandle_t handle, cusparseDirection_t dir, int mb, int nb, const cusparseMatDescr_t descrA, const cuComplex* bsrValA, const int* bsrRowPtrA, const int* bsrColIndA, int rowBlockDim, int colBlockDim, const cusparseMatDescr_t descrC, cuComplex* csrValC, int* csrRowPtrC, int* csrColIndC)

cusparseStatus_t cusparseZgebsr2csr(cusparseHandle_t handle, cusparseDirection_t dir, int mb, int nb, const cusparseMatDescr_t descrA, const cuDoubleComplex* bsrValA, const int* bsrRowPtrA, const int* bsrColIndA, int rowBlockDim, int colBlockDim, const cusparseMatDescr_t descrC, cuDoubleComplex* csrValC, int* csrRowPtrC, int* csrColIndC)
This function converts a sparse matrix in general BSR format that is defined by the three arrays `bsrValA`, `bsrRowPtrA`, and `bsrColIndA` into a sparse matrix in CSR format that is defined by arrays `csrValC`, `csrRowPtrC`, and `csrColIndC`.

Let \( m(=mb \times \text{rowBlockDim}) \) be number of rows of \( A \) and \( n(=nb \times \text{colBlockDim}) \) be number of columns of \( A \), then \( A \) and \( C \) are \( m \times n \) sparse matrices. The general BSR format of \( A \) contains \( \text{nnzb}(=\text{bsrRowPtrA}[mb] - \text{bsrRowPtrA}[0]) \) non-zero blocks, whereas sparse matrix \( A \) contains \( \text{nnz}(=\text{nnzb} \times \text{rowBlockDim} \times \text{colBlockDim}) \) elements. The user must allocate enough space for arrays `csrRowPtrC`, `csrColIndC`, and `csrValC`. The requirements are as follows:

- `csrRowPtrC` of \( m+1 \) elements
- `csrValC` of \( \text{nnz} \) elements
- `csrColIndC` of \( \text{nnz} \) elements

The general procedure is as follows:

```c
// Given general BSR format (bsrRowPtrA, bsrColIndA, bsrValA) and
// blocks of BSR format are stored in column-major order.

cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int m = mb*rowBlockDim;
int n = nb*colBlockDim;
int nnzb = bsrRowPtrA[mb] - bsrRowPtrA[0]; // number of blocks
int nnz  = nnzb * rowBlockDim * colBlockDim; // number of elements
cudaMalloc((void**)&csrRowPtrC, sizeof(int)*(m+1));
cudaMalloc((void**)&csrColIndC, sizeof(int)*nnz);
cudaMalloc((void**)&csrValC, sizeof(float)*nnz);
cusparseSgebsr2csr(handle, dir, mb, nb,
    descrA,
    bsrValA, bsrRowPtrA, bsrColIndA,
    rowBlockDim, colBlockDim,
    descrC,
    csrValC, csrRowPtrC, csrColIndC);
```

The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>dir</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>mb</td>
<td>number of block rows of sparse matrix A.</td>
</tr>
<tr>
<td>nb</td>
<td>number of block columns of sparse matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>bsrValA</td>
<td>&lt;type&gt; array of ( \text{nnzb} \times \text{rowBlockDim} \times \text{colBlockDim} ) non-zero elements of matrix A.</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>bsrRowPtrA</td>
<td>integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix A.</td>
</tr>
<tr>
<td>bsrColIndA</td>
<td>integer array of nnzb column indices of the non-zero blocks of matrix A.</td>
</tr>
<tr>
<td>rowBlockDim</td>
<td>number of rows within a block of A.</td>
</tr>
<tr>
<td>colBlockDim</td>
<td>number of columns within a block of A.</td>
</tr>
<tr>
<td>descrC</td>
<td>the descriptor of matrix C. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrValC</td>
<td>&lt;type&gt; array of nnz non-zero elements of matrix C.</td>
</tr>
<tr>
<td>csrRowPtrC</td>
<td>integer array of m+1 elements that contains the start of every row and the end of the last row plus one of matrix C.</td>
</tr>
<tr>
<td>csrColIndC</td>
<td>integer array of nnz column indices of the non-zero elements of matrix C.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status
13.5. `cusparse<t>csr2gebsr()`

cusparseStatus_t
cusparseScsr2gebsr_bufferSize(cusparseHandle_t         handle,
                          cusparseDirection_t      dir,
                          int                      m,
                          int                      n,
                          const cusparseMatDescr_t descrA,
                          const float*             csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          int                      rowBlockDim,
                          int                      colBlockDim,
                          int*                     pBufferSize)

cusparseStatus_t
cusparseDcsr2gebsr_bufferSize(cusparseHandle_t         handle,
                          cusparseDirection_t      dir,
                          int                      m,
                          int                      n,
                          const cusparseMatDescr_t descrA,
                          const double*            csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          int                      rowBlockDim,
                          int                      colBlockDim,
                          int*                     pBufferSize)

cusparseStatus_t
cusparseCcsr2gebsr_bufferSize(cusparseHandle_t         handle,
                          cusparseDirection_t      dir,
                          int                      m,
                          int                      n,
                          const cusparseMatDescr_t descrA,
                          const cuComplex*         csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          int                      rowBlockDim,
                          int                      colBlockDim,
                          int*                     pBufferSize)

cusparseStatus_t
cusparseZcsr2gebsr_bufferSize(cusparseHandle_t         handle,
                          cusparseDirection_t      dir,
                          int                      m,
                          int                      n,
                          const cusparseMatDescr_t descrA,
                          const cuDoubleComplex*   csrValA,
                          const int*               csrRowPtrA,
                          const int*               csrColIndA,
                          int                      rowBlockDim,
                          int                      colBlockDim,
                          int*                     pBufferSize)

cusparseStatus_t
cusparseXcsr2gebsrNnz(cusparseHandle_t         handle,
                      cusparseDirection_t      dir,
                      int                      m,
                      int                      n,
                      const cusparseMatDescr_t descrA,
                      const int*               csrRowPtrA,
                      const int*               csrColIndA,
                      const cusparseMatDescr_t descrC,
                      int*                     bsrRowPtrC,
This function converts a sparse matrix $A$ in CSR format (that is defined by arrays $\text{csrValA}$, $\text{csrRowPtrA}$, and $\text{csrColIndA}$) into a sparse matrix $C$ in general BSR format (that is defined by the three arrays $\text{bsrValC}$, $\text{bsrRowPtrC}$, and $\text{bsrColIndC}$).

The matrix $A$ is a $m \times n$ sparse matrix and matrix $C$ is a $(mb \times \text{rowBlockDim}) \times (nb \times \text{colBlockDim})$ sparse matrix, where $mb = (m + \text{rowBlockDim}-1)/\text{rowBlockDim}$ is the number of block rows of $C$, and $nb = (n + \text{colBlockDim}-1)/\text{colBlockDim}$ is the number of block columns of $C$.

The block of $C$ is of size $\text{rowBlockDim} \times \text{colBlockDim}$. If $m$ is not multiple of $\text{rowBlockDim}$ or $n$ is not multiple of $\text{colBlockDim}$, zeros are filled in.

The implementation adopts a two-step approach to do the conversion. First, the user allocates $\text{bsrRowPtrC}$ of $mb+1$ elements and uses function $\text{cusparseXcsr2gebsrNnz()}$ to determine the number of nonzero block columns per block row. Second, the user gathers $nnzb$ (number of nonzero block columns of matrix $C$) from either $(nnzb=*\text{nnzTotalDevHostPtr})$ or $(nnzb=\text{bsrRowPtrC}[mb]-\text{bsrRowPtrC}[0])$ and allocates $\text{bsrValC}$ of $nnzb \times \text{rowBlockDim} \times \text{colBlockDim}$ elements and $\text{bsrColIndC}$ of $nnzb$ integers. Finally function $\text{cusparse[S|D|C|Z]csr2gebsr()}$ is called to complete the conversion.

The user must obtain the size of the buffer required by $\text{csr2gebsr()}$ by calling $\text{csr2gebsr_bufferSize()}$, allocate the buffer, and pass the buffer pointer to $\text{csr2gebsr()}$. 
The general procedure is as follows:

```c
// Given CSR format (csrRowPtrA, csrColIndA, csrValA) and
// blocks of BSR format are stored in column-major order.

cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int base, nnzb;
int mb = (m + rowBlockDim-1)/rowBlockDim;
int nb = (n + colBlockDim-1)/colBlockDim;
int bufferSize;
void *pBuffer;
cusparseScsr2gebsr_bufferSize(handle, dir, m, n,
descrA, csrValA, csrRowPtrA, csrColIndA,
rowBlockDim, colBlockDim, 
&bufferSize);
cudaMalloc((void**)&pBuffer, bufferSize);
cudaMalloc((void**)&bsrRowPtrC, sizeof(int) *(mb+1));

int *nnzTotalDevHostPtr = &nnzb;

if (NULL != nnzTotalDevHostPtr){
nzb = *nnzTotalDevHostPtr;
} else{
cudaMemcpy(&nnzb, bsrRowPtrC+mb, sizeof(int), cudaMemcpyDeviceToHost);
cudaMemcpy(&base, bsrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
nzb -= base;
}
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzb);
cudaMalloc((void**)&bsrValC, sizeof(float)*(rowBlockDim*colBlockDim)*nnzb);
cusparseScsr2gebsr(handle, dir, m, n,
descrA, 
csrValA, csrRowPtrA, csrColIndA,
descrC, bsrRowPtrC, rowBlockDim, colBlockDim,
bsrValC, bsrColIndC, 
rowBlockDim, colBlockDim,
pBuffer);
```

The routine `cusparseXcsr2gebsrNnz()` has the following properties:

- The routine requires no extra storage
- The routine does **not** support asynchronous execution
- The routine does **not** support CUDA graph capture

The routine `cusparse<t>csr2gebsr()` has the following properties:

- The routine requires no extra storage if `pBuffer != NULL`
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>dir</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of sparse matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>number of columns of sparse matrix A.</td>
</tr>
</tbody>
</table>
**descrA**
the descriptor of matrix A. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.

**csrValA**
 `<type>` array of `nnz` nonzero elements of matrix A.

**csrRowPtrA**
integer array of `m+1` elements that contains the start of every row and the end of the last row plus one of matrix A.

**csrColIndA**
integer array of `nnz` column indices of the nonzero elements of matrix A.

**descrC**
the descriptor of matrix C. The supported matrix type is `CUSPARSE_MATRIX_TYPE_GENERAL`. Also, the supported index bases are `CUSPARSE_INDEX_BASE_ZERO` and `CUSPARSE_INDEX_BASE_ONE`.

**rowBlockDim**
number of rows within a block of C.

**colBlockDim**
number of columns within a block of C.

**pBuffer**
buffer allocated by the user, the size is return by `csr2gebsr_bufferSize()`.

**Output**

**bsrValC**
 `<type>` array of `nnzb*rowBlockDim*colBlockDim` nonzero elements of matrix C.

**bsrRowPtrC**
integer array of `mb+1` elements that contains the start of every block row and the end of the last block row plus one of matrix C.

**bsrColIndC**
integer array of `nnzb` column indices of the nonzero blocks of matrix C.

**nnzTotalDevHostPtr**
total number of nonzero blocks of matrix C. Pointer `nnzTotalDevHostPtr` can point to a device memory or host memory.

See `cusparseStatus_t` for the description of the return status.

### 13.6. `cusparse<t>coo2csr()`

```
cusparseStatus_t
cusparseXcoo2csr(cusparseHandle_t   handle,
                const int*         cooRowInd,
                int                nnz,
                int                m,
                int*               csrRowPtr,
                cusparseIndexBase_t idxBase)
```
This function converts the array containing the uncompressed row indices (corresponding to COO format) into an array of compressed row pointers (corresponding to CSR format).

It can also be used to convert the array containing the uncompressed column indices (corresponding to COO format) into an array of column pointers (corresponding to CSC format).

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Handle</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>cooRowInd</td>
<td>integer array of ( nnz ) uncompressed row indices.</td>
</tr>
<tr>
<td>nnz</td>
<td>number of non-zeros of the sparse matrix (that is also the length of array cooRowInd).</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>idxBase</td>
<td>CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>CsRowPtr</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrRowPtr</td>
<td>integer array of ( m+1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
13.7. cusparse<t>csc2dense()

This function converts the sparse matrix in CSC format that is defined by the three arrays \( \text{cscValA} \), \( \text{cscColPtrA} \), and \( \text{cscRowIndA} \) into the matrix \( A \) in dense format. The dense matrix \( A \) is filled in with the values of the sparse matrix and with zeros elsewhere.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input
| **handle** | handle to the cuSPARSE library context. |
| **m** | number of rows of matrix \( A \). |
| **n** | number of columns of matrix \( A \). |
| **descrA** | the descriptor of matrix \( A \). The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE. |
| **cscValA** | \(<\text{type}>\) array of \( nnz = \text{cscColPtrA}(m) - \text{cscColPtrA}(0) \) nonzero elements of matrix \( A \). |
| **cscRowIndA** | integer array of \( nnz = \text{cscColPtrA}(m) - \text{cscColPtrA}(0) \) row indices of the nonzero elements of matrix \( A \). |
| **cscColPtrA** | integer array of \( n+1 \) elements that contains the start of every row and the end of the last column plus one. |
| **lda** | leading dimension of dense array \( A \). |

**Output**

| **A** | array of dimensions \((\text{lda}, n)\) that is filled in with the values of the sparse matrix. |

See `cusparseStatus_t` for the description of the return status.
13.8. `cusparse<t>csr2bsr()`

cusparseStatus_t

cusparseXcsr2bsrNnz(cusparseHandle_t handle,
  cusparseDirection_t dir,
  int m,
  int n,
  const cusparseMatDescr_t descrA,
  const int* csrRowPtrA,
  const int* csrColIndA,
  int blockDim,
  const cusparseMatDescr_t descrC,
  int* bsrRowPtrC,
  int* nnzTotalDevHostPtr)

cusparseStatus_t

cusparseScsr2bsr(cusparseHandle_t handle,
  cusparseDirection_t dir,
  int m,
  int n,
  const cusparseMatDescr_t descrA,
  const float* csrValA,
  const int* csrRowPtrA,
  const int* csrColIndA,
  int blockDim,
  const cusparseMatDescr_t descrC,
  float* bsrValC,
  int* bsrRowPtrC,
  int* bsrColIndC)

cusparseStatus_t

cusparseDcsr2bsr(cusparseHandle_t handle,
  cusparseDirection_t dir,
  int m,
  int n,
  const cusparseMatDescr_t descrA,
  const double* csrValA,
  const int* csrRowPtrA,
  const int* csrColIndA,
  int blockDim,
  const cusparseMatDescr_t descrC,
  double* bsrValC,
  int* bsrRowPtrC,
  int* bsrColIndC)

cusparseStatus_t

cusparseCcsr2bsr(cusparseHandle_t handle,
  cusparseDirection_t dir,
  int m,
  int n,
  const cusparseMatDescr_t descrA,
  const cuComplex* csrValA,
  const int* csrRowPtrA,
  const int* csrColIndA,
  int blockDim,
  const cusparseMatDescr_t descrC,
  cuComplex* bsrValC,
  int* bsrRowPtrC,
  int* bsrColIndC)

cusparseStatus_t

cusparseZcsr2bsr(cusparseHandle_t handle,
  cusparseDirection_t dir,
  int m,
  int n,
  const cusparseMatDescr_t descrA,
  const cuDoubleComplex* csrValA,
  const int* csrRowPtrA,
  const int* csrColIndA,
  int blockDim,
  const cusparseMatDescr_t descrC,
  cuDoubleComplex* bsrValC,
  int* bsrRowPtrC,
  int* bsrColIndC)
This function converts a sparse matrix in CSR format that is defined by the three arrays `csrValA`, `csrRowPtrA`, and `csrColIndA` into a sparse matrix in BSR format that is defined by arrays `bsrValC`, `bsrRowPtrC`, and `bsrColIndC`.

A is an \( m \times n \) sparse matrix. The BSR format of A has \( mb \) block rows, \( nb \) block columns, and \( nnzb \) nonzero blocks, where \( mb = \frac{(m + \text{blockDim}-1)}{\text{blockDim}} \) and \( nb = \frac{(n + \text{blockDim}-1)}{\text{blockDim}} \).

If \( m \) or \( n \) is not a multiple of \( \text{blockDim} \), zeros are filled in.

The conversion in cuSPARSE entails a two-step approach. First, the user allocates `bsrRowPtrC` of \( mb+1 \) elements and uses function `cusparseXcsr2bsrNnz()` to determine the number of nonzero block columns per block row. Second, the user gathers `nnzb` (number of non-zero block columns of matrix C) from either \( (\text{nnzb} = \text{nnzTotalDevHostPtr}) \) or \( (\text{nnzb} = \text{bsrRowPtrC}[mb] - \text{bsrRowPtrC}[0]) \) and allocates `bsrValC` of `nnzb*blockDim*blockDim` elements and `bsrColIndC` of `nnzb` elements. Finally function `cusparse[S|D|C|Z]csr2bsr90` is called to complete the conversion.

The general procedure is as follows:

```c
// Given CSR format (csrRowPtrA, csrColIndA, csrValA) and
// blocks of BSR format are stored in column-major order.
cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int base, nnzb;
int mb = (m + blockDim-1)/blockDim;
cudaMalloc((void**)&bsrRowPtrC, sizeof(int) * (mb+1));
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzb;
cusparseXcsr2bsrNnz(handle, dir, m, n,
    descrA, csrRowPtrA, csrColIndA,
    blockDim,
    descrC, bsrRowPtrC,
    nnzTotalDevHostPtr);
if (NULL != nnzTotalDevHostPtr){
    nnzb = *nnzTotalDevHostPtr;
} else{
    cudaMemcpy(&nnzb, bsrRowPtrC+mb, sizeof(int), cudaMemcpyDeviceToHost);
    cudaMemcpy(&base, bsrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
    nnzb -= base;
}
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzb);
cudaMalloc((void**)&bsrValC, sizeof(float)*(blockDim*blockDim)*nnzb);
cusparseScsr2bsr(handle, dir, m, n,
    descrA, descrC, csrRowPtrA, csrColIndA,
    blockDim,
    descrC, descrA, bsrRowPtrC, bsrColIndC,
    blockDim,
    bsrValC, bsrRowPtrC, bsrColIndC);
```

The routine `cusparse<t>csr2bsr()` has the following properties:

- This function requires temporary extra storage that is allocated internally if `blockDim > 16`
- The routine does **not** support asynchronous execution if `blockDim == 1`
- The routine does **not** support CUDA graph capture if `blockDim == 1`

The routine `cusparseXcsr2bsrNnz()` has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine does **not** support asynchronous execution
The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>dir</td>
<td>storage format of blocks, either CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of sparse matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>number of columns of sparse matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A.</td>
</tr>
<tr>
<td>csrValA</td>
<td>&lt;type&gt; array of nnz(=csrRowPtrA[m] - csrRowPtr[0]) non-zero elements of matrix A.</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of m+1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of nnz column indices of the non-zero elements of matrix A.</td>
</tr>
<tr>
<td>blockDim</td>
<td>block dimension of sparse matrix A. The range of blockDim is between 1 and min(m, n).</td>
</tr>
<tr>
<td>descrC</td>
<td>the descriptor of matrix C.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bsrValC</td>
<td>&lt;type&gt; array of nnzb<em>blockDim</em>blockDim nonzero elements of matrix C.</td>
</tr>
<tr>
<td>bsrRowPtrC</td>
<td>integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix C.</td>
</tr>
<tr>
<td>bsrColIndC</td>
<td>integer array of nnzb column indices of the non-zero blocks of matrix C.</td>
</tr>
<tr>
<td>nnzTotalDevHostPtr</td>
<td>total number of nonzero elements in device or host memory. It is equal to (bsrRowPtrC[mb] - bsrRowPtrC[0]).</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

### 13.9. `cusparse<t>csr2coo()`

```c
cusparseStatus_t
cusparseXcsr2coo(cusparseHandle_t handle,
    const int* csrRowPtr,
    int nnz,
    int m,
    int* cooRowInd,
    cusparseIndexBase_t idxBase)
```
This function converts the array containing the compressed row pointers (corresponding to CSR format) into an array of uncompressed row indices (corresponding to COO format).

It can also be used to convert the array containing the compressed column indices (corresponding to CSC format) into an array of uncompressed column indices (corresponding to COO format).

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrRowPtr</td>
<td>integer array of ( m+1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>nnz</td>
<td>number of nonzeros of the sparse matrix (that is also the length of array cooRowInd).</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>idxBase</td>
<td>CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
</tbody>
</table>

**Output**

| cooRowInd     | integer array of \( nnz \) uncompressed row indices. |

See `cusparseStatus_t` for the description of the return status.
13.10. **cusparseCsr2cscEx2()**

```c
 cusparseStatus_t cusparseCsr2cscEx2_bufferSize(cusparseHandle_t handle, int m, int n, int nnz, const void* csrVal, const int* csrRowPtr, const int* csrColInd, void* cscVal, int* cscColPtr, int* cscRowInd, cudaMemcpyDataType valType, cusparseAction_t copyValues, cusparseIndexBase_t idxBase, cusparseCsr2CscAlg_t alg, size_t* bufferSize)

 cusparseStatus_t cusparseCsr2cscEx2(cusparseHandle_t handle, int m, int n, int nnz, const void* csrVal, const int* csrRowPtr, const int* csrColInd, void* cscVal, int* cscColPtr, int* cscRowInd, cudaMemcpyDataType valType, cusparseAction_t copyValues, cusparseIndexBase_t idxBase, cusparseCsr2CscAlg_t alg, void* buffer)
```

This function converts a sparse matrix in CSR format (that is defined by the three arrays `csrVal`, `csrRowPtr`, and `csrColInd`) into a sparse matrix in CSC format (that is defined by arrays `cscVal`, `cscRowInd`, and `cscColPtr`). The resulting matrix can also be seen as the transpose of the original sparse matrix. Notice that this routine can also be used to convert a matrix in CSC format into a matrix in CSR format.

For alg **CUSPARSE_CSR2CSC_ALG1**: it requires extra storage proportional to the number of nonzero values `nnz`. It provides in output always the same matrix.

For alg **CUSPARSE_CSR2CSC_ALG2**: it requires extra storage proportional to the number of rows `m`. It does not ensure always the same ordering of CSC column indices and values. Also, it provides better performance then **CUSPARSE_CSR2CSC_ALG1** for regular matrices.

It is executed asynchronously with respect to the host, and it may return control to the application on the host before the result is ready.
The function `cusparseCsr2cscEx2_bufferSize()` returns the size of the workspace needed by `cusparseCsr2cscEx2()` User needs to allocate a buffer of this size and give that buffer to `cusparseCsr2cscEx2()` as an argument.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine does not support CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>handle to the cuSPARSE library context</td>
</tr>
<tr>
<td><code>m</code></td>
<td>number of rows of the CSR input matrix; number of columns of the CSC output matrix</td>
</tr>
<tr>
<td><code>n</code></td>
<td>number of columns of the CSR input matrix; number of rows of the CSC output matrix</td>
</tr>
<tr>
<td><code>nnz</code></td>
<td>number of nonzero elements of the CSR and CSC matrices</td>
</tr>
<tr>
<td><code>csrVal</code></td>
<td>value array of size <code>nnz</code> of the CSR matrix; of same type as <code>valType</code></td>
</tr>
<tr>
<td><code>csrRowPtr</code></td>
<td>integer array of size <code>m + 1</code> that contains the CSR row offsets</td>
</tr>
<tr>
<td><code>csrColInd</code></td>
<td>integer array of size <code>nnz</code> that contains the CSR column indices</td>
</tr>
<tr>
<td><code>valType</code></td>
<td>value type for both CSR and CSC matrices</td>
</tr>
<tr>
<td><code>copyValues</code></td>
<td><code>CUSPARSE_ACTION_SYMBOLIC</code> or <code>CUSPARSE_ACTION_NUMERIC</code></td>
</tr>
<tr>
<td><code>idxBase</code></td>
<td>Index base <code>CUSPARSE_INDEX_BASE_ZERO</code> or <code>CUSPARSE_INDEX_BASE_ONE</code></td>
</tr>
<tr>
<td><code>alg</code></td>
<td>algorithm implementation. see <code>cusparseCsr2CscAlg_t</code> for possible values.</td>
</tr>
<tr>
<td><code>bufferSize</code></td>
<td>number of bytes of workspace needed by <code>cusparseCsr2cscEx2()</code></td>
</tr>
<tr>
<td><code>buffer</code></td>
<td>pointer to workspace buffer</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
13.11. cusparse<t>csr2dense()

cusparseStatus_t cusparseScsr2dense(cusparseHandle_t handle, int m, int n, const cusparseMatDescr_t descrA, const float* csrValA, const int* csrRowPtrA, const int* csrColIndA, float* A, int lda)

cusparseStatus_t cusparseDcsr2dense(cusparseHandle_t handle, int m, int n, const cusparseMatDescr_t descrA, const double* csrValA, const int* csrRowPtrA, const int* csrColIndA, double* A, int lda)

cusparseStatus_t cusparseCcsr2dense(cusparseHandle_t handle, int m, int n, const cusparseMatDescr_t descrA, const cuComplex* csrValA, const int* csrRowPtrA, const int* csrColIndA, cuComplex* A, int lda)

cusparseStatus_t cusparseZcsr2dense(cusparseHandle_t handle, int m, int n, const cusparseMatDescr_t descrA, const cuDoubleComplex* csrValA, const int* csrRowPtrA, const int* csrColIndA, cuDoubleComplex* A, int lda)

This function converts the sparse matrix in CSR format (that is defined by the three arrays csrValA, csrRowPtrA, and csrColIndA) into the matrix A in dense format. The dense matrix A is filled in with the values of the sparse matrix and with zeros elsewhere.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input
<table>
<thead>
<tr>
<th><strong>handle</strong></th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>m</strong></td>
<td>number of rows of matrix $A$.</td>
</tr>
<tr>
<td><strong>n</strong></td>
<td>number of columns of matrix $A$.</td>
</tr>
<tr>
<td><strong>descrA</strong></td>
<td>the descriptor of matrix $A$. The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code>. Also, the supported index bases are <code>CUSPARSE_INDEX_BASE_ZERO</code> and <code>CUSPARSE_INDEX_BASE_ONE</code>.</td>
</tr>
<tr>
<td><strong>csrValA</strong></td>
<td>&lt;type&gt; array of $\text{nnz} = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0)$ nonzero elements of matrix $A$.</td>
</tr>
<tr>
<td><strong>csrRowPtrA</strong></td>
<td>integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td><strong>csrColIndA</strong></td>
<td>integer array of $\text{nnz} = \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0)$ column indices of the nonzero elements of matrix $A$.</td>
</tr>
<tr>
<td><strong>lda</strong></td>
<td>leading dimension of array matrix $A$.</td>
</tr>
</tbody>
</table>

**Output**

| **A** | array of dimensions $(\text{lda}, n)$ that is filled in with the values of the sparse matrix. |

See `cusparseStatus_t` for the description of the return status.
13.12. cusparse<t>csr2csr_compress()

cusparseStatus_t
cusparseScsr2csr_compress(cusparseHandle_t         handle,
int                      m,
int                      n,
const cusparseMatDescr_t descrA,
const float*             csrValA,
const int*               csrColIndA,
const int*               csrRowPtrA,
int                      nnzA,
const int*               nnzPerRow,
float*                   csrValC,
int*                     csrColIndC,
int*                     csrRowPtrC,
float                    tol)

cusparseStatus_t
cusparseDcsr2csr_compress(cusparseHandle_t         handle,
int                      m,
int                      n,
const cusparseMatDescr_t descrA,
const double*            csrValA,
const int*               csrColIndA,
const int*               csrRowPtrA,
int                      nnzA,
const int*               nnzPerRow,
double*                  csrValC,
int*                     csrColIndC,
int*                     csrRowPtrC,
double                   tol)

cusparseStatus_t
cusparseCcsr2csr_compress(cusparseHandle_t         handle,
int                      m,
int                      n,
const cusparseMatDescr_t descrA,
const cuComplex*         csrValA,
const int*               csrColIndA,
const int*               csrRowPtrA,
int                      nnzA,
const int*               nnzPerRow,
cuComplex*               csrValC,
int*                     csrColIndC,
int*                     csrRowPtrC,
cuComplex                tol)

cusparseStatus_t
cusparseZcsr2csr_compress(cusparseHandle_t         handle,
int                      m,
int                      n,
const cusparseMatDescr_t descrA,
const cuDoubleComplex*   csrValA,
const int*               csrColIndA,
const int*               csrRowPtrA,
int                      nnzA,
const int*               nnzPerRow,
cuDoubleComplex*        csrValC,
int*                     csrColIndC,
int*                     csrRowPtrC,
cuDoubleComplex         tol)
This function compresses the sparse matrix in CSR format into compressed CSR format. Given a sparse matrix $A$ and a non-negative value threshold (in the case of complex values, only the magnitude of the real part is used in the check), the function returns a sparse matrix $C$, defined by

$$C(i,j) = A(i,j) \text{ if } |A(i,j)| > \text{threshold}$$

The implementation adopts a two-step approach to do the conversion. First, the user allocates $\text{csrRowPtrC}$ of $m+1$ elements and uses function $\text{cusparse<t>nnz_compress()}$ to determine $\text{nnzPerRow}$ (the number of nonzeros columns per row) and $\text{nnzC}$ (the total number of nonzeros). Second, the user allocates $\text{csrValC}$ of $\text{nnzC}$ elements and $\text{csrColIndC}$ of $\text{nnzC}$ integers. Finally function $\text{cusparse<t>csr2csr_compress()}$ is called to complete the conversion.

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>number of rows of matrix $A$.</td>
</tr>
<tr>
<td>$n$</td>
<td>number of columns of matrix $A$.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix $A$. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>csrValA</td>
<td>$&lt;\text{type}&gt;$ array of $\text{nnz} (= \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0))$ elements of matrix $A$.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of $\text{nnz} (= \text{csrRowPtrA}(m) - \text{csrRowPtrA}(0))$ column indices of the elements of matrix $A$.</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>nnzA</td>
<td>number of nonzero elements in matrix $A$.</td>
</tr>
<tr>
<td>nnzPerRow</td>
<td>this array contains the number of elements kept in the compressed matrix, by row.</td>
</tr>
<tr>
<td>tol</td>
<td>on input, this contains the non-negative tolerance value used for compression. Any values in matrix $A$ less than or equal to this value will be dropped during compression.</td>
</tr>
</tbody>
</table>

**Output**

| csrValC | on output, this array contains the typed values of elements kept in the compressed matrix. Size = $\text{nnzC}$. |
| `csrColIndC` | on output, this integer array contains the column indices of elements kept in the compressed matrix. Size = nnzC. |
| `csrRowPtrC` | on output, this integer array contains the row pointers for elements kept in the compressed matrix. Size = m+1 |

See `cusparseStatus_t` for the description of the return status

The following is a sample code to show how to use this API.

```c
#include <stdio.h>
#include <sys/time.h>
#include <cusparse.h>
#define ERR_NE(X,Y) do { if ((X) != (Y)) { 
    fprintf(stderr,"Error in %s at %s:%d
",func__,__FILE__,__LINE__); 
    exit(-1);}} while(0)
#define CUDA_CALL(X) ERR_NE((X),cudaSuccess)
#define CUSPARSE_CALL(X) ERR_NE((X),CUSPARSE_STATUS_SUCCESS)
int main(){
    int m = 6, n = 5;
    cusparseHandle_t handle;
    CUSPARSE_CALL( cusparseCreate(&handle) );
    cusparseMatDescr_t descrX;
    CUSPARSE_CALL(cusparseCreateMatDescr(&descrX));
    // Initialize sparse matrix
    float *X;
    CUDA_CALL(cudaMallocManaged( &X, sizeof(float) * m * n ));
    memset( X, 0, sizeof(float) * m * n );
    X[0 + 0*m] = 1.0;  X[0 + 1*m] = 3.0;
    X[1 + 1*m] = -4.0; X[1 + 2*m] = 5.0;
    X[2 + 0*m] = 2.0;  X[2 + 3*m] = 7.0;  X[2 + 4*m] = 8.0;
    X[3 + 2*m] = 6.0;  X[3 + 4*m] = 9.0;
    X[4 + 3*m] = 3.5;  X[4 + 4*m] = 5.5;
    X[5 + 0*m] = 6.5;  X[5 + 2*m] = -9.9;
    // Initialize total_nnz, and nnzPerRowX for cusparseSdense2csr()
    int total_nnz = 13;
    int *nnzPerRowX;
    CUDA_CALL( cudaMallocManaged( &nnzPerRowX, sizeof(int) * m ) );
    nnzPerRowX[0] = 2; nnzPerRowX[1] = 2; nnzPerRowX[2] = 3;
    float *csrValX;
    int *csrRowPtrX;
    int *csrColIndX;
    CUDA_CALL( cudaMallocManaged( &csrValX, sizeof(float) * total_nnz ) );
    CUDA_CALL( cudaMallocManaged( &csrRowPtrX, sizeof(int) * (m+1) ) );
    CUDA_CALL( cudaMallocManaged( &csrColIndX, sizeof(int) * total_nnz ) );
```
Before calling this API, call two APIs to prepare the input.

```c
/** Call cusparseSdense2csr to generate CSR format as the inputs for
cusparseScsr2csr_compress **/
CUSPARSE_CALL( cusparseSdense2csr( handle, m, n, descrX, X,
m, nnzPerRowX, csrValX,
csrRowPtrX, csrColIndX )) ;

float tol = 3.5;
int *nnzPerRowY;
int *testNNZTotal;
CUDA_CALL( cudaMallocManaged( &nnzPerRowY, sizeof(int) * m ));
CUDA_CALL( cudaMallocManaged( &testNNZTotal, sizeof(int)));
memset( nnzPerRowY, 0, sizeof(int) * m );
// cusparseSnnz_compress generates nnzPerRowY and testNNZTotal
CUSPARSE_CALL( cusparseSnnz_compress(handle, m, descrX, csrValX,
csrRowPtrX, nnzPerRowY,
testNNZTotal, tol));

float *csrValY;
int *csrRowPtrY;
int *csrColIndY;
CUDA_CALL( cudaMallocManaged( &csrValY, sizeof(float) * (*testNNZTotal)));
CUDA_CALL( cudaMallocManaged( &csrRowPtrY, sizeof(int) * (m+1)));
CUDA_CALL( cudaMallocManaged( &csrColIndY, sizeof(int) * (*testNNZTotal)));

CUSPARSE_CALL( cusparseScsr2csr_compress( handle, m, n, descrX, csrValX,
csrColIndX, csrRowPtrX,
total_nnz, nnzPerRowY,
csrValY, csrColIndY,
csrRowPtrY, tol));

/* Expect results
nnzPerRowY:  0 2 2 2 1 2
csrValY:     -4 5 7 8 6 9 5.5 6.5 -9.9
csrColIndY:  1 2 3 4 2 4 4 0 2
csrRowPtrY:  0 0 2 4 6 7 9
*/
cudaFree(X);
cusparseDestroy(handle);
cudaFree(nnzPerRowX);
cudaFree(csrValX);
cudaFree(csrRowPtrX);
cudaFree(csrColIndX);
cudaFree(csrValY);
cudaFree(nnzPerRowY);
cudaFree(testNNZTotal);
cudaFree(csrRowPtrY);
cudaFree(csrColIndY);
return 0;
```
13.13. cusparse<t>dense2csc()

This function converts the matrix $A$ in dense format into a sparse matrix in CSC format. All the parameters are assumed to have been pre-allocated by the user, and the arrays are filled in based on `nnzPerCol`, which can be precomputed with `cusparse<t>nnz()`.

- This function requires temporary extra storage that is allocated internally
- The routine does **not** support asynchronous execution
The routine does not support CUDA graph capture

Input

<table>
<thead>
<tr>
<th>Input</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>number of columns of matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>A</td>
<td>array of dimensions (lda, n).</td>
</tr>
<tr>
<td>lda</td>
<td>leading dimension of dense array A.</td>
</tr>
<tr>
<td>nnzPerCol</td>
<td>array of size n containing the number of nonzero elements per column.</td>
</tr>
</tbody>
</table>

Output

<table>
<thead>
<tr>
<th>Output</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>cscValA</td>
<td>&lt;type&gt; array of nnz (= cscRowPtrA(m) - cscRowPtrA(0)) nonzero elements of matrix A. It is only filled in if copyValues is set to CUSPARSE_ACTION_NUMERIC.</td>
</tr>
<tr>
<td>cscRowIndA</td>
<td>integer array of nnz (= cscRowPtrA(m) - cscRowPtrA(0)) row indices of the nonzero elements of matrix A.</td>
</tr>
<tr>
<td>cscColPtrA</td>
<td>integer array of n+1 elements that contains the start of every column and the end of the last column plus one.</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status.
13.14. cusparse<t>dense2csr()

This function converts the matrix $A$ in dense format into a sparse matrix in CSR format. All the parameters are assumed to have been pre-allocated by the user and the arrays are filled in based on $\text{nnzPerRow}$, which can be pre-computed with $\text{cusparse<t>nnz()}$.

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.
This function requires temporary extra storage that is allocated internally. The routine does not support asynchronous execution. The routine does not support CUDA graph capture.

**Input**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>number of rows of matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>number of columns of matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>A</td>
<td>array of dimensions (lda, n).</td>
</tr>
<tr>
<td>lda</td>
<td>leading dimension of dense array A.</td>
</tr>
<tr>
<td>nnzPerRow</td>
<td>array of size n containing the number of non-zero elements per row.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrValA</td>
<td>&lt;type&gt; array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) nonzero elements of matrix A.</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>integer array of m+1 elements that contains the start of every column and the end of the last column plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column indices of the non-zero elements of matrix A.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
13.15. cusparse<t>nnz()

This function computes the number of nonzero elements per row or column and the total number of nonzero elements in a dense matrix.

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**dirA**  
direction that specifies whether to count nonzero elements by CUSPARSE_DIRECTION_ROW or by CUSPARSE_DIRECTION_COLUMN.

**m**  
number of rows of matrix A.

**n**  
number of columns of matrix A.

**descrA**  
the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.

**A**  
array of dimensions (lda, n).

**lda**  
leading dimension of dense array A.

**Output**

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>nnzPerRowColumn</td>
<td>host</td>
<td>array of size m or n containing the number of nonzero elements per row or column, respectively.</td>
</tr>
<tr>
<td>nnzTotalDevHostPtr</td>
<td>device</td>
<td>total number of nonzero elements in device or host memory.</td>
</tr>
</tbody>
</table>

See **cusparseStatus_t** for the description of the return status

### 13.16. cusparseCreateIdentityPermutation()

```
cusparseStatus_t  
cusparseCreateIdentityPermutation(cusparseHandle_t handle,  
  int              n,  
  int*             p);  
```

This function creates an identity map. The output parameter p represents such map by \( p = 0:1:(n-1) \).

This function is typically used with **coosort**, **csrsort**, **cscsort**.

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>host</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>n</td>
<td>host</td>
<td>size of the map.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>device</td>
<td>integer array of dimensions n.</td>
</tr>
</tbody>
</table>
See `cusparseStatus_t` for the description of the return status

### 13.17. cusparseXcoosort()

This function sorts COO format. The sorting is in-place. Also the user can sort by row or sort by column.

**A** is an $m \times n$ sparse matrix that is defined in COO storage format by the three arrays `cooVals`, `cooRows`, and `cooCols`.

There is no assumption for the base index of the matrix. *coosort* uses stable sort on signed integer, so the value of `cooRows` or `cooCols` can be negative.

This function *coosort()* requires buffer size returned by *coosort_bufferSizeExt()*.

The address of `pBuffer` must be multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

The parameter `P` is both input and output. If the user wants to compute sorted `cooVal`, `P` must be set as 0:1:(nnz-1) before *coosort()* and after *coosort()*.

Remark: the dimension $m$ and $n$ are not used. If the user does not know the value of $m$ or $n$, just passes a value positive. This usually happens if the user only reads a COO array first and needs to decide the dimension $m$ or $n$ later.

Appendix section provides a simple example of *coosort()*.
The routine requires no extra storage if pBuffer != NULL
The routine does not support asynchronous execution
The routine does not support CUDA graph capture

Input

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>host</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>host</td>
<td>number of rows of matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>host</td>
<td>number of columns of matrix A.</td>
</tr>
<tr>
<td>nnz</td>
<td>host</td>
<td>number of nonzero elements of matrix A.</td>
</tr>
<tr>
<td>cooRows</td>
<td>device</td>
<td>integer array of nnz unsorted row indices of A.</td>
</tr>
<tr>
<td>cooCols</td>
<td>device</td>
<td>integer array of nnz unsorted column indices of A.</td>
</tr>
<tr>
<td>P</td>
<td>device</td>
<td>integer array of nnz unsorted map indices. To construct cooVal, the user has to set P=0:1:(nnz-1).</td>
</tr>
<tr>
<td>pBuffer</td>
<td>device</td>
<td>buffer allocated by the user; the size is returned by coosort_bufferSizeExt().</td>
</tr>
</tbody>
</table>

Output

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>cooRows</td>
<td>device</td>
<td>integer array of nnz sorted row indices of A.</td>
</tr>
<tr>
<td>cooCols</td>
<td>device</td>
<td>integer array of nnz sorted column indices of A.</td>
</tr>
<tr>
<td>P</td>
<td>device</td>
<td>integer array of nnz sorted map indices.</td>
</tr>
<tr>
<td>pBufferSizeInBytes</td>
<td>host</td>
<td>number of bytes of the buffer.</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status
13.18. `cusparseXcsrsort()`

```c
 cusparseStatus_t
cusparseXcsrsort_bufferSizeExt(cusparseHandle_t handle,
    int m,
    int n,
    int nnz,
    const int* csrRowPtr,
    const int* csrColInd,
    size_t* pBufferSizeInBytes)
```

```c
 cusparseStatus_t
cusparseXcsrsort(cusparseHandle_t handle,
    int m,
    int n,
    int nnz,
    const cusparseMatDescr_t descrA,
    const int* csrRowPtr,
    int* csrColInd,
    int* P,
    void* pBuffer)
```

This function sorts CSR format. The stable sorting is in-place.

The matrix type is regarded as `CUSPARSE_MATRIX_TYPE_GENERAL` implicitly. In other words, any symmetric property is ignored.

This function `csrsort()` requires buffer size returned by `csrsort_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If not, `CUSPARSE_STATUS_INVALID_VALUE` is returned.

The parameter `P` is both input and output. If the user wants to compute sorted `csrVal`, `P` must be set as 0:1:(nnz-1) before `csrsort()`, and after `csrsort()`, new sorted value array satisfies `csrVal_sorted = csrVal(P)`. 
The general procedure is as follows:

```c
// A is a 3x3 sparse matrix, base-0
// | 1 2 3 |
// | 4 5 6 |
// | 7 8 9 |
const int m = 3;
const int n = 3;
const int nnz = 9;
csrRowPtr[m+1] = { 0, 3, 6, 9}; // on device
csrColInd[nnz] = { 2, 1, 0, 0, 2,1, 1, 2, 0}; // on device
csrVal[nnz] = { 3, 2, 1, 4, 6, 5, 8, 9, 7}; // on device
size_t pBufferSizeInBytes = 0;
void *pBuffer = NULL;
int *P = NULL;
// step 1: allocate buffer
cusparseXcsrsort_bufferSizeExt(handle, m, n, nnz, csrRowPtr, csrColInd,
&pBufferSizeInBytes);
cudaMalloc( &pBuffer, sizeof(char)* pBufferSizeInBytes);
// step 2: setup permutation vector P to identity
cudaMalloc( (void**)&P, sizeof(int)*nnz);
cusparseCreateIdentityPermutation(handle, nnz, P);
// step 3: sort CSR format
cusparseXcsrsort(handle, m, n, nnz, descrA, csrRowPtr, csrColInd, P, pBuffer);
// step 4: gather sorted csrVal
cusparseDgthr(handle, nnz, csrVal, csrVal_sorted, P, CUSPARSE_INDEX_BASE_ZERO);
```

- The routine requires no extra storage if `pBuffer != NULL`
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>host</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>host</td>
<td>number of rows of matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>host</td>
<td>number of columns of matrix A.</td>
</tr>
<tr>
<td>nnz</td>
<td>host</td>
<td>number of nonzero elements of matrix A.</td>
</tr>
<tr>
<td>csrRowsPtr</td>
<td>device</td>
<td>integer array of m+1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColInd</td>
<td>device</td>
<td>integer array of nnz unsorted column indices of A.</td>
</tr>
<tr>
<td>P</td>
<td>device</td>
<td>integer array of nnz unsorted map indices. To construct csrVal, the user has to set P=0:1:(nnz-1).</td>
</tr>
<tr>
<td>pBuffer</td>
<td>device</td>
<td>buffer allocated by the user; the size is returned by csrsort_bufferSizeExt().</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrColInd</td>
<td>device</td>
<td>integer array of nnz sorted column indices of A.</td>
</tr>
</tbody>
</table>
### 13.19. `cusparseXcscsort()`

This function sorts CSC format. The stable sorting is in-place.

The matrix type is regarded as CUSPARSE_MATRIX_TYPE_GENERAL implicitly. In other words, any symmetric property is ignored.

This function `cscsort()` requires buffer size returned by `cscsort_bufferSizeExt()`. The address of `pBuffer` must be multiple of 128 bytes. If not, CUSPARSE_STATUS_INVALID_VALUE is returned.

The parameter `P` is both input and output. If the user wants to compute sorted `cscVal`, `P` must be set as 0:1:(nnz-1) before `cscsort()`, and after `cscsort()`, new sorted value array satisfies `cscVal_sorted = cscVal(P)`.

```c
#include <cusparse.h>

cusparseStatus_t cusparseXcscsort_bufferSizeExt(cusparseHandle_t handle,
                                              int m,
                                              int n,
                                              int nnz,
                                              const int* cscColPtr,
                                              const int* cscRowInd,
                                              size_t* pBufferSizeInBytes)

cusparseStatus_t cusparseXcscsort(cusparseHandle_t handle,
                                  int m,
                                  int n,
                                  int nnz,
                                  const cusparseMatDescr_t descrA,
                                  const int* cscColPtr,
                                  int* cscRowInd,
                                  int* P,
                                  void* pBuffer)
```
The general procedure is as follows:

```c
// A is a 3x3 sparse matrix, base-0
//     | 1 2 |
// A = | 4 0 |
//     | 0 8 |
const int m = 3;
const int n = 2;
const int nnz = 4;
cscColPtr[n+1] = { 0, 2, 4}; // on device
cscRowInd[nnz] = { 1, 0, 2, 0}; // on device
cscVal[nnz]    = { 4.0, 1.0, 8.0, 2.0 }; // on device
size_t pBufferSizeInBytes = 0;
void *pBuffer = NULL;
int *P = NULL;

// step 1: allocate buffer
cusparseXcscsort_bufferSizeExt(handle, m, n, nnz, cscColPtr, cscRowInd,
&pBufferSizeInBytes);
cudaMalloc( &pBuffer, sizeof(char)* pBufferSizeInBytes);

// step 2: setup permutation vector P to identity
cudaMalloc( (void**)&P, sizeof(int)*nnz);
cusparseCreateIdentityPermutation(handle, nnz, P);

// step 3: sort CSC format
cusparseXcscsort(handle, m, n, nnz, descrA, cscColPtr, cscRowInd, P, pBuffer);

// step 4: gather sorted cscVal
cusparseDgthr(handle, nnz, cscVal, cscVal_sorted, P, CUSPARSE_INDEX_BASE_ZERO);
```

- The routine requires no extra storage if `pBuffer != NULL`
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

### Input

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>host</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>host</td>
<td>number of rows of matrix A.</td>
</tr>
<tr>
<td>n</td>
<td>host</td>
<td>number of columns of matrix A.</td>
</tr>
<tr>
<td>nnz</td>
<td>host</td>
<td>number of nonzero elements of matrix A.</td>
</tr>
<tr>
<td>cscColPtr</td>
<td>device</td>
<td>integer array of n+1 elements that contains the start of every column and the end of the last column plus one.</td>
</tr>
<tr>
<td>cscRowInd</td>
<td>device</td>
<td>integer array of nnz unsorted row indices of A.</td>
</tr>
<tr>
<td>P</td>
<td>device</td>
<td>integer array of nnz unsorted map indices. To construct cscVal, the user has to set P=0:1:(nnz-1).</td>
</tr>
<tr>
<td>pBuffer</td>
<td>device</td>
<td>buffer allocated by the user; the size is returned by cscsort_bufferSizeExt().</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>cscRowInd</td>
<td>device</td>
<td>integer array of nnz sorted row indices of A.</td>
</tr>
<tr>
<td>P</td>
<td>device</td>
<td>integer array of nnz sorted map indices.</td>
</tr>
<tr>
<td>-----------</td>
<td>--------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>pBufferSizeInBytes</td>
<td>host</td>
<td>number of bytes of the buffer.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
13.20. `cusparseXcsru2csr()`

```c
 cusparseStatus_t
 cusparseCreateCsru2csrInfo(csru2csrInfo_t *info);

cusparseStatus_t
 cusparseDestroyCsru2csrInfo(csru2csrInfo_t info);

cusparseStatus_t
 cusparseScsru2csr_bufferSizeExt(cusparseHandle_t handle,
 int m,
 int n,
 int nnz,
 float* csrVal,
 const int* csrRowPtr,
 int* csrColInd,
 csru2csrInfo_t info,
 size_t* pBufferSizeInBytes)

cusparseStatus_t
 cusparseDcsru2csr_bufferSizeExt(cusparseHandle_t handle,
 int m,
 int n,
 int nnz,
 double* csrVal,
 const int* csrRowPtr,
 int* csrColInd,
 csru2csrInfo_t info,
 size_t* pBufferSizeInBytes)

cusparseStatus_t
 cusparseCcsru2csr_bufferSizeExt(cusparseHandle_t handle,
 int m,
 int n,
 int nnz,
 cuComplex* csrVal,
 const int* csrRowPtr,
 int* csrColInd,
 csru2csrInfo_t info,
 size_t* pBufferSizeInBytes)

cusparseStatus_t
 cusparseZcsru2csr_bufferSizeExt(cusparseHandle_t handle,
 int m,
 int n,
 int nnz,
 cuDoubleComplex* csrVal,
 const int* csrRowPtr,
 int* csrColInd,
 csru2csrInfo_t info,
 size_t* pBufferSizeInBytes)

cusparseStatus_t
 cusparseScsr2csru(cusparseHandle_t handle,
 int m,
 int n,
 int nnz,
 const cusparseMatDescr_t descrA,
 float* csrVal,
 const int* csrRowPtr,
 int* csrColInd,
 csru2csrInfo_t info,
 void* pBuffer)

cusparseStatus_t
 cusparseDcsr2csru(cusparseHandle_t handle,
 int m,
 int n,
 int nnz,
 const cusparseMatDescr_t descrA,
 double* csrVal,
 const int* csrRowPtr,
 int* csrColInd,
 csru2csrInfo_t info,
 void* pBuffer)

cusparseStatus_t
 cusparseCcsr2csru(cusparseHandle_t handle,
 int m,
 int n,
 int nnz,
 const cusparseMatDescr_t descrA,
 cuComplex* csrVal,
 const int* csrRowPtr,
 int* csrColInd,
 csru2csrInfo_t info,
 void* pBuffer)

cusparseStatus_t
 cusparseZcsr2csru(cusparseHandle_t handle,
 int m,
 int n,
 int nnz,
 const cusparseMatDescr_t descrA,
 cuDoubleComplex* csrVal,
 const int* csrRowPtr,
 int* csrColInd,
 csru2csrInfo_t info,
 void* pBuffer)
```
This function transfers unsorted CSR format to CSR format, and vice versa. The operation is in-place.

This function is a wrapper of csrsort and gthr. The usecase is the following scenario.

If the user has a matrix \( A \) of CSR format which is unsorted, and implements his own code (which can be CPU or GPU kernel) based on this special order (for example, diagonal first, then lower triangle, then upper triangle), and wants to convert it to CSR format when calling CUSPARSE library, and then convert it back when doing something else on his/her kernel. For example, suppose the user wants to solve a linear system \( Ax=b \) by the following iterative scheme

\[
\chi^{(k+1)} = \chi^{(k)} + L^{(-1)} \cdot (b - A \chi^{(k)})
\]

The code heavily uses SpMv and triangular solve. Assume that the user has an in-house design of SpMV (Sparse Matrix-Vector multiplication) based on special order of \( A \). However the user wants to use CUSAPRSE library for triangular solver. Then the following code can work.

```plaintext
do
   step 1: compute residual vector \( r = b - A \chi^{(k)} \) by
   step 2: \( B := \text{sort}(A) \), and \( L \) is lower triangular part of \( B \)
         (only sort A once and keep the permutation vector)
   step 3: solve \( z = L^{(-1)} \cdot b \)
   step 4: add correction \( \chi^{(k+1)} = \chi^{(k)} + z \)
   step 5: \( A := \text{unsort}(B) \)
   (use permutation vector to get back the unsorted CSR)
until convergence
```

The requirements of step 2 and step 5 are

1. In-place operation.
2. The permutation vector \( P \) is hidden in an opaque structure.
3. No \texttt{cudaMalloc} inside the conversion routine. Instead, the user has to provide the buffer explicitly.
4. The conversion between unsorted CSR and sorted CSR may needs several times, but the function only generates the permutation vector \( P \) once.
5. The function is based on csrsort, gather and scatter operations.

The operation is called \texttt{csru2csr}, which means unsorted CSR to sorted CSR. Also we provide the inverse operation, called \texttt{csr2csru}.

In order to keep the permutation vector invisible, we need an opaque structure called \texttt{csru2csrInfo}. Then two functions (\texttt{cusparseCreateCsru2csrInfo}, \texttt{cusparseDestroyCsru2csrInfo}) are used to initialize and to destroy the opaque structure.

\texttt{cusparse[S|D|C|Z]csru2csr_bufferSizeExt} returns the size of the buffer.

The permutation vector \( P \) is also allocated inside \texttt{csru2csrInfo}. The lifetime of the permutation vector is the same as the lifetime of \texttt{csru2csrInfo}.
**cusparse[S|D|C|Z]csru2csr** performs forward transformation from unsorted CSR to sorted CSR. First call uses csrsort to generate the permutation vector \( P \), and subsequent call uses \( P \) to do transformation.

**cusparse[S|D|C|Z]csr2csru** performs backward transformation from sorted CSR to unsorted CSR. \( P \) is used to get unsorted form back.

The routine **cusparse\(<t>csru2csr()** has the following properties:

- The routine requires no extra storage if \( pBuffer \neq NULL \)
- The routine does **not** support asynchronous execution
- The routine does **not** support CUDA graph capture

The routine **cusparse\(<t>csr2csru()** has the following properties if \( pBuffer \neq NULL \):

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

The following tables describe parameters of **csr2csru_bufferSizeExt** and **csr2csru**.

### Input

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>host</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>host</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>n</td>
<td>host</td>
<td>number of columns of matrix ( A ).</td>
</tr>
<tr>
<td>nnz</td>
<td>host</td>
<td>number of nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>descrA</td>
<td>host</td>
<td>the descriptor of matrix ( A ). The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>csrVal</td>
<td>device</td>
<td>(&lt;type&gt;) array of nnz unsorted nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>csrRowsPtr</td>
<td>device</td>
<td>integer array of ( m+1 ) elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColInd</td>
<td>device</td>
<td>integer array of nnz unsorted column indices of ( A ).</td>
</tr>
<tr>
<td>info</td>
<td>host</td>
<td>opaque structure initialized using cusparseCreateCsru2csrInfo().</td>
</tr>
<tr>
<td>pBuffer</td>
<td>device</td>
<td>buffer allocated by the user; the size is returned by csr2csr_bufferSizeExt().</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>csrVal</td>
<td>device</td>
<td>(&lt;type&gt;) array of nnz sorted nonzero elements of matrix ( A ).</td>
</tr>
<tr>
<td>csrColInd</td>
<td>device</td>
<td>integer array of nnz sorted column indices of ( A ).</td>
</tr>
<tr>
<td>pBufferSizeInBytes</td>
<td>host</td>
<td>number of bytes of the buffer.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
13.21. cusparseXpruneDense2csr()

cusparseStatus_t
cusparseHpruneDense2csr_bufferSizeExt(cusparseHandle_t handle,
int m,
int n,
const __half* A,
int lda,
threshold,
const cusparseMatDescr_t descrC,
const __half* csrValC,
const int* csrRowPtrC,
const int* csrColIndC,
size_t* pBufferSizeInBytes)

cusparseStatus_t
cusparseSpruneDense2csr_bufferSizeExt(cusparseHandle_t handle,
int m,
int n,
const float* A,
int lda,
threshold,
const cusparseMatDescr_t descrC,
const float* csrValC,
const int* csrRowPtrC,
const int* csrColIndC,
size_t* pBufferSizeInBytes)

cusparseStatus_t
cusparseDpruneDense2csr_bufferSizeExt(cusparseHandle_t handle,
int m,
int n,
const double* A,
int lda,
threshold,
const cusparseMatDescr_t descrC,
const double* csrValC,
const int* csrRowPtrC,
const int* csrColIndC,
size_t* pBufferSizeInBytes)

cusparseStatus_t
cusparseHpruneDense2csrNnz(cusparseHandle_t handle,
int m,
int n,
const __half* A,
int lda,
threshold,
const cusparseMatDescr_t descrC,
__half* csrValC,
const int* csrRowPtrC,
int* nnzTotalDevHostPtr,
void* pBuffer)

cusparseStatus_t
cusparseSpruneDense2csrNnz(cusparseHandle_t handle,
int m,
int n,
const float* A,
int lda,
threshold,
const cusparseMatDescr_t descrC,
float* csrValC,
const int* csrRowPtrC,
int* nnzTotalDevHostPtr,
void* pBuffer)

cusparseStatus_t
cusparseDpruneDense2csrNnz(cusparseHandle_t handle,
int m,
int n,
const double* A,
int lda,
threshold,
const cusparseMatDescr_t descrC,
double* csrValC,
const int* csrRowPtrC,
int* nnzTotalDevHostPtr,
void* pBuffer)
This function prunes a dense matrix to a sparse matrix with CSR format.

Given a dense matrix \( A \) and a non-negative value \( \text{threshold} \), the function returns a sparse matrix \( C \), defined by

\[
C(i,j) = A(i,j) \quad \text{if} \quad |A(i,j)| > \text{threshold}
\]

The implementation adopts a two-step approach to do the conversion. First, the user allocates \( \text{csrRowPtrC} \) of \( m+1 \) elements and uses function \( \text{pruneDense2csrNnz()} \) to determine the number of nonzeros columns per row. Second, the user gathers \( \text{nnzC} \) (number of nonzeros of matrix \( C \)) from either \( (\text{nnzC} = *\text{nnzTotalDevHostPtr}) \) or \( (\text{nnzC} = \text{csrRowPtrC}[m] - \text{csrRowPtrC}[0]) \) and allocates \( \text{csrValC} \) of \( \text{nnzC} \) elements and \( \text{csrColIndC} \) of \( \text{nnzC} \) integers. Finally function \( \text{pruneDense2csr()} \) is called to complete the conversion.

The user must obtain the size of the buffer required by \( \text{pruneDense2csr()} \) by calling \( \text{pruneDense2csr_bufferSizeExt()} \), allocate the buffer, and pass the buffer pointer to \( \text{pruneDense2csr()} \).

Appendix section provides a simple example of \( \text{pruneDense2csr()} \).

The routine \( \text{cusparse}<t>\text{pruneDense2csrNnz()} \) has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

The routine \( \text{cusparse}<t>\text{DpruneDense2csr()} \) has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>host</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>m</td>
<td>host</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>n</td>
<td>host</td>
<td>number of columns of matrix ( A ).</td>
</tr>
<tr>
<td>A</td>
<td>device</td>
<td>array of dimension (lda, n).</td>
</tr>
<tr>
<td>lda</td>
<td>device</td>
<td>leading dimension of ( A ). It must be at least max(1, m).</td>
</tr>
<tr>
<td>threshold</td>
<td>host or device</td>
<td>a value to drop the entries of ( A ). ( \text{threshold} ) can point to a device memory or host memory.</td>
</tr>
<tr>
<td>descrC</td>
<td>host</td>
<td>the descriptor of matrix ( C ). The supported matrix type is ( \text{CUSPARSE_MATRIX_TYPE_GENERAL} ). Also, the supported index bases are ( \text{CUSPARSE_INDEX_BASE_ZERO} ) and ( \text{CUSPARSE_INDEX_BASE_ONE} ).</td>
</tr>
<tr>
<td>pBuffer</td>
<td>device</td>
<td>buffer allocated by the user; the size is returned by ( \text{pruneDense2csr_bufferSizeExt()} ).</td>
</tr>
</tbody>
</table>

**Output**
<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>nnzTotalDevHostPtr</td>
<td>device or host</td>
<td>total number of nonzero of matrix C. nnzTotalDevHostPtr can point to a device memory or host memory.</td>
</tr>
<tr>
<td>csrValC</td>
<td>device</td>
<td>&lt;type&gt; array of nnzC nonzero elements of matrix C.</td>
</tr>
<tr>
<td>csrRowsPtrC</td>
<td>device</td>
<td>integer array of m+1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndC</td>
<td>device</td>
<td>integer array of nnzC column indices of C.</td>
</tr>
<tr>
<td>pBufferSizeInBytes</td>
<td>host</td>
<td>number of bytes of the buffer.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
13.22. cusparseXpruneCsr2csr()

cusparseStatus_t

cusparseDpruneCsr2csr_bufferSizeExt(cusparseHandle_t handle,
    int m, int n, int nnzA,
    const cusparseMatDescr_t descrA,
    const __half* csrValA,
    const int* csrRowPtrA,
    const int* csrColIndA,
    const __half* threshold,
    const cusparseMatDescr_t descrC,
    const __half* csrValC,
    const int* csrRowPtrC,
    const int* csrColIndC,
    size_t* pBufferSizeInBytes)

cusparseStatus_t

cusparseSpruneCsr2csr_bufferSizeExt(cusparseHandle_t handle,
    int m, int n, int nnzA,
    const cusparseMatDescr_t descrA,
    const float* csrValA,
    const int* csrRowPtrA,
    const int* csrColIndA,
    const float* threshold,
    const cusparseMatDescr_t descrC,
    const float* csrValC,
    const int* csrRowPtrC,
    const int* csrColIndC,
    size_t* pBufferSizeInBytes)

cusparseStatus_t

cusparseDpruneCsr2csr_bufferSizeExt(cusparseHandle_t handle,
    int m, int n, int nnzA,
    const cusparseMatDescr_t descrA,
    const double* csrValA,
    const int* csrRowPtrA,
    const int* csrColIndA,
    const double* threshold,
    const cusparseMatDescr_t descrC,
    const double* csrValC,
    const int* csrRowPtrC,
    const int* csrColIndC,
    size_t* pBufferSizeInBytes)

cusparseStatus_t

cusparseHpruneCsr2csrNnzNnz(cusparseHandle_t handle,
    int m, int n, int nnzA,
    const cusparseMatDescr_t descrA,
    const __half* csrValA,
    const int* csrRowPtrA,
    const int* csrColIndA,
    const __half* threshold,
    const cusparseMatDescr_t descrC,
    const int* pBufferSizeInBytes)
This function prunes a sparse matrix to a sparse matrix with CSR format.

Given a sparse matrix $A$ and a non-negative value $\text{threshold}$, the function returns a sparse matrix $C$, defined by

$$C(i,j) = A(i,j) \text{ if } |A(i,j)| > \text{threshold}$$

The implementation adopts a two-step approach to do the conversion. First, the user allocates $\text{csrRowPtrC}$ of $m+1$ elements and uses function $\text{pruneCsr2csrNnz()}$ to determine the number of nonzeros columns per row. Second, the user gathers $\text{nnzC}$ (number of nonzeros of matrix $C$) from either $(\text{nnzC} = \asternzTotalDevHostPtr)$ or $(\text{nnzC} = \text{csrRowPtrC}[m] - \text{csrRowPtrC}[0])$ and allocates $\text{csrValC}$ of $\text{nnzC}$ elements and $\text{csrColIndC}$ of $\text{nnzC}$ integers. Finally function $\text{pruneCsr2csr()}$ is called to complete the conversion.

The user must obtain the size of the buffer required by $\text{pruneCsr2csr()}$ by calling $\text{pruneCsr2csr_bufferSizeExt()}$, allocate the buffer, and pass the buffer pointer to $\text{pruneCsr2csr()}$.

Appendix section provides a simple example of $\text{pruneCsr2csr()}$.

The routine $\text{cusparse\langle t\rangle pruneCsr2csrNnz()}$ has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

The routine $\text{cusparse\langle t\rangle pruneCsr2csr()}$ has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>host</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>$m$</td>
<td>host</td>
<td>number of rows of matrix $A$.</td>
</tr>
<tr>
<td>$n$</td>
<td>host</td>
<td>number of columns of matrix $A$.</td>
</tr>
<tr>
<td>$\text{nnzA}$</td>
<td>host</td>
<td>number of nonzeros of matrix $A$.</td>
</tr>
<tr>
<td>descrA</td>
<td>host</td>
<td>the descriptor of matrix $A$. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>$\text{csrValA}$</td>
<td>device</td>
<td>$\langle$type$\rangle$ array of $\text{nnzA}$ nonzero elements of matrix $A$.</td>
</tr>
<tr>
<td>csrRowsPtrA</td>
<td>device</td>
<td>integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>device</td>
<td>integer array of $\text{nnzA}$ column indices of $A$.</td>
</tr>
<tr>
<td>threshold</td>
<td>host or device</td>
<td>a value to drop the entries of $A$. threshold can point to a device memory or host memory.</td>
</tr>
</tbody>
</table>
descrC | host | the descriptor of matrix C. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.

pBuffer | device | buffer allocated by the user; the size is returned by pruneCsr2csr_bufferSizeExt().

### Output

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>nnzTotalDevHostPtr</td>
<td>device or host</td>
<td>total number of nonzero of matrix C. nnzTotalDevHostPtr can point to a device memory or host memory.</td>
</tr>
<tr>
<td>csrValC</td>
<td>device</td>
<td>&lt;type&gt; array of nnzC nonzero elements of matrix C.</td>
</tr>
<tr>
<td>csrRowsPtrC</td>
<td>device</td>
<td>integer array of m+1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndC</td>
<td>device</td>
<td>integer array of nnzC column indices of C.</td>
</tr>
<tr>
<td>pBufferSizeInBytes</td>
<td>host</td>
<td>number of bytes of the buffer.</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status.
13.23. cusparseXpruneDense2csrPercentage()

cusparseStatus_t
cusparseHpruneDense2csrByPercentage_bufferSizeExt(cusparseHandle_t handle, int m, int n, const __half* A, int lda, float percentage, const cusparseMatDescr_t descrC, const __half* csrValC, const int* csrRowPtrC, const int* csrColIndC, pruneInfo_t info, size_t* pBufferSizeInBytes)

cusparseStatus_t
cusparseSpruneDense2csrByPercentage_bufferSizeExt(cusparseHandle_t handle, int m, int n, const float* A, int lda, float percentage, const cusparseMatDescr_t descrC, const float* csrValC, const int* csrRowPtrC, const int* csrColIndC, pruneInfo_t info, size_t* pBufferSizeInBytes)

cusparseStatus_t
cusparseDpruneDense2csrByPercentage_bufferSizeExt(cusparseHandle_t handle, int m, int n, const double* A, int lda, float percentage, const cusparseMatDescr_t descrC, const double* csrValC, const int* csrRowPtrC, const int* csrColIndC, pruneInfo_t info, size_t* pBufferSizeInBytes)
This function prunes a dense matrix to a sparse matrix by percentage.

Given a dense matrix $A$ and a non-negative value $\text{percentage}$, the function computes sparse matrix $C$ by the following three steps:

Step 1: sort absolute value of $A$ in ascending order.

$$\text{key} := \text{sort}( |A| )$$

Step 2: choose threshold by the parameter $\text{percentage}$

$$\text{pos} = \text{ceil}(m \times n \times (\text{percentage}/100)) - 1$$
$$\text{pos} = \text{min}(\text{pos}, m \times n - 1)$$
$$\text{pos} = \text{max}(\text{pos}, 0)$$
$$\text{threshold} = \text{key}[\text{pos}]$$

Step 3: call $\text{pruneDense2csr()}$ by with the parameter $\text{threshold}$.

The implementation adopts a two-step approach to do the conversion. First, the user allocates $\text{csrRowPtrC}$ of $m+1$ elements and uses function $\text{pruneDense2csrNnzByPercentage()}$ to determine the number of nonzeros columns per row. Second, the user gathers $\text{nnzC}$ (number of nonzeros of matrix $C$) from either ($\text{nnzC} = *\text{nnzTotalDevHostPtr}$) or ($\text{nnzC} = \text{csrRowPtrC}[m] - \text{csrRowPtrC}[0]$) and allocates $\text{csrValC}$ of $\text{nnzC}$ elements and $\text{csrColIndC}$ of $\text{nnzC}$ integers. Finally function $\text{pruneDense2csrByPercentage()}$ is called to complete the conversion.

The user must obtain the size of the buffer required by $\text{pruneDense2csrByPercentage()}$ by calling $\text{pruneDense2csrByPercentage_bufferSizeExt()}$, allocate the buffer, and pass the buffer pointer to $\text{pruneDense2csrByPercentage()}$.

Remark 1: the value of $\text{percentage}$ must be not greater than 100. Otherwise, $\text{CUSPARSE\_STATUS\_INVALID\_VALUE}$ is returned.

Remark 2: the zeros of $A$ are not ignored. All entries are sorted, including zeros. This is different from $\text{pruneCsr2csrByPercentage()}$

Appendix section provides a simple example of $\text{pruneDense2csrNnzByPercentage()}$.

The routine $\text{cusparse<}{t}\text{pruneDense2csrNnzByPercentage()}$ has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

The routine $\text{cusparse<}{t}\text{pruneDense2csrByPercentage()}$ has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

Input

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
</table>

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cuSPARSE Library

DU-06709-001_v11.0 | 237
### Input Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>handle</code></td>
<td>host</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td><code>m</code></td>
<td>host</td>
<td>number of rows of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>n</code></td>
<td>host</td>
<td>number of columns of matrix <code>A</code>.</td>
</tr>
<tr>
<td><code>A</code></td>
<td>device</td>
<td>array of dimension <code>(lda, n)</code>.</td>
</tr>
<tr>
<td><code>lda</code></td>
<td>device</td>
<td>leading dimension of <code>A</code>. It must be at least <code>max(1, m)</code>.</td>
</tr>
<tr>
<td><code>percentage</code></td>
<td>host</td>
<td>percentage &lt;=100 and percentage &gt;= 0</td>
</tr>
<tr>
<td><code>descrC</code></td>
<td>host</td>
<td>the descriptor of matrix <code>C</code>. The supported matrix type is <code>CUSPARSE_MATRIX_TYPE_GENERAL</code>. Also, the supported index bases are <code>CUSPARSE_INDEX_BASE_ZERO</code> and <code>CUSPARSE_INDEX_BASE_ONE</code>.</td>
</tr>
<tr>
<td><code>pBuffer</code></td>
<td>device</td>
<td>buffer allocated by the user; the size is returned by <code>pruneDense2csrByPercentage_bufferSizeExt()</code>.</td>
</tr>
</tbody>
</table>

### Output Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>nnzTotalDevHostPtr</code></td>
<td>device or host</td>
<td>total number of nonzero of matrix <code>C</code>. <code>nnzTotalDevHostPtr</code> can point to a device memory or host memory.</td>
</tr>
<tr>
<td><code>csrValC</code></td>
<td>device</td>
<td><code>&lt;type&gt; array of nnzC nonzero elements of matrix </code>C`.</td>
</tr>
<tr>
<td><code>csrRowsPtrC</code></td>
<td>device</td>
<td>integer array of <code>m+1</code> elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td><code>csrColIndC</code></td>
<td>device</td>
<td>integer array of <code>nnzC</code> column indices of <code>C</code>.</td>
</tr>
<tr>
<td><code>pBufferSizeInBytes</code></td>
<td>host</td>
<td>number of bytes of the buffer.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
13.24. cusparseXpruneCsr2csrByPercentage()

cusparseStatus_t
cusparseXpruneCsr2csrByPercentage_bufferSizeExt(cusparseHandle_t
handle,
    m,
    n,
    nnzA,
    descrA,
    csrValA,
    csrRowPtrA,
    csrColIndA,
    percentage,
    descrC,
    csrValC,
    csrRowPtrC,
    csrColIndC,
    info,
    pBufferSizeInBytes)

cusparseStatus_t
cusparseSpruneCsr2csrByPercentage_bufferSizeExt(cusparseHandle_t
handle,
    m,
    n,
    nnzA,
    descrA,
    csrValA,
    csrRowPtrA,
    csrColIndA,
    percentage,
    descrC,
    csrValC,
    csrRowPtrC,
    csrColIndC,
    info,
    pBufferSizeInBytes)
This function prunes a sparse matrix to a sparse matrix by percentage.

Given a sparse matrix \( A \) and a non-negative value \( \text{percentage} \), the function computes sparse matrix \( C \) by the following three steps:

Step 1: sort absolute value of \( A \) in ascending order.

\[
\text{key} := \text{sort}( |\text{csrVal}| )
\]

Step 2: choose threshold by the parameter \( \text{percentage} \)

\[
\begin{align*}
pos &= \text{ceil}(\text{nnzA} \times (\text{percentage}/100)) - 1 \\
pos &= \min(pos, \text{nnzA} - 1) \\
pos &= \max(pos, 0) \\
\text{threshold} &= \text{key}[pos]
\end{align*}
\]

Step 3: call \( \text{pruneCsr2csr()} \) by with the parameter \( \text{threshold} \).

The implementation adopts a two-step approach to do the conversion. First, the user allocates \( \text{csrRowPtrC} \) of \( m+1 \) elements and uses function \( \text{pruneCsr2csrNnzByPercentage()} \) to determine the number of nonzeros columns per row. Second, the user gathers \( \text{nnzC} \) (number of nonzeros of matrix \( C \)) from either \( (\text{nnzC}=\text{nnzTotalDevHostPtr}) \) or \( (\text{nnzC}=	ext{csrRowPtrC}[m]−\text{csrRowPtrC}[0]) \) and allocates \( \text{csrValC} \) of \( \text{nnzC} \) elements and \( \text{csrColIndC} \) of \( \text{nnzC} \) integers. Finally function \( \text{pruneCsr2csrByPercentage()} \) is called to complete the conversion.

The user must obtain the size of the buffer required by \( \text{pruneCsr2csrByPercentage()} \) by calling \( \text{pruneCsr2csrByPercentage\_bufferSizeExt()} \), allocate the buffer, and pass the buffer pointer to \( \text{pruneCsr2csrByPercentage()} \).

Remark 1: the value of \( \text{percentage} \) must be not greater than 100. Otherwise, \( \text{CUSPARSE\_STATUS\_INVALID\_VALUE} \) is returned.

Appendix section provides a simple example of \( \text{pruneCsr2csrByPercentage()} \).

The routine \( \text{cusparse<t>pruneCsr2csrNnzByPercentage()} \) has the following properties:

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
- The routine does not support CUDA graph capture

The routine \( \text{cusparse<t>pruneCsr2csrByPercentage()} \) has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>host</td>
<td>handle to the cuSPARSE library context.</td>
</tr>
<tr>
<td>( m )</td>
<td>host</td>
<td>number of rows of matrix ( A ).</td>
</tr>
<tr>
<td>( n )</td>
<td>host</td>
<td>number of columns of matrix ( A ).</td>
</tr>
</tbody>
</table>
cuSPARSE Format Conversion Reference

<table>
<thead>
<tr>
<th>nnzA</th>
<th>host</th>
<th>number of nonzeros of matrix A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>descrA</td>
<td>host</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>csrValA</td>
<td>device</td>
<td>&lt;type&gt; array of nnzA nonzero elements of matrix A.</td>
</tr>
<tr>
<td>csrRowsPtrA</td>
<td>device</td>
<td>integer array of m+1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndA</td>
<td>device</td>
<td>integer array of nnzA column indices of A.</td>
</tr>
<tr>
<td>percentage</td>
<td>host</td>
<td>percentage &lt;=100 and percentage &gt;= 0</td>
</tr>
<tr>
<td>descrC</td>
<td>host</td>
<td>the descriptor of matrix C. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>pBuffer</td>
<td>device</td>
<td>buffer allocated by the user; the size is returned by pruneCsr2csrByPercentage_bufferSizeExt().</td>
</tr>
</tbody>
</table>

Output

<table>
<thead>
<tr>
<th>parameter</th>
<th>device or host</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>nnzTotalDevHostPtr</td>
<td>device or host</td>
<td>total number of nonzero of matrix C. nnzTotalDevHostPtr can point to a device memory or host memory.</td>
</tr>
<tr>
<td>csrValC</td>
<td>device</td>
<td>&lt;type&gt; array of nnzC nonzero elements of matrix C.</td>
</tr>
<tr>
<td>csrRowsPtrC</td>
<td>device</td>
<td>integer array of m+1 elements that contains the start of every row and the end of the last row plus one.</td>
</tr>
<tr>
<td>csrColIndC</td>
<td>device</td>
<td>integer array of nnzC column indices of C.</td>
</tr>
<tr>
<td>pBufferSizeInBytes</td>
<td>host</td>
<td>number of bytes of the buffer.</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status
13.25. cusparse<t>nnz_compress() 

```c

cusparseStatus_t 
cusparseSnnz_compress(cusparseHandle_t handle, 
    int m, 
    const cusparseMatDescr_t descr, 
    const float* csrValA, 
    const int* csrRowPtrA, 
    int* nnzPerRow, 
    int* nnzC, 
    float tol) 

cusparseStatus_t 
cusparseDnnz_compress(cusparseHandle_t handle, 
    int m, 
    const cusparseMatDescr_t descr, 
    const double* csrValA, 
    const int* csrRowPtrA, 
    int* nnzPerRow, 
    int* nnzC, 
    double tol) 

cusparseStatus_t 
cusparseCnnz_compress(cusparseHandle_t handle, 
    int m, 
    const cusparseMatDescr_t descr, 
    const cuComplex* csrValA, 
    const int* csrRowPtrA, 
    int* nnzPerRow, 
    int* nnzC, 
    cuComplex tol) 

cusparseStatus_t 
cusparseZnnz_compress(cusparseHandle_t handle, 
    int m, 
    const cusparseMatDescr_t descr, 
    const cuDoubleComplex* csrValA, 
    const int* csrRowPtrA, 
    int* nnzPerRow, 
    int* nnzC, 
    cuDoubleComplex tol) 
```

This function is the step one to convert from csr format to compressed csr format.

Given a sparse matrix \(A\) and a non-negative value threshold, the function returns \(\text{nnzPerRow}\) (the number of nonzeros columns per row) and \(\text{nnzC}\) (the total number of nonzeros) of a sparse matrix \(C\), defined by

\[
C(i,j) = A(i,j) \quad \text{if } |A(i,j)| > \text{threshold}
\]

A key assumption for the cuComplex and cuDoubleComplex case is that this tolerance is given as the real part. For example tol = 1e-8 + 0*i and we extract cureal, that is the x component of this struct.

- This function requires temporary extra storage that is allocated internally
- The routine does not support asynchronous execution
The routine does **not** support CUDA graph capture

**Input**

<table>
<thead>
<tr>
<th>handle</th>
<th>handle to the cuSPARSE library context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>number of rows of matrix A.</td>
</tr>
<tr>
<td>descrA</td>
<td>the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.</td>
</tr>
<tr>
<td>csrValA</td>
<td>csr noncompressed values array</td>
</tr>
<tr>
<td>csrRowPtrA</td>
<td>the corresponding input noncompressed row pointer.</td>
</tr>
<tr>
<td>tol</td>
<td>non-negative tolerance to determine if a number less than or equal to it.</td>
</tr>
</tbody>
</table>

**Output**

<table>
<thead>
<tr>
<th>nnzPerRow</th>
<th>this array contains the number of elements whose absolute values are greater than tol per row.</th>
</tr>
</thead>
<tbody>
<tr>
<td>nnzC</td>
<td>host/device pointer of the total number of elements whose absolute values are greater than tol.</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status
Chapter 14.
CUSPARSE GENERIC API REFERENCE

The cuSPARSE Generic APIs allow computing the most common sparse linear algebra operations, such as sparse matrix-vector (SpMV) and sparse matrix-matrix multiplication (SpMM), in a flexible way. The new APIs have the following capabilities and features:

- Set matrix data layouts, number of batches, and storage formats (for example, CSR, COO, and so on)
- Set input/output/compute data types. This also allows mixed data-type computation
- Set types of sparse matrix indices
- Choose the algorithm for the computation
- Provide external device memory for internal operations
- Provide extensive consistency checks across input matrices and vectors for a given routine. This includes the validation of matrix sizes, data types, layout, allowed operations, etc.

14.1. Generic Types Reference

The cuSPARSE generic type references are described in this section.

14.1.1. cudaDataType_t

The section describes the types shared by multiple CUDA Libraries and defined in the header file `library_types.h`. The `cudaDataType` type is an enumerator to specify the data precision. It is used when the data reference does not carry the type itself (e.g. `void*`). For example, it is used in the routine `cusparseSpMM()`.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
<th>Data Type</th>
<th>Header</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
<td>The data type is 16-bit IEEE-754 floating-point</td>
<td>__half</td>
<td>cuda_fp16.h</td>
</tr>
<tr>
<td>CUDA_C_16F</td>
<td>The data type is 16-bit complex IEEE-754 floating-point</td>
<td>__half2</td>
<td>cuda_fp16.h</td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
<td>The data type is 16-bit bfloat floating-point</td>
<td>__nv_bfloat16</td>
<td>cuda_bf16.h</td>
</tr>
<tr>
<td>Value</td>
<td>Meaning</td>
<td>Data Type</td>
<td>Header</td>
</tr>
<tr>
<td>-------------</td>
<td>----------------------------------------------</td>
<td>-------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>CUDA_C_16BF</td>
<td>The data type is 16-bit complex bfloat</td>
<td>__nv_bfloat162</td>
<td>cuda_bf16.h</td>
</tr>
<tr>
<td></td>
<td>floating-point</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CUDA_R_32F</td>
<td>The data type is 32-bit IEEE-754 floating-point</td>
<td>float</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_32F</td>
<td>The data type is 32-bit complex IEEE-754</td>
<td>cuComplex</td>
<td>cuComplex.h</td>
</tr>
<tr>
<td></td>
<td>floating-point</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CUDA_R_64F</td>
<td>The data type is 64-bit IEEE-754 floating-point</td>
<td>double</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_64F</td>
<td>The data type is 64-bit complex IEEE-754</td>
<td>cuDoubleComplex</td>
<td>cuComplex.h</td>
</tr>
<tr>
<td></td>
<td>floating-point</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CUDA_R_8I</td>
<td>The data type is 8-bit integer</td>
<td>int8_t</td>
<td>stdint.h</td>
</tr>
<tr>
<td>CUDA_R_32I</td>
<td>The data type is 32-bit integer</td>
<td>int32_t</td>
<td>stdint.h</td>
</tr>
</tbody>
</table>

**IMPORTANT:** The Generic API routines allow all data types reported in the respective section of the documentation only on GPU architectures with *native* support for them. If a specific GPU model does not provide *native* support for a given data type, the routine returns `CUSPARSE_STATUS_ARCH_MISMATCH` error.

Unsupported data types and Compute Capability (CC):
- `__half` on GPUs with `CC < 53` (e.g. Kepler)
- `__nv_bfloat16` on GPUs with `CC < 80` (e.g. Kepler, Maxwell, Pascal, Volta, Turing)

see https://developer.nvidia.com/cuda-gpus

### 14.1.2. cusparseFormat_t

This type indicates the format of the sparse matrix.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_FORMAT_COO</td>
<td>The matrix is stored in Coordinate (COO) format organized in Structure of Arrays (SoA) layout</td>
</tr>
<tr>
<td>CUSPARSE_FORMAT_COO_AOS</td>
<td>The matrix is stored in Coordinate (COO) format organized in Array of Structures (SoA) layout</td>
</tr>
<tr>
<td>CUSPARSE_FORMAT_CSR</td>
<td>The matrix is stored in Compressed Sparse Row (CSR) format</td>
</tr>
</tbody>
</table>

### 14.1.3. cusparseOrder_t

This type indicates the memory layout of a dense matrix. Currently, only column-major layout is supported.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_ORDER_ROW</td>
<td>The matrix is stored in row-major</td>
</tr>
<tr>
<td>CUSPARSE_ORDER_COL</td>
<td>The matrix is stored in column-major</td>
</tr>
</tbody>
</table>
14.1.4. cusparseIndexType_t

This type indicates the index type for representing the sparse matrix indices.

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_INDEX_16U</td>
<td>16-bit unsigned integer [1, 65535]</td>
</tr>
<tr>
<td>CUSPARSE_INDEX_32I</td>
<td>32-bit signed integer [1, 2^31 - 1]</td>
</tr>
<tr>
<td>CUSPARSE_INDEX_64I</td>
<td>64-bit signed integer [1, 2^63 - 1]</td>
</tr>
</tbody>
</table>

14.2. Sparse Vector APIs

The cuSPARSE helper functions for sparse vector descriptor are described in this section.

14.2.1. cusparseCreateSpVec()

```c
 cusparseStatus_t
cusparseCreateSpVec(cusparseSpVecDescr_t* spVecDescr,
                   int64_t               size,
                   int64_t               nnz,
                   void*                 indices,
                   void*                 values,
                   cusparseIndexType_t   idxType,
                   cusparseIndexBase_t   idxBase,
                   cudaDataType          valueType)
```

This function initializes the sparse matrix descriptor `spVecDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spVecDescr</td>
<td>HOST</td>
<td>OUT</td>
<td>Sparse vector descriptor</td>
</tr>
<tr>
<td>size</td>
<td>HOST</td>
<td>IN</td>
<td>Size of the sparse vector</td>
</tr>
<tr>
<td>nnz</td>
<td>HOST</td>
<td>IN</td>
<td>Number of non-zero entries of the sparse vector</td>
</tr>
<tr>
<td>indices</td>
<td>DEVICE</td>
<td>IN</td>
<td>Indices of the sparse vector. Array of size nnz</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>IN</td>
<td>Values of the sparse vector. Array of size nnz</td>
</tr>
<tr>
<td>idxType</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the data type of indices</td>
</tr>
<tr>
<td>idxBase</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the the base index of indices</td>
</tr>
<tr>
<td>valueType</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the datatype of values</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

14.2.2. cusparseDestroySpVec()

```c
 cusparseStatus_t
cusparseDestroySpVec(cusparseSpVecDescr_t spVecDescr)
```
This function releases the host memory allocated for the sparse vector descriptor `spVecDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spVecDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse vector descriptor</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.

### 14.2.3. cusparseSpVecGet()

```c
 cusparseStatus_t
cusparseSpVecGet(const cusparseSpVecDescr_t spVecDescr,
                  int64_t*                size,
                  int64_t*                nnz,
                  void**                 indices,
                  void**                 values,
                  cusparseIndexType_t*   idxType,
                  cusparseIndexBase_t*   idxBase,
                  cudaDataType*          valueType)
```

This function returns the fields of the sparse vector descriptor `spVecDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spVecDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse vector descriptor</td>
</tr>
<tr>
<td>size</td>
<td>HOST</td>
<td>OUT</td>
<td>Size of the sparse vector</td>
</tr>
<tr>
<td>nnz</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of non-zero entries of the sparse vector</td>
</tr>
<tr>
<td>indices</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Indices of the sparse vector. Array of size nnz</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Values of the sparse vector. Array of size nnz</td>
</tr>
<tr>
<td>idxType</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the data type of indices</td>
</tr>
<tr>
<td>idxBase</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the the base index of indices</td>
</tr>
<tr>
<td>valueType</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the datatype of values</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.

### 14.2.4. cusparseSpVecGetIndexBase()

```c
 cusparseStatus_t
cusparseSpVecGetIndexBase(const cusparseSpVecDescr_t spVecDescr,
                           cusparseIndexBase_t*       idxBase)
```

This function returns the `idxBase` field of the sparse vector descriptor `spVecDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spVecDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse vector descriptor</td>
</tr>
<tr>
<td>idxBase</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the the base index of indices</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
14.2.5. cusparseSpVecGetValues()

This function returns the values field of the sparse vector descriptor spVecDescr.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spVecDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse vector descriptor</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Values of the sparse vector. Array of size nnz</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status.

14.2.6. cusparseSpVecSetValues()

This function set the values field of the sparse vector descriptor spVecDescr.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spVecDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse vector descriptor</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>IN</td>
<td>Values of the sparse vector. Array of size nnz</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status.

14.3. Sparse Matrix APIs

The cuSPARSE helper functions for sparse matrix descriptor are described in this section.

14.3.1. cusparseCreateCoo()

This function initializes the sparse matrix descriptor spMatDescr in the COO format (Structure of Arrays layout).
### Param. | Memory | In/out | Meaning
---|---|---|---
spMatDescr | HOST | OUT | Sparse matrix descriptor
rows | HOST | IN | Number of rows of the sparse matrix
cols | HOST | IN | Number of columns of the sparse matrix
nnz | HOST | IN | Number of non-zero entries of the sparse matrix
cooRowInd | DEVICE | IN | Row indices of the sparse matrix. Array of size nnz
cooColInd | DEVICE | IN | Column indices of the sparse matrix. Array of size nnz
cooValues | DEVICE | IN | Values of the sparse matrix. Array of size nnz
cooIdxType | HOST | IN | Enumerator specifying the data type of cooRowInd and cooColInd
idxBase | HOST | IN | Enumerator specifying the base index of cooRowInd and cooColInd
valueType | HOST | IN | Enumerator specifying the datatype of cooValues

See `cusparseStatus_t` for the description of the return status

### 14.3.2. `cusparseCreateCooAoS()`

```c

void* cooiInd,
void* cooValues,
cusparseIndexType_t cooIdxType,
cusparseIndexBase_t idxBase,
cudaDataType valueType)
```

This function initializes the sparse matrix descriptor `spMatDescr` in the COO format (Array of Structures layout).

### Param. | Memory | In/out | Meaning
---|---|---|---
spMatDescr | HOST | OUT | Sparse matrix descriptor
rows | HOST | IN | Number of rows of the sparse matrix
cols | HOST | IN | Number of columns of the sparse matrix
nnz | HOST | IN | Number of non-zero entries of the sparse matrix
cooInd | DEVICE | IN | <Row, Column> indices of the sparse matrix. Array of size nnz
cooValues | DEVICE | IN | Values of the sparse matrix. Array of size nnz
cooIdxType | HOST | IN | Enumerator specifying the data type of cooInd
idxBase | HOST | IN | Enumerator specifying the base index of cooInd
valueType | HOST | IN | Enumerator specifying the datatype of cooValues
This function initializes the sparse matrix descriptor `spMatDescr` in the CSR format.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>spMatDescr</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td><code>rows</code></td>
<td>HOST</td>
<td>IN</td>
<td>Number of rows of the sparse matrix</td>
</tr>
<tr>
<td><code>cols</code></td>
<td>HOST</td>
<td>IN</td>
<td>Number of columns of the sparse matrix</td>
</tr>
<tr>
<td><code>nnz</code></td>
<td>HOST</td>
<td>IN</td>
<td>Number of non-zero entries of the sparse matrix</td>
</tr>
<tr>
<td><code>csrRowOffsets</code></td>
<td>DEVICE</td>
<td>IN</td>
<td>Row offsets of the sparse matrix. Array of size <code>rows + 1</code></td>
</tr>
<tr>
<td><code>csrColInd</code></td>
<td>DEVICE</td>
<td>IN</td>
<td>Column indices of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
<tr>
<td><code>csrValues</code></td>
<td>DEVICE</td>
<td>IN</td>
<td>Values of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
<tr>
<td><code>csrRowOffsetsType</code></td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the data type of <code>csrRowOffsets</code></td>
</tr>
<tr>
<td><code>csrColIndType</code></td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the data type of <code>csrColInd</code></td>
</tr>
<tr>
<td><code>idxBase</code></td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the base index of <code>csrRowOffsets</code> and <code>csrColInd</code></td>
</tr>
<tr>
<td><code>valueType</code></td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the datatype of <code>csrValues</code></td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status
14.3.5. cusparseCooGet()

This function returns the fields of the sparse matrix descriptor `spMatDescr` stored in COO format (Array of Structures layout).

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>spMatDescr</code></td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td><code>rows</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Number of rows of the sparse matrix</td>
</tr>
<tr>
<td><code>cols</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Number of columns of the sparse matrix</td>
</tr>
<tr>
<td><code>nnz</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Number of non-zero entries of the sparse matrix</td>
</tr>
<tr>
<td><code>cooRowInd</code></td>
<td>DEVICE</td>
<td>OUT</td>
<td>Row indices of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
<tr>
<td><code>cooColInd</code></td>
<td>DEVICE</td>
<td>OUT</td>
<td>Column indices of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
<tr>
<td><code>cooValues</code></td>
<td>DEVICE</td>
<td>OUT</td>
<td>Values of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
<tr>
<td><code>cooIdxType</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the data type of <code>cooRowInd</code> and <code>cooColInd</code></td>
</tr>
<tr>
<td><code>idxBase</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the base index of <code>cooRowInd</code> and <code>cooColInd</code></td>
</tr>
<tr>
<td><code>valueType</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the datatype of <code>cooValues</code></td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

14.3.6. cusparseCooAosGet()

This function returns the fields of the sparse matrix descriptor `spMatDescr` stored in COO format (Structure of Arrays layout).
### 14.3.7. cusparseCsrGet()

```c
cusparseStatus_t CUSPARSEAPI
cusparseCsrGet(const cusparseSpMatDescr_t spMatDescr,
                int64_t* rows,                  
                int64_t* cols,                  
                int64_t* nnz,                   
                void** csrRowOffsets,          
                void** csrColInd,              
                void** csrValues,              
                cusparseIndexType_t* csrRowOffsetsType,  
                cusparseIndexType_t* csrColIndType,   
                cusparseIndexBase_t* idxBase,     
                cudaDataType* valueType);
```

This function returns the fields of the sparse matrix descriptor `spMatDescr` stored in CSR format.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td>rows</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of rows of the sparse matrix</td>
</tr>
<tr>
<td>cols</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of columns of the sparse matrix</td>
</tr>
<tr>
<td>nnz</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of non-zero entries of the sparse matrix</td>
</tr>
<tr>
<td>csrRowOffsets</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Row offsets of the sparse matrix. Array of size <code>rows + 1</code></td>
</tr>
<tr>
<td>csrColInd</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Column indices of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
<tr>
<td>csrValues</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Values of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
<tr>
<td>csrRowOffsetsType</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the data type of <code>csrRowOffsets</code></td>
</tr>
<tr>
<td>csrColIndType</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the data type of <code>csrColInd</code></td>
</tr>
<tr>
<td>idxBase</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the base index of <code>csrColInd</code></td>
</tr>
<tr>
<td>valueType</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the datatype of <code>csrValues</code></td>
</tr>
</tbody>
</table>
### 14.3.8. cusparseCsrSetPointers() 

```c
 cusparseStatus_t CUSPARSEAPI
cusparseCsrSetPointers(cusparseSpMatDescr_t spMatDescr,
 void* csrRowOffsets,
 void* csrColInd,
 void* csrValues);
```

This function sets the pointers of the sparse matrix descriptor `spMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>spMatDescr</code></td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td><code>csrRowOffsets</code></td>
<td>DEVICE</td>
<td>IN</td>
<td>Row offsets of the sparse matrix. Array of size <code>rows + 1</code></td>
</tr>
<tr>
<td><code>csrColInd</code></td>
<td>DEVICE</td>
<td>IN</td>
<td>Column indices of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
<tr>
<td><code>csrValues</code></td>
<td>DEVICE</td>
<td>IN</td>
<td>Values of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.

### 14.3.9. cusparseSpMatGetSize() 

```c
 cusparseStatus_t CUSPARSEAPI
cusparseSpMatGetSize(cusparseSpMatDescr_t spMatDescr,
 int64_t* rows,
 int64_t* cols,
 int64_t* nnz);
```

This function returns the sizes of the sparse matrix `spMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>spMatDescr</code></td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td><code>rows</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Number of rows of the sparse matrix</td>
</tr>
<tr>
<td><code>cols</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Number of columns of the sparse matrix</td>
</tr>
<tr>
<td><code>nnz</code></td>
<td>HOST</td>
<td>OUT</td>
<td>Number of non-zero entries of the sparse matrix</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.

### 14.3.10. cusparseSpMatGetFormat() 

```c
 cusparseStatus_t
cusparseSpMatGetFormat(const cusparseSpMatDescr_t spMatDescr,
 cusparseFormat_t* format);
```

This function returns the `format` field of the sparse matrix descriptor `spMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
</table>
### Param. | Memory | In/out | Meaning
--- | --- | --- | ---
spMatDescr | HOST | IN | Sparse matrix descriptor
format | HOST | OUT | Enumerator specifying the storage format of the sparse matrix

See [cusparseStatus_t](#) for the description of the return status

### 14.3.11. cusparseSpMatGetIndexBase()

```c
cusparseStatus_t
cusparseSpMatGetIndexBase(const cusparseSpMatDescr_t spMatDescr,
cusparseIndexBase_t* idxBase)
```

This function returns the `idxBase` field of the sparse matrix descriptor `spMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td>idxBase</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the base index of the sparse matrix</td>
</tr>
</tbody>
</table>

See [cusparseStatus_t](#) for the description of the return status

### 14.3.12. cusparseSpMatGetValues()

```c
cusparseStatus_t CUSPARSEAPI
cusparseSpMatGetValues(cusparseSpMatDescr_t spMatDescr,
void** values)
```

This function returns the `values` field of the sparse matrix descriptor `spMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Values of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
</tbody>
</table>

See [cusparseStatus_t](#) for the description of the return status

### 14.3.13. cusparseSpMatSetValues()

```c
cusparseStatus_t CUSPARSEAPI
cusparseSpMatSetValues(cusparseSpMatDescr_t spMatDescr,
void* values)
```

This function sets the `values` field of the sparse matrix descriptor `spMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>IN</td>
<td>Values of the sparse matrix. Array of size <code>nnz</code></td>
</tr>
</tbody>
</table>

See [cusparseStatus_t](#) for the description of the return status
14.3.14. cusparseSpMatGetStridedBatch()

cusparseStatus_t cusparseSpMatGetStridedBatch(const cusparseSpMatDescr_t spMatDescr, int* batchCount)

This function returns the batchCount field of the sparse matrix descriptor spMatDescr.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td>batchCount</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of batches of the sparse matrix</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status.

14.3.15. cusparseSpMatSetStridedBatch() [DEPRECATED]

[[DEPRECATED]] use cusparseSpMatSetCsrStridedBatch(), cusparseSpMatSetCooStridedBatch() instead. The routine will be removed in the next major release.

cusparseStatus_t cusparseSpMatSetStridedBatch(cusparseSpMatDescr_t spMatDescr, int batchCount)

This function sets the batchCount field of the sparse matrix descriptor spMatDescr.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td>batchCount</td>
<td>HOST</td>
<td>IN</td>
<td>Number of batches of the sparse matrix</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status.

14.3.16. cusparseCooSetStridedBatch()

cusparseStatus_t cusparseCooSetStridedBatch(cusparseSpMatDescr_t spMatDescr, int batchCount, int64_t batchStride)

This function sets the batchCount and the batchStride fields of the sparse matrix descriptor spMatDescr.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td>batchCount</td>
<td>HOST</td>
<td>IN</td>
<td>Number of batches of the sparse matrix</td>
</tr>
<tr>
<td>batchStride</td>
<td>HOST</td>
<td>IN</td>
<td>address offset between consecutive batches</td>
</tr>
</tbody>
</table>

See cusparseStatus_t for the description of the return status.
14.3.17. cusparseCsrSetStridedBatch()

```
cusparseStatus_t
cusparseCsrSetStridedBatch(cusparseSpMatDescr_t spMatDescr,  
    int                 batchCount,                  
    int64_t             offsetsBatchStride,        
    int64_t             columnsValuesBatchStride)```

This function sets the `batchCount` and the `batchStride` fields of the sparse matrix descriptor `spMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>spMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix descriptor</td>
</tr>
<tr>
<td>batchCount</td>
<td>HOST</td>
<td>IN</td>
<td>Number of batches of the sparse matrix</td>
</tr>
<tr>
<td>offsetsBatchStride</td>
<td>HOST</td>
<td>IN</td>
<td>Address offset between consecutive batches for the row offset array</td>
</tr>
<tr>
<td>offsetsColumnsValuesBatchStride</td>
<td>HOST</td>
<td>IN</td>
<td>Address offset between consecutive batches for the column and value arrays</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

14.4. Dense Vector APIs

The cuSPARSE helper functions for dense vector descriptor are described in this section.

14.4.1. cusparseCreateDnVec()

```
cusparseStatus_t
cusparseCreateDnVec(cusparseDnVecDescr_t* dnVecDescr,   
    int64_t               size,                      
    void*                 values,                     
    cudaDataType          valueType)```

This function initializes the dense vector descriptor `dnVecDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnVecDescr</td>
<td>HOST</td>
<td>OUT</td>
<td>Dense vector descriptor</td>
</tr>
<tr>
<td>size</td>
<td>HOST</td>
<td>IN</td>
<td>Size of the dense vector</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>IN</td>
<td>Values of the dense vector. Array of size <code>size</code></td>
</tr>
<tr>
<td>valueType</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the datatype of <code>values</code></td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status
14.4.2. cusparseDestroyDnVec()

```c
 cusparseStatus_t
cusparseDestroyDnVec(cusparseDnVecDescr_t dnVecDescr)
```

This function releases the host memory allocated for the dense vector descriptor `dnVecDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnVecDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense vector descriptor</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.

14.4.3. cusparseDnVecGet()

```c
 cusparseStatus_t
 cusparseDnVecGet(const cusparseDnVecDescr_t dnVecDescr,
                  int64_t* size,
                  void** values,
                  cudaDataType* valueType)
```

This function returns the fields of the dense vector descriptor `dnVecDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnVecDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense vector descriptor</td>
</tr>
<tr>
<td>size</td>
<td>HOST</td>
<td>OUT</td>
<td>Size of the dense vector</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Values of the dense vector. Array of size <code>nnz</code></td>
</tr>
<tr>
<td>valueType</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the datatype of <code>values</code></td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.

14.4.4. cusparseDnVecGetValues()

```c
 cusparseStatus_t
 cusparseDnVecGetValues(const cusparseDnVecDescr_t dnVecDescr,
                        void** values)
```

This function returns the `values` field of the dense vector descriptor `dnVecDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnVecDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense vector descriptor</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Values of the dense vector</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status.
14.4.5. cusparseDnVecSetValues()

```c
 cusparseStatus_t
cusparseDnVecSetValues(cusparseDnVecDescr_t dnVecDescr,
                        void* values)
```

This function sets the `values` field of the dense vector descriptor `dnVecDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnVecDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense vector descriptor</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>IN</td>
<td>Values of the dense vector. Array of size <code>size</code></td>
</tr>
</tbody>
</table>

The possible error values returned by this function and their meanings are listed below:

See `cusparseStatus_t` for the description of the return status

14.5. Dense Matrix APIs

The cuSPARSE helper functions for dense matrix descriptor are described in this section.

14.5.1. cusparseCreateDnMat()

```c
 cusparseStatus_t
cusparseCreateDnMat(cusparseDnMatDescr_t* dnMatDescr,
                     int64_t rows,
                     int64_t cols,
                     int64_t ld,
                     void* values,
                     cudaDataType valueType,
                     cusparseOrder_t order)
```

The function initializes the dense matrix descriptor `dnMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnMatDescr</td>
<td>HOST</td>
<td>OUT</td>
<td>Dense matrix descriptor</td>
</tr>
<tr>
<td>rows</td>
<td>HOST</td>
<td>IN</td>
<td>Number of rows of the dense matrix</td>
</tr>
<tr>
<td>cols</td>
<td>HOST</td>
<td>IN</td>
<td>Number of columns of the dense matrix</td>
</tr>
<tr>
<td>ld</td>
<td>HOST</td>
<td>IN</td>
<td>Leading dimension of the dense matrix</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>IN</td>
<td>Values of the dense matrix. Array of size <code>size</code></td>
</tr>
<tr>
<td>valueType</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the datatype of <code>values</code></td>
</tr>
<tr>
<td>order</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the memory layout of the dense matrix</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status
14.5.2. cusparseDestroyDnMat()

cusparseStatus_t
 cusparseDestroyDnMat(cusparseDnMatDescr_t dnMatDescr)

This function releases the host memory allocated for the dense matrix descriptor `dnMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense matrix descriptor</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

14.5.3. cusparseDnMatGet()

cusparseStatus_t
 cusparseDnMatGet(const cusparseDnMatDescr_t dnMatDescr,
                  int64_t*                   rows,
                  int64_t*                   cols,
                  int64_t*                   ld,
                  void**                     values,
                  cudaDataType*              valueType,
                  cusparseOrder_t*           order)

This function returns the fields of the dense matrix descriptor `dnMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense matrix descriptor</td>
</tr>
<tr>
<td>rows</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of rows of the dense matrix</td>
</tr>
<tr>
<td>cols</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of columns of the dense matrix</td>
</tr>
<tr>
<td>ld</td>
<td>HOST</td>
<td>OUT</td>
<td>Leading dimension of the dense matrix</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Values of the dense matrix. Array of size ld * cols</td>
</tr>
<tr>
<td>valueType</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the datatype of values</td>
</tr>
<tr>
<td>order</td>
<td>HOST</td>
<td>OUT</td>
<td>Enumerator specifying the memory layout of the dense matrix</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

14.5.4. cusparseDnMatGetValues()

cusparseStatus_t CUSPARSEAPI
cusparseDnMatGetValues(const cusparseDnMatDescr_t dnMatDescr,
                        void**                     values)

This function returns the `values` field of the dense matrix descriptor `dnMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense matrix descriptor</td>
</tr>
<tr>
<td>Param.</td>
<td>Memory</td>
<td>In/out</td>
<td>Meaning</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Values of the dense matrix. Array of size ( ld \times cols )</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

### 14.5.5. `cusparseDnSetValues()`

```c
cusparseStatus_t CUSPARSEAPI cusparseDnMatSetValues(cusparseDnMatDescr_t dnMatDescr, void* values)
```

This function sets the `values` field of the dense matrix descriptor `dnMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense matrix descriptor</td>
</tr>
<tr>
<td>values</td>
<td>DEVICE</td>
<td>IN</td>
<td>Values of the dense matrix. Array of size ( ld \times cols )</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

### 14.5.6. `cusparseDnMatGetStridedBatch()`

```c
cusparseStatus_t cusparseDnMatGetStridedBatch(const cusparseDnMatDescr_t dnMatDescr, int* batchCount, int64_t* batchStride)
```

The function returns the number of batches and the batch stride of the dense matrix descriptor `dnMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense matrix descriptor</td>
</tr>
<tr>
<td>batchCount</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of batches of the dense matrix</td>
</tr>
<tr>
<td>batchStride</td>
<td>HOST</td>
<td>OUT</td>
<td>Address offset between a matrix and the next one in the batch</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

### 14.5.7. `cusparseDnMatSetStridedBatch()`

```c
cusparseStatus_t cusparseDnMatSetStridedBatch(cusparseDnMatDescr_t dnMatDescr, int batchCount, int64_t batchStride)
```

The function sets the number of batches and the batch stride of the dense matrix descriptor `dnMatDescr`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>dnMatDescr</td>
<td>HOST</td>
<td>IN</td>
<td>Dense matrix descriptor</td>
</tr>
<tr>
<td>Param.</td>
<td>Memory</td>
<td>In/out</td>
<td>Meaning</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>--------</td>
<td>-------------------------------------------------------------------------</td>
</tr>
<tr>
<td>batchCount</td>
<td>HOST</td>
<td>IN</td>
<td>Number of batches of the dense matrix</td>
</tr>
<tr>
<td>batchStride</td>
<td>HOST</td>
<td>IN</td>
<td>Address offset between a matrix and the next one in the batch. batchStride ≥ ld * cols if the matrix uses column-major layout, batchStride ≥ ld * rows otherwise</td>
</tr>
</tbody>
</table>

See `cusparseStatus_t` for the description of the return status

## 14.6. Generic API Functions

### 14.6.1. `cusparseAxpby()`

```c
cusparseStatus_t
cusparseAxpby(CusparseHandle_t handle,
              const void* alpha,
              cusparseSpVecDescr_t vecX,
              const void* beta,
              cusparseDnVecDescr_t vecY)
```

The function computes the sum of a sparse vector `vecX` and a dense vector `vecY`

\[ Y = \alpha X + \beta Y \]

In other words,

\[
\text{for } i = 0 \text{ to } \text{nnz-1} \\
\quad Y[\text{X_indices}[i]] = \text{alpha} \times \text{X_values}[i] + \beta \times Y[\text{X_indices}[i]]
\]

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>HOST</td>
<td>IN</td>
<td>Handle to the cuSPARSE library context</td>
</tr>
<tr>
<td>alpha</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>(\alpha) scalar used for multiplication</td>
</tr>
<tr>
<td>vecX</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse vector <code>X</code></td>
</tr>
<tr>
<td>beta</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>(\beta) scalar used for multiplication</td>
</tr>
<tr>
<td>vecY</td>
<td>HOST</td>
<td>IN/OUT</td>
<td>Dense vector <code>Y</code></td>
</tr>
</tbody>
</table>

`cusparseAxpby` supports the following index type for representing the sparse vector `vecX`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseAxpby` supports the following datatypes:

<table>
<thead>
<tr>
<th>X/Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
</tr>
</tbody>
</table>
cusparseAxpy() has the following constraints:
- The arrays representing the sparse vector vecX must be aligned to 16 bytes

cusparseAxpy() has the following properties:
- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the the sparse vector vecX indices are distinct

cusparseAxpy() supports the following optimizations:
- CUDA graph capture
- Hardware Memory Compression

See cusparseStatus_t for the description of the return status

Please visit cuSPARSE Library Samples - cusparseAxpy for a code example.

14.6.2. cusparseGather()

cusparseStatus_t
    cusparseGather(cusparseHandle_t handle, 
                   cusparseDnVecDescr_t vecY, 
                   cusparseSpVecDescr_t vecX)

The function gathers the elements of the dense vector vecY into the sparse vector vecX
In other words,

```plaintext
for i=0 to nnz-1
    X_values[i] = Y[X_indices[i]]
```

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>HOST</td>
<td>IN</td>
<td>Handle to the cuSPARSE library context</td>
</tr>
<tr>
<td>vecX</td>
<td>HOST</td>
<td>OUT</td>
<td>Sparse vector X</td>
</tr>
<tr>
<td>vecY</td>
<td>HOST</td>
<td>IN</td>
<td>Dense vector Y</td>
</tr>
</tbody>
</table>
**cusparseGather** supports the following index type for representing the sparse vector `vecX`:

- 32-bit indices (**CUSPARSE_INDEX_32I**)
- 64-bit indices (**CUSPARSE_INDEX_64I**)

**cusparseGather** supports the following datatypes:

<table>
<thead>
<tr>
<th>X/Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
</tr>
<tr>
<td>CUDA_R_32F</td>
</tr>
<tr>
<td>CUDA_R_64F</td>
</tr>
<tr>
<td>CUDA_C_16F</td>
</tr>
<tr>
<td>CUDA_C_16BF</td>
</tr>
<tr>
<td>CUDA_C_32F</td>
</tr>
<tr>
<td>CUDA_C_64F</td>
</tr>
</tbody>
</table>

**cusparseGather** has the following constraints:

- The arrays representing the sparse vector `vecX` must be aligned to 16 bytes

**cusparseGather** has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the sparse vector `vecX` indices are distinct

**cusparseGather** supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See **cusparseStatus_t** for the description of the return status

Please visit cuSPARSE Library Samples - cusparseGather for a code example.

### 14.6.3. **cusparseScatter()**

```c
cusparseStatus_t
 cusparseScatter(cusparseHandle_t handle,
                 cusparseSpVecDescr_t vecX,
                 cusparseDnVecDescr_t vecY)
```

The function scatters the elements of the sparse vector `vecX` into the dense vector `vecY`. 
In other words,

\[ \text{for } i=0 \text{ to } \text{nnz-1} \]
\[ Y[X\text{\_indices}[i]] = X\text{\_values}[i] \]

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>HOST</td>
<td>IN</td>
<td>Handle to the cuSPARSE library context</td>
</tr>
<tr>
<td>vecX</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse vector ( X )</td>
</tr>
<tr>
<td>vecY</td>
<td>HOST</td>
<td>OUT</td>
<td>Dense vector ( Y )</td>
</tr>
</tbody>
</table>

cusparseScatter supports the following index type for representing the sparse vector \( vecX \):

- 32-bit indices (CUSPARSE_INDEX_32I)
- 64-bit indices (CUSPARSE_INDEX_64I)

cusparseScatter supports the following datatypes:

<table>
<thead>
<tr>
<th>( X/Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
</tr>
<tr>
<td>CUDA_R_32F</td>
</tr>
<tr>
<td>CUDA_R_64F</td>
</tr>
<tr>
<td>CUDA_C_16F</td>
</tr>
<tr>
<td>CUDA_C_16BF</td>
</tr>
<tr>
<td>CUDA_C_32F</td>
</tr>
<tr>
<td>CUDA_C_64F</td>
</tr>
</tbody>
</table>

cusparseScatter() has the following constraints:

- The arrays representing the sparse vector \( vecX \) must be aligned to 16 bytes

cusparseScatter() has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the sparse vector \( vecX \) indices are distinct

cusparseScatter() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See cuSpaReSta tus_t for the description of the return status

Please visit cuSPARSE Library Samples - cusparseScatter for a code example.
14.6.4. cusparseRot()

cusparseStatus_t
cusparseRot(cusparseHandle_t handle,
const void* c_coeff,
const void* s_coeff,
cusparseSpVecDescr_t vecX,
cusparseDnVecDescr_t vecY)

The function computes the Givens rotation matrix

\[
G = \begin{bmatrix}
c & s \\
-s & c
\end{bmatrix}
\]

to a sparse \textbf{vecX} and a dense vector \textbf{vecY}

In other words,

\[
\text{for } i=0 \text{ to } \text{nnz}-1 \\
\quad \text{vecY}[\text{X_indices}[i]] = c \times \text{vecY}[\text{X_indices}[i]] - s \times \text{vecX}[i] \\
\quad \text{vecX}[i] = c \times \text{vecX}[i] + s \times \text{vecY}[\text{X_indices}[i]]
\]

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>HOST</td>
<td>IN</td>
<td>Handle to the cuSPARSE library context</td>
</tr>
<tr>
<td>c_coeff</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>cosine element of the rotation matrix</td>
</tr>
<tr>
<td>vecX</td>
<td>HOST</td>
<td>IN/OUT</td>
<td>Sparse vector \textbf{X}</td>
</tr>
<tr>
<td>s_coeff</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>sine element of the rotation matrix</td>
</tr>
<tr>
<td>vecY</td>
<td>HOST</td>
<td>IN/OUT</td>
<td>Dense vector \textbf{Y}</td>
</tr>
</tbody>
</table>

cusparseRot supports the following index type for representing the sparse vector vecX:

- 32-bit indices (CUSPARSE_INDEX_32I)
- 64-bit indices (CUSPARSE_INDEX_64I)

cusparseRot supports the following datatypes:

<table>
<thead>
<tr>
<th>X/Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
</tr>
<tr>
<td>CUDA_R_32F</td>
</tr>
<tr>
<td>CUDA_R_64F</td>
</tr>
<tr>
<td>CUDA_C_16F</td>
</tr>
<tr>
<td>CUDA_C_16BF</td>
</tr>
<tr>
<td>CUDA_C_32F</td>
</tr>
<tr>
<td>CUDA_C_64F</td>
</tr>
</tbody>
</table>
cusparseRot() has the following constraints:

- The arrays representing the sparse vector vecX must be aligned to 16 bytes

cusparseRot() has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the the sparse vector vecX indices are distinct

cusparseRot() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See cusparseStatus_t for the description of the return status

Please visit cuSPARSE Library Samples - cusparseRot for a code example.

### 14.6.5. cusparseSpVV()

```c
cusparseStatus_t
cusparseSpVV_bufferSize(cusparseHandle_t handle,
cusparseOperation_t opX,
cusparseSpVecDescr_t vecX,
cusparseDnVecDescr_t vecY,
void* result,
cudaDataType computeType,
size_t* bufferSize)
```

```c
cusparseStatus_t
cusparseSpVV(cusparseHandle_t handle,
cusparseOperation_t opX,
cusparseSpVecDescr_t vecX,
cusparseDnVecDescr_t vecY,
void* result,
cudaDataType computeType,
void* externalBuffer)
```

The function computes the inner dot product of a sparse vector vecX and a dense vector vecY

\[ \text{result} = X \cdot Y \]

In other words,

```c
cresult = 0;
for i=0 to nnz-1
    result += X_values[i] * Y[X_indices[i]]
```

\[ \text{op}(X) = \begin{cases} X & \text{if op}(X) == \text{CUSPARSEOPERATION\_NON\_TRANSPOSE} \\
X & \text{if op}(X) == \text{CUSPARSEOPERATION\_CONJUGATE\_TRANSPOSE} \end{cases} \]
The function `cusparseSpVV_bufferSize()` returns the size of the workspace needed by `cusparseSpVV()`.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>HOST</td>
<td>IN</td>
<td>Handle to the cuSPARSE library context</td>
</tr>
<tr>
<td>opX</td>
<td>HOST</td>
<td>IN</td>
<td>Operation op(X) that is non-transpose or conjugate transpose</td>
</tr>
<tr>
<td>vecX</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse vector X</td>
</tr>
<tr>
<td>vecY</td>
<td>HOST</td>
<td>IN</td>
<td>Dense vector Y</td>
</tr>
<tr>
<td>result</td>
<td>HOST or DEVICE</td>
<td>OUT</td>
<td>The resulting dot product</td>
</tr>
<tr>
<td>computeType</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the datatype in which the computation is executed</td>
</tr>
<tr>
<td>bufferSize</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of bytes of workspace needed by <code>cusparseSpVV</code></td>
</tr>
<tr>
<td>externalBuf</td>
<td>DEVICE</td>
<td>IN</td>
<td>Pointer to workspace buffer</td>
</tr>
</tbody>
</table>

`cusparseSpVV` supports the following index type for representing the sparse vector `vecX`:

- 32-bit indices (`CUSPARSE_INDEX_32I`)
- 64-bit indices (`CUSPARSE_INDEX_64I`)

The datatypes combinations currently supported for `cusparseSpVV` are listed below:

### Uniform-precision computation:

<table>
<thead>
<tr>
<th>X/Y</th>
<th>computeType</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_32F</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_64F</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_16F</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_16BF</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_32F</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_64F</td>
<td></td>
</tr>
</tbody>
</table>

### Mixed-precision computation:

<table>
<thead>
<tr>
<th>X/Y</th>
<th>computeType/result</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_8I</td>
<td>CUDA_R_32I</td>
</tr>
<tr>
<td>CUDA_R_8I</td>
<td>CUDA_R_32F</td>
</tr>
<tr>
<td>CUDA_R_16F</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
<td></td>
</tr>
</tbody>
</table>

`cusparseSpVV()` has the following constraints:
The arrays representing the sparse vector \( \text{vecX} \) must be aligned to 16 bytes.

\text{cusparseSpVV()} has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run if the sparse vector \( \text{vecX} \) indices are distinct

\text{cusparseSpVV()} supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See \text{cusparseStatus_t} for the description of the return status.

Please visit cuSPARSE Library Samples - \text{cusparseSpVV} for a code example.

### 14.6.6. \text{cusparseSpMV()}

```c
cusparseStatus_t cusparseSpMV_bufferSize(cusparseHandle_t handle,
                                        cusparseOperation_t opA,
                                        const void* alpha,
                                        const cusparseSpMatDescr_t matA,
                                        const cusparseDnVecDescr_t vecX,
                                        const void* beta,
                                        const cusparseDnVecDescr_t vecY,
                                        cudaDataType computeType,
                                        cusparseSpMVAlg_t alg,
                                        size_t* bufferSize)
```

```c
cusparseStatus_t cusparseSpMV(cusparseHandle_t handle,
                             cusparseOperation_t opA,
                             const void* alpha,
                             const cusparseSpMatDescr_t matA,
                             const cusparseDnVecDescr_t vecX,
                             const void* beta,
                             const cusparseDnVecDescr_t vecY,
                             cudaDataType computeType,
                             cusparseSpMVAlg_t alg,
                             void* externalBuffer)
```

This function performs the multiplication of a sparse matrix \( \text{matA} \) and a dense vector \( \text{vecX} \)

\[ Y = \alpha \text{op}(A) \cdot X + \beta Y \]

where

- \( \text{op}(A) \) is a sparse matrix of size \( m \times k \)
- \( X \) is a dense vector of size \( k \)
- \( Y \) is a dense vector of size \( m \)
- \( \alpha \) and \( \beta \) are scalars

Also, for matrix \( A \)

\[
\begin{align*}
\text{op}(A) &= \begin{cases} 
A & \text{if } \text{op}(A) = \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\
A^T & \text{if } \text{op}(A) = \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\
A^H & \text{if } \text{op}(A) = \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE}
\end{cases}
\end{align*}
\]

When using the (conjugate) transpose of the sparse matrix \( A \), this routine may produce slightly different results during different runs with the same input parameters.

The function \texttt{cusparseSpMV\_bufferSize()} returns the size of the workspace needed by \texttt{cusparseSpMV()}

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>HOST</td>
<td>IN</td>
<td>Handle to the cuSPARSE library context</td>
</tr>
<tr>
<td>\texttt{opA}</td>
<td>HOST</td>
<td>IN</td>
<td>Operation ( \text{op}(A) )</td>
</tr>
<tr>
<td>\texttt{alpha}</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>( \alpha ) scalar used for multiplication</td>
</tr>
<tr>
<td>\texttt{matA}</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix ( A )</td>
</tr>
<tr>
<td>\texttt{vecX}</td>
<td>HOST</td>
<td>IN</td>
<td>Dense vector ( X )</td>
</tr>
<tr>
<td>\texttt{beta}</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>( \beta ) scalar used for multiplication</td>
</tr>
<tr>
<td>\texttt{vecY}</td>
<td>HOST</td>
<td>IN/OUT</td>
<td>Dense vector ( Y )</td>
</tr>
<tr>
<td>computeType</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the datatype in which the computation is executed</td>
</tr>
<tr>
<td>\texttt{alg}</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the algorithm for the computation</td>
</tr>
<tr>
<td>\texttt{bufferSize}</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of bytes of workspace needed by \texttt{cusparseSpMV}</td>
</tr>
<tr>
<td>externalBu</td>
<td>DEVICE</td>
<td>OUT</td>
<td>Pointer to workspace buffer</td>
</tr>
</tbody>
</table>

\texttt{cusparseSpMV} supports the following index type for representing the sparse vector \texttt{vecX}:

- 32-bit indices (\texttt{CUSPARSE\_INDEX\_32I})
- 64-bit indices (\texttt{CUSPARSE\_INDEX\_64I})

\texttt{cusparseSpMV} supports the following datatypes:

Uniform-precision computation:

<table>
<thead>
<tr>
<th>( A/X/Y/\text{computeType} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
</tr>
<tr>
<td>CUDA_R_32F</td>
</tr>
<tr>
<td>CUDA_R_64F</td>
</tr>
<tr>
<td>CUDA_C_16F</td>
</tr>
<tr>
<td>CUDA_C_32F</td>
</tr>
<tr>
<td>CUDA_C_64F</td>
</tr>
</tbody>
</table>
Mixed-precision computation:

<table>
<thead>
<tr>
<th>A/X</th>
<th>Y</th>
<th>computeType</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_8I</td>
<td>CUDA_R_32I</td>
<td>CUDA_R_32I</td>
</tr>
<tr>
<td>CUDA_R_8I</td>
<td>CUDA_R_32F</td>
<td>CUDA_R_32F</td>
</tr>
<tr>
<td>CUDA_R_16F</td>
<td>CUDA_R_32F</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_16F</td>
<td>CUDA_R_16F</td>
<td></td>
</tr>
</tbody>
</table>

The sparse matrix formats currently supported are listed below:

<table>
<thead>
<tr>
<th>Format</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_FORMAT_COO</td>
<td>May produce slightly different results during different runs with the same input parameters</td>
</tr>
<tr>
<td>CUSPARSE_FORMAT_COO_AOS</td>
<td>May produce slightly different results during different runs with the same input parameters</td>
</tr>
<tr>
<td>CUSPARSE_FORMAT_CSR</td>
<td>Provides deterministic (bit-wise) results for each run</td>
</tr>
</tbody>
</table>

`cusparseSpMV` supports the following algorithms:

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_MV_ALG_DEFAULT</td>
<td>Default algorithm for any sparse matrix format</td>
</tr>
<tr>
<td>CUSPARSE_COOMV_ALG</td>
<td>Default algorithm for COO sparse matrix format</td>
</tr>
<tr>
<td>CUSPARSE_CSRMV_ALG1</td>
<td>Default algorithm for CSR sparse matrix format</td>
</tr>
<tr>
<td>CUSPARSE_CSRMV_ALG2</td>
<td>Algorithm 2 for CSR sparse matrix format. May provide better performance for irregular matrices</td>
</tr>
</tbody>
</table>

The function has the following limitations:

- Half-precision is not supported with 64-bit indices (`CUSPARSE_INDEX_64I`)

`cusparseSpMV()` has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution

`cusparseSpMV()` supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See `cusparseStatus_t` for the description of the return status.

Please visit cuSPARSE Library Samples - cusparseSpMV CSR and cusparseSpMV COO for a code example.
14.6.7. cuSparseSpMM()

```c
 cuSparseStatus_t
cuSparseSpMM_bufferSize(cusparseHandle_t     handle,
cusparseOperation_t  opA,
cusparseOperation_t  opB,
const void*          alpha,
cusparseSpMatDescr_t matA,
cusparseDnMatDescr_t matB,
const void*          beta,
cusparseDnMatDescr_t matC,
cudaDataType         computeType,
cusparseSpMMAlg_t    alg,
size_t*              bufferSize)
```

```c
 cuSparseStatus_t

```
cuSparseSpMM(cusparseHandle_t     handle,
cusparseOperation_t  opA,
cusparseOperation_t  opB,
const void*          alpha,
cusparseSpMatDescr_t matA,
cusparseDnMatDescr_t matB,
const void*          beta,
cusparseDnMatDescr_t matC,
cudaDataType         computeType,
cusparseSpMMAlg_t    alg,
void*                externalBuffer)
```

The function performs the multiplication of a sparse matrix $\text{matA}$ and a dense matrix $\text{matB}$

$$ C = \alpha \text{op}(A) \cdot \text{op}(B) + \beta C $$

where

- $\text{op}(A)$ is a sparse matrix of size $m \times k$
- $\text{op}(B)$ is a dense matrix of size $k \times n$
- $C$ is a dense matrix of size $m \times n$
- $\alpha$ and $\beta$ are scalars

The routine can be also used to perform the multiplication of a dense matrix $\text{matB}$ and a sparse matrix $\text{matA}$ by switching the dense matrices layout:

$$ C_C = B_C \cdot A + \beta C_C $$

$$ C_R = A^T \cdot B_R + \beta C_R $$

where $B_C$, $C_C$ indicate column-major layout, while $B_R$, $C_R$ refer to row-major layout.

Also, for matrix $A$ and $B$

$$ \text{op}(A) = \begin{cases} A & \text{if \text{op}(A) == CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\
A^T & \text{if \text{op}(A) == CUSPARSE\_OPERATION\_TRANSPOSE} \\
A^H & \text{if \text{op}(A) == CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases} $$
When using the (conjugate) transpose of the sparse matrix $A$, this routine may produce slightly different results during different runs with the same input parameters.

The function `cusparseSpMM_bufferSize()` returns the size of the workspace needed by `cusparseSpMM()`

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>HOST</td>
<td>IN</td>
<td>Handle to the cuSPARSE library context</td>
</tr>
<tr>
<td>opA</td>
<td>HOST</td>
<td>IN</td>
<td>Operation $\text{op}(A)$</td>
</tr>
<tr>
<td>alpha</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>$\alpha$ scalar used for multiplication</td>
</tr>
<tr>
<td>matA</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix $A$</td>
</tr>
<tr>
<td>matB</td>
<td>HOST</td>
<td>IN</td>
<td>Dense matrix $B$</td>
</tr>
<tr>
<td>beta</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>$\beta$ scalar used for multiplication</td>
</tr>
<tr>
<td>matC</td>
<td>HOST</td>
<td>IN/OUT</td>
<td>Dense matrix $C$</td>
</tr>
<tr>
<td>computeType</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the datatype in which the computation is executed</td>
</tr>
<tr>
<td>alg</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the algorithm for the computation</td>
</tr>
<tr>
<td>bufferSize</td>
<td>HOST</td>
<td>OUT</td>
<td>Number of bytes of workspace needed by <code>cusparseSpMM</code></td>
</tr>
<tr>
<td>externalBuffer</td>
<td>DEVICE</td>
<td>IN</td>
<td>Pointer to workspace buffer</td>
</tr>
</tbody>
</table>

`cusparseSpMM` supports the following sparse matrix formats:

- CUSPARSE_FORMAT_COO
- CUSPARSE_FORMAT_CSR

`cusparseSpMM` supports the following index type for representing the sparse vector $\text{vecX}$:

- 32-bit indices (CUSPARSE_INDEX_32I)
- 64-bit indices (CUSPARSE_INDEX_64I) only with CUSPARSE_SPMM_COO_ALG4 and CUSPARSE_SPMM_CSR_ALG2 algorithms

`cusparseSpMM` supports the following datatypes:

Uniform-precision computation:

<table>
<thead>
<tr>
<th>A/B/C/computeType</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_32F</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_64F</td>
<td></td>
</tr>
</tbody>
</table>
Mixed-precision computation:

<table>
<thead>
<tr>
<th>A/B</th>
<th>C</th>
<th>computeType</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_8I</td>
<td>CUDA_R_32I</td>
<td>CUDA_R_32I</td>
</tr>
<tr>
<td>CUDA_R_8I</td>
<td>CUDA_R_32I</td>
<td>CUDA_R_32F</td>
</tr>
<tr>
<td>CUDA_R_16F</td>
<td>CUDA_R_32F</td>
<td>CUDA_R_32F</td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
<td>CUDA_R_16F</td>
<td>CUDA_R_16BF</td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
<td>CUDA_R_16BF</td>
<td>CUDA_R_16BF</td>
</tr>
</tbody>
</table>

NOTE: CUDA_R_16BF/CUDA_C_16BF data types are supported only with CUSPARSE_SPMM_COO_ALG4 and CUSPARSE_SPMM_CSR_ALG2 algorithms.

cusparseSpMM supports the following algorithms:

[D]: deprecated

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_MM_ALG_DEFAULT</td>
<td>Default algorithm for any sparse matrix format</td>
</tr>
</tbody>
</table>
| CUSPARSE_COOMM_ALG1 | Algorithm 1 for COO sparse matrix format
  - May provide better performance for small number of nnz
  - It supports only column-major layout
  - It supports batched computation
  - May produce slightly different results during different runs with the same input parameters |
| CUSPARSE_COOMM_ALG2 | Algorithm 2 for COO sparse matrix format
  - In general, slower than Algorithm 1 and 2
  - It supports only column-major layout
  - It supports batched computation
  - It provides deterministic result
  - It requires additional memory |
| CUSPARSE_COOMM_ALG3 | Algorithm 3 for COO sparse matrix format
  - May provide better performance for large number of nnz
  - It supports only column-major layout |
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_COO_MM_ALG4</td>
<td>May produce slightly different results during different runs with the same input parameters</td>
</tr>
<tr>
<td>CUSPARSE_SPM_M_COO_ALG4</td>
<td>Algorithm 4 for COO sparse matrix format</td>
</tr>
<tr>
<td></td>
<td>‣ Provide the best performance with row-major layout</td>
</tr>
<tr>
<td></td>
<td>‣ It supports batched computation</td>
</tr>
<tr>
<td></td>
<td>‣ May produce slightly different results during different runs with the same input parameters</td>
</tr>
<tr>
<td>CUSPARSE_CSR_MM_ALG1</td>
<td>Algorithm 1 for CSR sparse matrix format</td>
</tr>
<tr>
<td>CUSPARSE_SPM_M_CSR_ALG1</td>
<td>‣ It provides deterministic result</td>
</tr>
<tr>
<td></td>
<td>‣ It supports only column-major layout</td>
</tr>
<tr>
<td>CUSPARSE_SPM_M_CSR_ALG2</td>
<td>Algorithm 2 for CSR sparse matrix format</td>
</tr>
<tr>
<td></td>
<td>‣ Provide the best performance with row-major layout</td>
</tr>
<tr>
<td></td>
<td>‣ It supports batched computation</td>
</tr>
<tr>
<td></td>
<td>‣ May produce slightly different results during different runs with the same input parameters</td>
</tr>
</tbody>
</table>

**Performance notes:**

- Row-major layout provides higher performance than column-major.
- CUSPARSE_SPM_M_COO_ALG4 and CUSPARSE_SPM_M_CSR_ALG2 should be used with row-major layout, while CUSPARSE_SPM_M_COO_ALG1, CUSPARSE_SPM_M_COO_ALG3, and CUSPARSE_SPM_M_CSR_ALG1 with column-major layout.
- For beta != 1, the output matrix is scaled before the actual computation

`cusparseSpMM()` with CUSPARSE_SPM_M_COO_ALG4 and CUSPARSE_SPM_M_CSR_ALG2 support the following batch modes:

- \( C_i = A \cdot B_i \)
- \( C_i = A_i \cdot B \)
- \( C_i = A_i \cdot B_i \)

The number of batches and their strides can be set by using `cusparseCooSetStridedBatch`, `cusparseCsrSetStridedBatch`, and `cusparseDnMatSetStridedBatch`.

`cusparseSpMM()` has the following properties:

- The routine requires no extra storage for CUSPARSE_SPM_M_COO_ALG1, CUSPARSE_SPM_M_COO_ALG3, CUSPARSE_SPM_M_COO_ALG4, and CUSPARSE_SPM_M_CSR_ALG1
- The routine supports asynchronous execution
- Provides deterministic (bit-wise) results for each run only for CUSPARSE_SPM_M_COO_ALG2 and CUSPARSE_SPM_M_CSR_ALG1 algorithms, and `opA == CUSPARSE_OPERATION_NON_TRANSPOSE`

`cusparseSpMM()` supports the following optimizations:
14.6.8. cusparseConstrainedGeMM()

cusparseStatus_t cusparseConstrainedGeMM(cusparseHandle_t handle, cusparseOperation_t opA, cusparseOperation_t opB, const void* alpha, cusparseDnMatDescr_t matA, cusparseDnMatDescr_t matB, const void* beta, cusparseSpMatDescr_t matC, cudaDataType computeType, void* externalBuffer)
cusparseStatus_t cusparseConstrainedGeMM_bufferSize(cusparseHandle_t handle, cusparseOperation_t opA, cusparseOperation_t opB, const void* alpha, cusparseDnMatDescr_t matA, cusparseDnMatDescr_t matB, const void* beta, cusparseSpMatDescr_t matC, cudaDataType computeType, size_t* bufferSize)

This function performs the multiplication of matA and matB, followed by an element-wise multiplication with the sparsity pattern of matC. Formally, it performs the following operation:

\[ C = \alpha \text{op}(A) \cdot \text{op}(B) \odot \text{spy}(C) + \beta C \]

where op(A) is a dense matrix of size \(m \times k\), op(B) is a dense matrix of size \(k \times n\), \(C\) is a sparse matrix of size \(m \times n\), \(\alpha\) and \(\beta\) are scalars, \(\odot\) denotes the Hadamard (entry-wise) matrix product, and \(\text{spy}(C)\) is the sparsity pattern matrix of \(C\) defined as:

\[
\text{spy}(C)_{ij} = \begin{cases} 
0 & \text{if } C_{ij} = 0 \\
1 & \text{otherwise} 
\end{cases}
\]

Matrices op(A) and op(B) are defined as

\[
\text{op}(A) = \begin{cases} 
A & \text{if } \text{opA} = = \text{CUSPARSE_OPERATION_NON_TRANSPOSE} \\
A^T & \text{if } \text{opA} = = \text{CUSPARSE_OPERATION_TRANSPOSE} \\
A^H & \text{if } \text{opA} = = \text{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE}
\end{cases}
\]

See cusparseStatus_t for the description of the return status.

Please visit cuSPARSE Library Samples - cusparseSpMM CSR and cusparseSpMM COO for a code example.
The function has the following limitations:

- Only 32-bit indices `CUSPARSE_INDEX_32I` is supported.

The datatypes combinations currently supported for `cusparseSpMM` are listed below:

Uniform-precision computation:

<table>
<thead>
<tr>
<th>A/X/ Y/computeType</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
</tr>
<tr>
<td>CUDA_R_32F</td>
</tr>
<tr>
<td>CUDA_R_64F</td>
</tr>
<tr>
<td>CUDA_C_16F</td>
</tr>
<tr>
<td>CUDA_C_32F</td>
</tr>
<tr>
<td>CUDA_C_64F</td>
</tr>
</tbody>
</table>

Currently supported sparse matrix formats:

<table>
<thead>
<tr>
<th>Format</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_FORMAT_CSR</td>
<td>The column indices in each row must be sorted</td>
</tr>
</tbody>
</table>
cusparseConstrainedGeMM() has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution

cusparseConstrainedGeMM() supports the following optimizations:

- CUDA graph capture
- Hardware Memory Compression

See `cusparseStatus_t` for the description of the return status
14.6.9. `cusparseSpGEMM()`

```c
 cusparseStatus_t CUSPARSEAPI
cusparseSpGEMM_createDescr(cusparseSpGEMMDescr_t* descr);

cusparseStatus_t CUSPARSEAPI
cusparseSpGEMM_destroyDescr(cusparseSpGEMMDescr_t descr);

cusparseStatus_t CUSPARSEAPI
cusparseSpGEMM_workEstimation(cusparseHandle_t      handle,
                                      cusparseOperation_t   opA,
                                      cusparseOperation_t   opB,
                                      const void*           alpha,
                                      cusparseSpMatDescr_t  matA,
                                      cusparseSpMatDescr_t  matB,
                                      const void*           beta,
                                      cusparseSpMatDescr_t  matC,
                                      cudaDataType          computeType,
                                      cusparseSpGEMMAlg_t   alg,
                                      cusparseSpGEMMDescr_t spgemmDescr,
                                      size_t*               bufferSize1,
                                      void*                 externalBuffer1);

cusparseStatus_t CUSPARSEAPI
cusparseSpGEMM_compute(cusparseHandle_t      handle,
                                      cusparseOperation_t   opA,
                                      cusparseOperation_t   opB,
                                      const void*           alpha,
                                      cusparseSpMatDescr_t  matA,
                                      cusparseSpMatDescr_t  matB,
                                      const void*           beta,
                                      cusparseSpMatDescr_t  matC,
                                      cudaDataType          computeType,
                                      cusparseSpGEMMAlg_t   alg,
                                      cusparseSpGEMMDescr_t spgemmDescr,
                                      void*                 externalBuffer1,
                                      size_t*               bufferSize2,
                                      void*                 externalBuffer2);

cusparseStatus_t CUSPARSEAPI
cusparseSpGEMM_copy(cusparseHandle_t      handle,
                                      cusparseOperation_t   opA,
                                      cusparseOperation_t   opB,
                                      const void*           alpha,
                                      cusparseSpMatDescr_t  matA,
                                      cusparseSpMatDescr_t  matB,
                                      const void*           beta,
                                      cusparseSpMatDescr_t  matC,
                                      cudaDataType          computeType,
                                      cusparseSpGEMMAlg_t   alg,
                                      cusparseSpGEMMDescr_t spgemmDescr,
                                      void*                 externalBuffer2);
```

This function performs the multiplication of two sparse matrices `matA` and `matB` where

\[
C = \alpha op(A) \cdot op(B) + \beta C
\]

where \(\alpha\) and \(\beta\) are scalars.
The example CSR SpGEMM() shows the computation workflow for all steps.

The functions `cusparseSpGEMM_workEstimation()` and `cusparseSpGEMM_compute()` are used for both determining the buffer size and performing the actual computation.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Memory</th>
<th>In/out</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>handle</td>
<td>HOST</td>
<td>IN</td>
<td>Handle to the cuSPARSE library context</td>
</tr>
<tr>
<td>opA</td>
<td>HOST</td>
<td>IN</td>
<td>Operation op (A)</td>
</tr>
<tr>
<td>opB</td>
<td>HOST</td>
<td>IN</td>
<td>Operation op (B)</td>
</tr>
<tr>
<td>alpha</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>$\alpha$ scalar used for multiplication</td>
</tr>
<tr>
<td>matA</td>
<td>HOST</td>
<td>IN</td>
<td>Sparse matrix A</td>
</tr>
<tr>
<td>matB</td>
<td>HOST</td>
<td>IN</td>
<td>Dense matrix B</td>
</tr>
<tr>
<td>beta</td>
<td>HOST or DEVICE</td>
<td>IN</td>
<td>$\beta$ scalar used for multiplication</td>
</tr>
<tr>
<td>matC</td>
<td>HOST</td>
<td>IN/OUT</td>
<td>Dense matrix C</td>
</tr>
<tr>
<td>computeType</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the datatype in which the computation is executed</td>
</tr>
<tr>
<td>alg</td>
<td>HOST</td>
<td>IN</td>
<td>Enumerator specifying the algorithm for the computation</td>
</tr>
<tr>
<td>spgemmDescr</td>
<td>HOST</td>
<td>IN/OUT</td>
<td>Opaque descriptor for storing internal data used across the three steps</td>
</tr>
<tr>
<td>bufferSize1</td>
<td>HOST</td>
<td>IN/OUT</td>
<td>Number of bytes of workspace needed by <code>cusparseSpGEMM_workEstimation</code> or <code>cusparseSpGEMM_compute</code></td>
</tr>
<tr>
<td>externalBuffer1</td>
<td>DEVICE</td>
<td>IN</td>
<td>Pointer to workspace buffer</td>
</tr>
</tbody>
</table>

MEMORY REQUIREMENT: the first invocation of `cusparseSpGEMM_compute` provides an *upper bound* of the memory required for the computation that is generally several times larger of the actual memory used. The user can provide an arbitrary buffer size `bufferSize2` in the second invocation. If it is not sufficient, the routine will return `CUSPARSE_STATUS_INSUFFICIENT_RESOURCES` status.

Currently, the function has the following limitations:

- Only 32-bit indices `CUSPARSE_INDEX_32I` is supported
- Only CSR format `CUSPARSE_FORMAT_CSR` is supported
- Only $opA$, $opB$ equal to `CUSPARSE_OPERATION_NON_TRANSPOSE` are supported

The datatypes combinations currently supported for `cusparseSpGEMM` are listed below:

**Uniform-precision computation:**

<table>
<thead>
<tr>
<th>A/B/ C/computeType</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_16F</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_16BF</td>
<td></td>
</tr>
<tr>
<td>CUDA_R_32F</td>
<td></td>
</tr>
</tbody>
</table>
A/B/ C/computeType

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CUDA_R_64F</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_16F</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_16BF</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_32F</td>
<td></td>
</tr>
<tr>
<td>CUDA_C_64F</td>
<td></td>
</tr>
</tbody>
</table>

**cusparseSpGEMM** routine runs for the following algorithm:

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSPARSE_SPGEMM_DEFAULT</td>
<td>Default algorithm. Provides deterministic (bit-wise) results for each run</td>
</tr>
</tbody>
</table>

**cusparseSpGEMM()** has the following properties:

- The routine requires no extra storage
- The routine supports asynchronous execution
- The routine supports does not support CUDA graph capture

**cusparseSpGEMM()** supports the following optimizations:

- Hardware Memory Compression

See **cusparseStatus_t** for the description of the return status

Please visit cuSPARSE Library Samples - cusparseSpGEMM for a code example.
The cuSPARSE library is implemented using the C-based CUDA toolchain, and it thus provides a C-style API that makes interfacing to applications written in C or C++ trivial. There are also many applications implemented in Fortran that would benefit from using cuSPARSE, and therefore a cuSPARSE Fortran interface has been developed.

Unfortunately, Fortran-to-C calling conventions are not standardized and differ by platform and toolchain. In particular, differences may exist in the following areas:

Symbol names (capitalization, name decoration)
Argument passing (by value or reference)
Passing of pointer arguments (size of the pointer)

To provide maximum flexibility in addressing those differences, the cuSPARSE Fortran interface is provided in the form of wrapper functions, which are written in C and are located in the file `cusparse_fortran.c`. This file also contains a few additional wrapper functions (for `cudaMalloc()`, `cudaMemset`, and so on) that can be used to allocate memory on the GPU.

The cuSPARSE Fortran wrapper code is provided as an example only and needs to be compiled into an application for it to call the cuSPARSE API functions. Providing this source code allows users to make any changes necessary for a particular platform and toolchain.

The cuSPARSE Fortran wrapper code has been used to demonstrate interoperability with the compilers g95 0.91 (on 32-bit and 64-bit Linux) and g95 0.92 (on 32-bit and 64-bit Mac OS X). In order to use other compilers, users have to make any changes to the wrapper code that may be required.

The direct wrappers, intended for production code, substitute device pointers for vector and matrix arguments in all cuSPARSE functions. To use these interfaces, existing applications need to be modified slightly to allocate and deallocate data structures in GPU memory space (using `CUDA_MALLOC()` and `CUDA_FREE()`) and to copy data between GPU and CPU memory spaces (using the `CUDA_MEMCPY()` routines). The sample wrappers provided in `cusparse_fortran.c` map device pointers to the OS-
dependent type `size_t`, which is 32 bits wide on 32-bit platforms and 64 bits wide on a 64-bit platforms.

One approach to dealing with index arithmetic on device pointers in Fortran code is to use C-style macros and to use the C preprocessor to expand them. On Linux and Mac OS X, preprocessing can be done by using the option `-cpp` with g95 or gfortran. The function `GET_SHIFTED_ADDRESS()`, provided with the cuSPARSE Fortran wrappers, can also be used, as shown in example B.

Example B shows the the C++ of example A implemented in Fortran 77 on the host. This example should be compiled with `ARCH_64` defined as 1 on a 64-bit OS system and as undefined on a 32-bit OS system. For example, on g95 or gfortran, it can be done directly on the command line using the option `-cpp -DARCH_64=1`. 

15.1. Fortran Application

```fortran
program cusparse_fortran_example
implicit none
integer cudaMalloc
external cudaFree
integer cudaMemcpy_c2fort_int
integer cudaMemcpy_c2fort_real
integer cudaMemcpy_fort2c_int
integer cudaMemcpy_fort2c_real
integer cudaMemcpy
integer cusparseCreate
integer cusparseDestroy
integer cusparseCreateMatDescr
integer cusparseDestroyMatDescr
integer cusparseCreateMatType
integer cusparseDestroyMatType
integer cusparseGetMatFillMode
integer cusparseGetMatDiagType
integercusparseSetMatIndexBase
integer cusparseGetMatIndexBase
integer cusparseXcoo2csr
integer cusparseDscsr
integer cusparseDcsrmv
integer cusparseDcsrmm
external get_shifted_address
#if ARCH_64
integer*8 handle
integer*8 descrA
integer*8 cooRowIndex
integer*8 cooColIndex
integer*8 cooVal
integer*8 xInd
integer*8 xVal
integer*8 y
integer*8 z
integer*8 csrRowPtr
integer*8 ynp1
#else
integer*4 handle
integer*4 descrA
integer*4 cooRowIndex
integer*4 cooColIndex
integer*4 cooVal
integer*4 xInd
integer*4 xVal
integer*4 y
integer*4 z
integer*4 csrRowPtr
integer*4 ynp1
#endif
integer status
integer cudaStat1,cudaStat2,cudaStat3
cudaStat4,cudaStat5,cudaStat6
integer n, nnz, nnz_vector
parameter (n=4, nnz=9, nnz_vector=3)
integer cooRowIndexHostPtr(nnz)
integer cooColIndexHostPtr(nnz)
real*8 cooValHostPtr(nnz)
integer xIndHostPtr(nnz_vector)
real*8 xValHostPtr(nnz_vector)
real*8 yHostPtr(2*n)
real*8 zHostPtr(2*(n+1))
integer i, j
integer version, mtype, fmode, dtype, ibase
real*8 dzero, dtwo, dthree, dfive
real*8 epsilon
```

write(*,*) "testing fortran example"
16.1. COO Sort

This chapter provides a simple example in the C programming language of sorting of COO format.

A is a 3x3 sparse matrix,
int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    cudaError_t cudaStat2 = cudaSuccess;
    cudaError_t cudaStat3 = cudaSuccess;
    cudaError_t cudaStat4 = cudaSuccess;
    cudaError_t cudaStat5 = cudaSuccess;
    cudaError_t cudaStat6 = cudaSuccess;

    /* A is a 3x3 sparse matrix
    | 1 2 0 |
    | 0 5 0 |
    | 0 8 0 |
    */
    const int m = 3;
    const int n = 3;
    const int nnz = 4;

#if 0
    /* index starts at 0 */
    int h_cooRows[nnz] = {2, 1, 0, 0};
    int h_cooCols[nnz] = {1, 1, 0, 1};
#else
    /* index starts at -2 */
    int h_cooRows[nnz] = {0, -1, -2, -2};
    int h_cooCols[nnz] = {-1, -1, -2, -1};
#endif
    double h_cooVals[nnz] = {8.0, 5.0, 1.0, 2.0};
    int h_P[nnz];

    int *d_cooRows = NULL;
    int *d_cooCols = NULL;
    int *d_P = NULL;
    double *d_cooVals = NULL;
    double *d_cooVals_sorted = NULL;
    size_t pBufferSizeInBytes = 0;
    void *pBuffer = NULL;

    printf("m = %d, n = %d, nnz=%d \n", m, n, nnz);

    /* step 1: create cusparse handle, bind a stream */
    cudaStat1 = cudaStreamCreateWithFlags(&stream, cudaStreamNonBlocking);
    assert(cudaSuccess == cudaStat1);

    status = cusparseCreate(&handle);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    status = cusparseSetStream(handle, stream);
    assert(CUSPARSE_STATUS_SUCCESS == status);

    /* step 2: allocate buffer */
    cudaStat6 = cudaMalloc( &pBuffer, sizeof(char)* pBufferSizeInBytes);
    cudaStat5 = cudaMalloc( &d_cooVals_sorted, sizeof(double)*nnz);
    cudaStat4 = cudaMalloc( &d_cooVals, sizeof(double)*nnz);
    cudaStat3 = cudaMalloc( &d_P, sizeof(int)*nnz);
    cudaStat2 = cudaMalloc( &d_cooCols, sizeof(int)*nnz);
    cudaStat1 = cudaMalloc( &d_cooRows, sizeof(int)*nnz);

    printf("pBufferSizeInBytes = %lld bytes \n", (long long)pBufferSizeInBytes);

    /* index starts at -2 */
    /* A = | 0 5 0 |
    | 0 8 0 |
    | 1 2 0 |
    */
    int h_cooRows[nnz] = {0, -1, -2, -2};
    int h_cooCols[nnz] = {-1, -1, -2, -1};
    double h_cooVals[nnz] = {8.0, 5.0, 1.0, 2.0};
    int h_P[nnz];
17.1. Prune Dense to Sparse

This section provides a simple example in the C programming language of pruning a dense matrix to a sparse matrix of CSR format.

A is a 4x4 dense matrix,
Appendix C: Examples of prune

\[
A = \begin{bmatrix}
10 & 0.0 & 2.0 & -3.0 \\
0.0 & 4.0 & 0.0 & 0.0 \\
5.0 & 0.0 & 6.0 & 7.0 \\
0.0 & 8.0 & 0.0 & 9.0
\end{bmatrix}
\]

/* How to compile (assume cuda is installed at /usr/local/cuda/)
 * nvcc -c -I/usr/local/cuda/include prunedense_example.cpp
 * g++ -o prunedense_example.cpp prunedense_example.o -L/usr/local/cuda/lib64
 * -lcusparse -lcudart
 */
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <cusparse.h>

void printMatrix(int m, int n, const float*A, int lda, const char* name)
{
    for(int row = 0 ; row < m ; row++)
        for(int col = 0 ; col < n ; col++)
            float Areg = A[row + col*lda];
        printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
}

void printCsr(
    int m,
    int n,
    int nnz,
    const cusparseMatDescr_t descrA,
    const float *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    const char* name)
{
    const int base = (cusparseGetMatIndexBase(descrA) ? CUSPARSE_INDEX_BASE_ONE) ? 0:1 ;
    printf("matrix %s is %d-by-%d, nnz=%d, base=%d\n", name, m, n, nnz, base);
    for(int row = 0 ; row < m ; row++)
        for(int col = 0 ; col < n ; col++)
            float Areg = csrValA[col];
        printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
}

int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrC = NULL;

cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    cudaError_t cudaStat2 = cudaSuccess;
    cudaError_t cudaStat3 = cudaSuccess;
    cudaError_t cudaStat4 = cudaSuccess;
    cudaError_t cudaStat5 = cudaSuccess;
    const int m = 4;
    const int n = 4;
    const int lda = m;
*/
17.2. Prune Sparse to Sparse

This section provides a simple example in the C programming language of pruning a sparse matrix to a sparse matrix of CSR format.

A is a 4x4 sparse matrix,
Appendix C: Examples of prune

\[
A = \begin{bmatrix}
1.0 & 0.0 & 2.0 & -3.0 \\
0.0 & 4.0 & 0.0 & 0.0 \\
5.0 & 0.0 & 6.0 & 7.0 \\
0.0 & 8.0 & 0.0 & 9.0
\end{bmatrix}
\]

/*  How to compile (assume cuda is installed at /usr/local/cuda/)  */
* nvcc -c -I/usr/local/cuda/include prunecsr_example.cpp
* g++ -o prunecsr_example.cpp prunecsr_example.o -L/usr/local/cuda/lib64 -lcusparse -lcudart
*/
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <cuda_runtime.h>
#include <assert.h>
#include <cusparse.h>

void printCsr(
    int m,
    int n,
    int nnz,
    const cusparseMatDescr_t descrC,
    const float *csrValC,
    const int *csrRowPtrC,
    const int *csrColIndC,
    const char* name)
{
    const int base = (cusparseGetMatIndexBase(descrC) != CUSPARSE_INDEX_BASE_ONE)? 0:1 ;
    printf("matrix %s is %d-by-%d, nnz=%d, base=%d, output base=1\n", name, m, n, nnz, base);
    for(int row = 0 ; row < m ; row++){
        const int start = csrRowPtrC[row ] - base;
        const int end = csrRowPtrC[row+1] - base;
        for(int colidx = start ; colidx < end ; colidx++){
            const int col = csrColIndC[colidx] - base;
            const float Areg = csrValC[colidx];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrC = NULL;
    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    const int m = 4;
    const int n = 4;
    const int nnzA = 9;
    
    const int csrRowPtrA[m+1] = { 1, 4, 5, 8, 10};
    const int csrColIndA[nnzA] = { 1, 3, 4, 2, 1, 3, 4, 2, 4};
    const float csrValA[nnzA] = {1, 2, -3, 4, 5, 6, 7, 8, 9};
    int* csrRowPtrC = NULL;
    int* csrColIndC = NULL;
    float* csrValC = NULL;
    int nnzC = 0;
    float threshold = 4.1; /*remove Aij <= 4.1*/
    // call cusparseSpruneCsr2csr
    status = cusparseCreateMatDescr(&descrC);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    status = cusparseSetMatIndexBase(descrC,CUSPARSE_INDEX_BASE_ZERO);
    /* C is base-0 */
    status = cusparseCreateMatDescr(&descrA);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    status = cusparseSetMatType(descrA, CUSPARSE_MATRIX_TYPE_GENERAL );
    cusparseSetMatIndexBase(descrA,CUSPARSE_INDEX_BASE_ONE);
    /* A is base-1*/
    assert(CUSPARSE_STATUS_SUCCESS == status);
    status = cusparseCreate(&handle);
    assert(cudaSuccess == cudaStat1);
    printf("threshold = %E \n", threshold);
    printf("example of prune\n");
    /* step 3: query workspace */
    status = cusparseSpruneCsr2csr_bufferSizeExt(
        descrA,nnzA,m,nnz,n, &lworkInBytes);
    cudaStat1 = cudaMalloc((void**)&d_work, lworkInBytes);
    if (NULL != d_work) { cudaFree(d_work); }
    printf("lworkInBytes (prune) = %lld \n", (long long)lworkInBytes);
    cudaStat1 = cudaMemcpy(d_csrRowPtrC, csrRowPtrA, sizeof(int)*(m+1),
                       cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(d_csrValA , csrValA   , sizeof(float)*nnzA,
                        cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(d_csrColIndA, csrColIndA,
                      cudaMemcpyHostToDevice);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrRowPtrC, sizeof(int)*(m+1) );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrValA   , sizeof(float)*nnzA );
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMalloc ((void**)&d_csrColIndC, sizeof(int)*nnzA);
    assert(cudaSuccess == cudaStat1);
    /* step 4: set stream, handle, and other contexts */
    status = cusparseSetStream(handle, stream);
    assert(CUSPARSE_STATUS_SUCCESS == status);
    status = cusparseCreate(&handle);
    assert(cudaSuccess == cudaStat1);
    printf("nnzC = %d\n", nnzC);
    printf("C is empty \n");
    /* step 5: compute csrColIndC and csrValC */
    cudaStat1 = cudaMemcpy(csrValC   , d_csrValC   , sizeof(float)*nnzC ,
                       cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(csrColIndC, d_csrColIndC, sizeof(int  )*nnzC ,
                       cudaMemcpyDeviceToHost);
    assert( NULL != csrColIndC);
    /* step 6: output C */
    printf("printCsr(m, n, nnzC, descrC, csrValC, csrRowPtrC, csrColIndC, "C"");
    cudaStat1 = cudaDeviceSynchronize();
    printCsr(m, n, nnzC, descrC, csrValC, csrRowPtrC, csrColIndC, "C");
    cudaStat1 = cudaMemcpy(csrValC   , d_csrValC   , sizeof(float)*nnzC ,
                       cudaMemcpyDeviceToHost);
    assert(cudaSuccess == cudaStat1);
    cudaStat1 = cudaMemcpy(csrColIndC, d_csrColIndC, sizeof(int  )*nnzC ,
                       cudaMemcpyDeviceToHost);
    assert( NULL != csrColIndC);
    cudaStat1 = cudaDeviceSynchronize();
    /* step 7: free workspace */
    if (CSR_SCRPTC) free(CSR_SCRPTC);
    if (CSR_SCRPTK) free(CSR_SCRPTK);
    if (CSR_SCRPTV) free(CSR_SCRPTV);
    if (CSR_SCRPTC1) free(CSR_SCRPTC1);
    if (CSR_SCRPTK1) free(CSR_SCRPTK1);
    if (CSR_SCRPTV1) free(CSR_SCRPTV1);
    if (CSR_SCRPTB) free(CSR_SCRPTB);
    if (CSR_SCRPTB1) free(CSR_SCRPTB1);
    return 0;
}
17.3. Prune Dense to Sparse by Percentage

This section provides a simple example in the C programming language of pruning a dense matrix to a sparse matrix by percentage.

A is a 4x4 dense matrix,

\[
A = \begin{bmatrix}
1.0 & 0.0 & 2.0 & -3.0 \\
0.0 & 4.0 & 0.0 & 0.0 \\
5.0 & 0.0 & 6.0 & 7.0 \\
0.0 & 8.0 & 0.0 & 9.0
\end{bmatrix}
\]
The percentage is 50, which means to prune 50 percent of the dense matrix. The matrix has 16 elements, so 8 out of 16 must be pruned out. Therefore 7 zeros are pruned out, and value 1.0 is also out because it is the smallest among 9 nonzero elements.

```c
/* How to compile (assume cuda is installed at /usr/local/cuda/)
   nvcc -c -I/usr/local/cuda/include prunedense2csrbyP.cpp
   g++ -o prunedense2csrbyP.o prunedense2csrbyP.o -L/usr/local/cuda/lib64 -lcusparse -lcudart */
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <cuda_runtime.h>
#include <cusparse.h>

void printMatrix(int m, int n, const float*A, int lda, const char* name)
{
    for(int row = 0 ; row < m ; row++){
        for(int col = 0 ; col < n ; col++){
            float Areg = A[row + col*lda];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

void printCsr(
    int m,
    int n,
    int nnz,
    const cusparseMatDescr_t descrC,
    const float*csrcValC,
    const int*csrcColIndC,
    const char* name)
{
    const int base = (cusparseGetMatIndexBase(descrC) !=
                     CUSPARSE_INDEX_BASE_ONE)? 0:1 ;

    printf("matrix %s is %d-by-%d, nnz=%d, base=%d, output base-1\n", name, m,
           n, nnz, base);
    for(int row = 0 ; row < m ; row++){
        const int start = csrRowPtrA[row ] - base;
        const int end = csrRowPtrA[row+1] - base;
        for(int colidx = start ; colidx < end ; colidx++){
            const int col = csrColIndA[colidx] - base;
            const float Areg = csrcValA[colidx];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrC = NULL;
    pruneInfo_t info = NULL;

    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    cudaError_t cudaStat2 = cudaSuccess;
    cudaError_t cudaStat3 = cudaSuccess;
    cudaError_t cudaStat4 = cudaSuccess;
    cudaError_t cudaStat5 = cudaSuccess;
    const int m = 4;
    const int n = 4;
    const int lda = m;
```
17.4. Prune Sparse to Sparse by Percentage

This section provides a simple example in the C programming language of pruning a sparse matrix to a sparse matrix by percentage.

A is a 4x4 sparse matrix,

\[
A = \begin{pmatrix}
1.0 & 0.0 & 2.0 & -3.0 \\
0.0 & 4.0 & 0.0 & 0.0 \\
5.0 & 0.0 & 6.0 & 7.0 \\
0.0 & 8.0 & 0.0 & 9.0 \\
\end{pmatrix}
\]
The percentage is 20, which means to prune 20 percent of the nonzeros. The sparse matrix has 9 nonzero elements, so 1.4 elements must be pruned out. The function removes 1.0 and 2.0 which are the first two smallest numbers of nonzeros.

```c
/* How to compile (assume cuda is installed at /usr/local/cuda/)
 * nvcc -c -I/usr/local/cuda/include pruneCSR2CSRByP.cpp
 * g++ -o pruneCSR2CSRByP pruneCSR2CSRByP.o -L/usr/local/cuda/lib64 -lcudart
 */
#include <stdio.h>
#include <stdlib.h>
#include <cassert.h>
#include <cuda_runtime.h>
#include <cusparse.h>

void printCsr(
    int m, int n,
    int nnz,
    const cusparseMatDescr_t descrA,
    const float *csrValA,
    const int *csrRowPtrA,
    const int *csrColIndA,
    const char* name)
{
    const int base = (cusparseGetMatIndexBase(descrA) != CUSPARSE_INDEX_BASE_ONE)? 0:1;

    printf("matrix %s is %d-by-%d, nnz=%d, base=%d, output base-1\n", name, m, n, nnz, base);
    for(int row = 0 ; row < m ; row++){
        for(int colidx = csrRowPtrA[row ]; colidx < end; colidx++){
            const int col = csrColIndA[colidx] - base;
            const float Areg = csrValA[colidx];
            printf("%s(%d,%d) = %f\n", name, row+1, col+1, Areg);
        }
    }
}

int main(int argc, char*argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrA = NULL;
    cusparseMatDescr_t descrC = NULL;
    pruneInfo_t info = NULL;

    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    const int m = 4;
    const int n = 4;
    const int nnzA = 9;
    const int csrRowPtrA[m+1] = { 1, 4, 5, 8, 10};
    const int csrColIndA[nnzA] = { 1, 3, 4, 2, 1, 3, 4, 2, 4};
    const float csrValA[nnzA] = {1.0, -2.0, -3.0, -4.0, -5.0, -6.0, -7.0, -8.0, -9.0};
    int* csrRowPtrC = NULL;
    int* csrColIndC = NULL;
    float* csrValC = NULL;
    */
```
18.1. Batched Penta-diagonal Solver

This section provides a simple example in the C programming language of `gpsvInterleavedBatch`.

The example solves two penta-diagonal systems and assumes data layout is NOT interleaved format. Before calling `gpsvInterleavedBatch`, `cublasXgeam` is used to
transform the data layout, from aggregate format to interleaved format. If the user can prepare interleaved format, no need to transpose the data.

```c
void residaul_eval(
    int n,
    const float *ds,
    const float *dl,
    const float *d,
    const float *du,
    const float *dw,
    const float *b,
    const float *x,
    float *r_nrminf_ptr)
{
    float r_nrminf = 0;
    for(int i = 0 ; i < n ; i++){
        float dot = 0;
        if (i > 1) {
            dot += ds[i]*x[i-2];
        }
        if (i > 0) {
            dot += dl[i]*x[i-1];
        }
        dot += d[i]*x[i];
        if (i < (n-1)) {
            dot += du[i]*x[i+1];
        }
        if (i < (n-2)) {
            dot += dw[i]*x[i+2];
        }
        float ri = b[i] - dot;
        r_nrminf = (r_nrminf > fabs(ri)) ? r_nrminf : fabs(ri);
    }
    *r_nrminf_ptr = r_nrminf;
}
```

```c
int main(int argc, char*argv[])
{
    cusparseHandle_t cusparseH = NULL;
    cublasHandle_t cublasH = NULL;
    cudaStream_t stream = NULL;

    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cublasStatus_t cublasStat = CUBLAS_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;

    const int n = 4;
    const int batchSize = 2;

    /*
     *     |  1   8   13   0  |   |  1  |
     * A1 =|  5   2   9   14  | , b1 = |  2  |
     *     | 11   6   3   10  |   |  3  |
     * r_nrminf | -0.0592 |   | 0.3428 |
     * r_nrminf | -0.1295 |   | 0.1982 |
     */
```
19.1. Forward Triangular Solver

This section provides a simple example in the C programming language of csrsm2.
Appendix E: Examples of csrsm2

The example solves a lower triangular system with 2 right hand side vectors.

```c
/*
 How to compile (assume cuda is installed at /usr/local/cuda/)
 nvcc -c -I/usr/local/cuda/include csrsm2.cpp
 g++ -o csrsm2 csrsm2.o -L/usr/local/cuda/lib64 -lcusparse -lcudart *
*/
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>
#include <cuda_runtime.h>
#include <cusparse.h>

/* compute | b - A*x|_inf */
void residaul_eval(
    int n,
    const cusparseMatDescr_t descrA,
    const float *csrVal,
    const int *csrRowPtr,
    const int *csrColInd,
    const float *b,
    const float *x,
    float *r_nrminf_ptr
)
{
    constant int base = (cusparseGetMatIndexBase(descrA) !=
        CUSPARSE_INDEX_BASE_ONE)? 0:1;
    constant int lower = (CUSPARSE_FILL_MODE_LOWER ==
        cusparseGetMatFillMode(descrA))? 1:0;
    constant int unit = (CUSPARSE_DIAG_TYPE_UNIT ==
        cusparseGetMatDiagType(descrA))? 1:0;

    float r_nrminf = 0;
    for (int row = 0 ; row < n ; row++){
        constant int start = csrRowPtr[row] - base;
        constant int end = csrRowPtr[row+1] - base;
        float dot = 0;
        for(int colidx = start ; colidx < end ; colidx++)
        {
            constant int col = csrColInd[colidx] - base;
            float Aij = csrVal[colidx];
            float xj = x[col];
            if ( (row == col) && unit )
            {
                Aij = 1.0;
            }
            int valid = (row >= col) && lower ||
                        (row <= col) && !lower ;
            if ( valid )
            {
                dot += Aij*xj;
            }
        }  
        float ri = b[row] - dot;
        r_nrminf = (r_nrminf > fabs(ri))? r_nrminf : fabs(ri);
    }
    *r_nrminf_ptr = r_nrminf;
}

int main(int argc, char* argv[])
{
    cusparseHandle_t handle = NULL;
    cudaStream_t stream = NULL;
    cusparseMatDescr_t descrA = NULL;
    csrsm2Info_t info = NULL;

    cusparseStatus_t status = CUSPARSE_STATUS_SUCCESS;
    cudaError_t cudaStat1 = cudaSuccess;
    constant int nrhs = 2;
    constant int n = 4;
    constant int nnzA = 9;
    constant cusparseSolvePolicy_t policy = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
    constant float h_one = 1.0;
    * | 1 0 2 -3 |
    * | 0 4 0 0 |
```
Chapter 20.  
APPENDIX F: ACKNOWLEDGEMENTS

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